1 Find the gradient of the line segment joining each pair of points.

a (3, 1) and (5, 5) **b** (4, 7) and (10, 9) **c** (6, 1) and (2, 5) **d** (–2, 2) and (2, 8)

e (1, 3) and (7, –1) **f** (4, 5) and (–5, –7) **g** (–2, 0) and (0, –8) **h** (8, 6) and (–7, –2)

2 Write down the gradient and y-intercept of each line.

a $y = 4x - 1$ **b** $y = \frac{1}{3}x + 3$ **c** $y = 6 - x$ **d** $y = -2x - \frac{3}{5}$

3 Find the gradient and y-intercept of each line.

a $x + y + 3 = 0$ **b** $x - 2y - 6 = 0$ **c** $3x + 3y - 2 = 0$ **d** $4x - 5y + 1 = 0$

4 Write down, in the form $y - y_1 = m(x - x_1)$, the equation of the straight line with the given gradient which passes through the given point.

a gradient 2, point (4, 1) **b** gradient 5, point (2, –5)

c gradient –3, point (–1, 1) **d** gradient $\frac{1}{2}$, point (1, 6)

e gradient –2, point $(\frac{3}{4}, -\frac{1}{4})$ **f** gradient $-\frac{1}{5}$, point (–3, –7)

5 Find, in the form $y = mx + c$, the equation of the straight line with the given gradient which passes through the given point.

a gradient 3, point (1, 2) **b** gradient –1, point (5, 3)

c gradient 4, point (–2, –3) **d** gradient –2, point (–4, 1)

e gradient $\frac{1}{3}$, point (–3, 1) **f** gradient $-\frac{5}{6}$, point (9, –2)

6 Find, in each case, the equation of the straight line with gradient m which passes through the point P. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $m = 1$, P (2, –4) **b** $m = \frac{1}{2}$, P (6, 1) **c** $m = -4$, P (–1, 8)

d $m = \frac{2}{5}$, P (–3, 5) **e** $m = -3$, $P\ (\frac{3}{2}, -\frac{1}{8})$ **f** $m = -\frac{3}{4}$, $P\ (\frac{2}{3}, -7)$

7 Find, in the form $y = mx + c$, the equation of the straight line passing through each pair of points.

a (0, 1) and (4, 13) **b** (2, 9) and (7, –1) **c** (–4, 3) and (2, 7)

d $(-\frac{1}{2}, -2)$ and (2, 8) **e** (3, –2) and (18, –5) **f** (–3.2, 4) and (–2, 0.4)

8 Find, in the form $ax + by + c = 0$, where a, b and c are integers, the equation of the straight line which passes through each pair of points.

a (3, 0) and (5, 2) **b** (–1, 8) and (5, –4) **c** (–5, 3) and (7, 5)

d (–4, –1) and (8, –17) **e** (2, –1.5) and (7, 0) **f** $(-\frac{3}{5}, \frac{1}{10})$ and (3, 1)

9 The straight line l passes through the points A (–6, 8) and B (3, 2).

a Find an equation of the line l.

b Show that the point C (9, –2) lies on l.

10 The point M $(k, 2k)$ lies on the line with equation $x - 3y + 15 = 0$.

Find the value of the constant k.

11 The point with coordinates $(4p, p^2)$ lies on the line with equation $2x - 4y + 5 = 0$.

Find the two possible values of the constant p.

12 Find the coordinates of the points at which each straight line crosses the coordinate axes.

a $y = 2x + 5$ **b** $x - 3y + 6 = 0$ **c** $2x + 4y - 3 = 0$ **d** $5x - 3y = 10$

13 The line l has the equation $5x - 18y - 30 = 0$.

a Find the coordinates of the points A and B where the line l crosses the coordinate axes.

b Find the area of triangle OAB where O is the origin.

14 Find the exact length of the line segment joining each pair of points, giving your answers in terms of surds where appropriate.

a (1, 1) and (4, 5) **b** (0, 0) and (3, 1) **c** (1, –4) and (9, 11)

d (7, –8) and (–9, 4) **e** (3, 12) and (1, 7) **f** (–6, –3) and (2, –7)

15 The points P (22, 15), Q (–13, c) and R (k, 24) all lie on a circle, centre (2, 0).

Find the radius of the circle and the possible values of the constants c and k.

16 The points A (–2, 7) and B (6, –3) lie at either end of the diameter of a circle.

Find the area of the circle, giving your answer as an exact multiple of π.

17 The corners of a triangle are the points P (4, 7), Q (–2, 5) and R (3, –10).

a Find the length of each side of triangle PQR, giving your answers in terms of surds.

b Hence, verify that triangle PQR contains a right-angle.

c Find the area of triangle PQR.

18 Find the coordinates of the mid-point of the line segment joining each pair of points.

a (0, 2) and (8, 4) **b** (1, 9) and (7, 5) **c** (–5, 1) and (3, –7)

d (–5, –7) and (7, –5) **e** (1, 0) and (2, 9) **f** (–1, –2) and (4, –5)

g (2.4, 3.1) and (0.6, 4.5) **h** (0, 3) and $(\frac{1}{2}, \frac{3}{2})$ **i** $(-\frac{5}{4}, 2)$ and $(-1, -\frac{3}{5})$

19 The straight line l_1 passes through the points P (–2, 1) and Q (4, –1).

a Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The straight line l_2 passes through the point R (2, 4) and through the mid-point of PQ.

b Find the equation of l_2 in the form $y = mx + c$.

20 Find the coordinates of the point of intersection of each pair of straight lines.

a $y = 2x + 1$, $y = 3x - 1$ **b** $y = x + 7$, $y = 4 - 2x$ **c** $y = 5x - 4$, $y = 3x - 1$

d $x + 2y - 4 = 0$, $3x - 2y + 4 = 0$ **e** $2x + y - 2 = 0$, $x + 3y + 9 = 0$ **f** $3x + 2y = 0$, $x + 4y - 2 = 0$

21 The line l with equation $x - 2y + 2 = 0$ crosses the y-axis at the point P. The line m with equation $3x + y - 15 = 0$ crosses the y-axis at the point Q and intersects l at the point R.

Find the area of triangle PQR.

1 Find the gradient of a straight line that is

a parallel to the line $y = 3 - 2x$,
b parallel to the line $2x - 5y + 1 = 0$,
c perpendicular to the line $y = 3x + 4$,
d perpendicular to the line $x + 2y - 3 = 0$.

2 Find, in the form $y = mx + c$, the equation of the straight line

a parallel to the line $y = 4x - 1$ which passes through the point with coordinates (1, 7),
b perpendicular to the line $y = 6 - x$ which passes through the point with coordinates (–4, 3),
c perpendicular to the line $x - 3y = 0$ which passes through the point with coordinates (–2, –2).

3 Find, in the form $ax + by + c = 0$, where a, b and c are integers, the equation of the straight line

a parallel to the line $2x - 3y + 5 = 0$ which passes through the point with coordinates (3, –1),
b perpendicular to the line $3x + 4y = 1$ which passes through the point with coordinates (2, 5),
c parallel to the line $3x + 5y = 6$ which passes through the point with coordinates (–4, –7).

4 Find, in the form $ax + by + c = 0$, where a, b and c are integers, the equation of the perpendicular bisector of the line segment joining each pair of points.

a (0, 4) and (8, 0)
b (2, 7) and (4, 1)
c (–3, –2) and (9, 1)

5 The vertices of a triangle are the points A (–6, –3), B (4, –1) and C (3, 4).

a Find the gradient of AB and the gradient of BC.
b Show that $\angle ABC = 90°$.

6 The line with equation $2x - 3y + 5 = 0$ is perpendicular to the line with equation $3x + ky - 1 = 0$.

Find the value of the constant k.

7 The straight line l passes through the points A (–5, 5) and B (1, 7).

a Find an equation of the line l. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

The point M is the mid-point of AB.

b Prove that the line OM, where O is the origin, is perpendicular to line l.

8 The straight line p has the equation $3x - 4y + 8 = 0$.

The straight line q is parallel to p and passes through the point with coordinates (8, 5).

a Find the equation of q in the form $y = mx + c$.

The straight line r is perpendicular to p and passes through the point with coordinates (–4, 6).

b Find the equation of r in the form $ax + by + c = 0$, where a, b and c are integers.
c Find the coordinates of the point where lines q and r intersect.

9 The straight line l_1 passes through the points with coordinates (–3, 7) and (1, –5).

a Find an equation of the line l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The line l_2 is perpendicular to l_1 and passes through the point with coordinates (4, 6).

b Find, in the form $k\sqrt{5}$, the distance from the origin of the point where l_1 and l_2 intersect.

1 The straight line l has gradient -3 and passes through the point with coordinates $(3, -5)$.

a Find an equation of the line l.

The straight line m passes through the points with coordinates $(-1, -2)$ and $(4, 1)$.

b Find the equation of m in the form $ax + by + c = 0$, where a, b and c are integers.

The lines l and m intersect at the point P.

c Find the coordinates of P.

2 Given that the straight line passing through the points A $(2, -3)$ and B $(7, k)$ has gradient $\frac{3}{2}$,

a find the value of k,

b show that the perpendicular bisector of AB has the equation $8x + 12y - 45 = 0$.

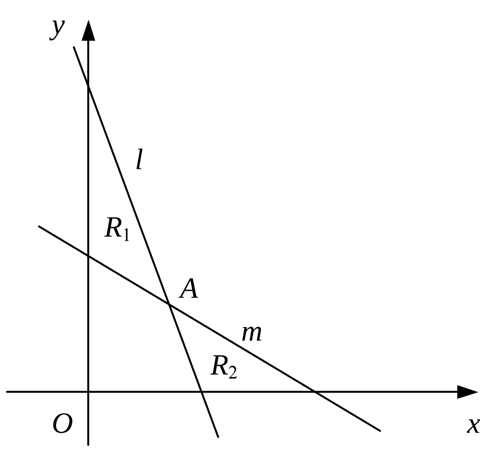
3 The vertices of a triangle are the points A $(5, 4)$, B $(-5, 8)$ and C $(1, 11)$.

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Find the coordinates of the point M, the mid-point of AC.

c Show that OM is perpendicular to AB, where O is the origin.

4



The line l with equation $3x + y - 9 = 0$ intersects the line m with equation $2x + 3y - 12 = 0$ at the point A as shown in the diagram above.

a Find, as exact fractions, the coordinates of the point A.

The region R_1 is bounded by l, m and the y-axis.

The region R_2 is bounded by l, m and the x-axis.

b Show that the ratio of the area of R_1 to the area of R_2 is $25 : 18$

5 The straight line l has the equation $2x + 5y + 10 = 0$.

The straight line m has the equation $6x - 5y - 30 = 0$.

a Sketch the lines l and m on the same set of axes showing the coordinates of any points at which each line crosses the coordinate axes.

The points where line m crosses the coordinate axes are denoted by A and B.

b Show that l passes through the mid-point of AB.

6 The straight line l passes through the points with coordinates $(-10, -4)$ and $(5, 4)$.

a Find the equation of l in the form $ax + by + c = 0$, where a, b and c are integers.

The line l crosses the coordinate axes at the points P and Q.

b Find, as an exact fraction, the area of triangle OPQ, where O is the origin.

c Show that the length of PQ is $2\frac{5}{6}$.

7 The point A has coordinates $(-8, 1)$ and the point B has coordinates $(-4, -5)$.

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Show that the distance of the mid-point of AB from the origin is $k\sqrt{10}$ where k is an integer to be found.

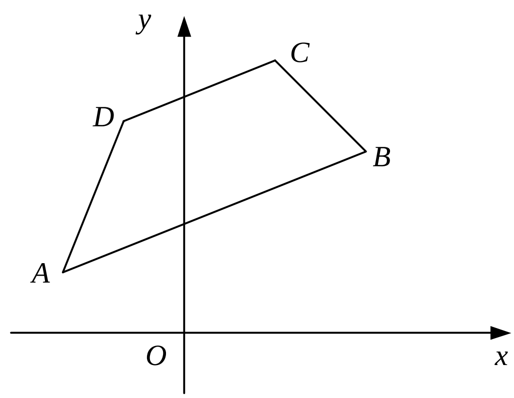
8 The straight line l_1 has gradient $\frac{1}{3}$ and passes through the point with coordinates $(-3, 4)$.

a Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The straight line l_2 has the equation $5x + py - 2 = 0$ and intersects l_1 at the point with coordinates $(q, 7)$.

b Find the values of the constants p and q.

9



The diagram shows trapezium $ABCD$ in which sides AB and DC are parallel. The point A has coordinates $(-4, 2)$ and the point B has coordinates $(6, 6)$.

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Given that the gradient of BC is -1,

b find an equation of the straight line passing through B and C.

Given also that the point D has coordinates $(-2, 7)$,

c find the coordinates of the point C,

d show that $\angle ACB = 90°$.

10 The straight line l passes through the points $A\,(1,\ 2\sqrt{3}\,)$ and $B\,(\sqrt{3}\,, 6)$.

a Find the gradient of l in its simplest form.

b Show that l also passes through the origin.

c Show that the straight line which passes through A and is perpendicular to l has equation

$$x + 2\sqrt{3}\,y - 13 = 0.$$

1 The straight line l has the equation $y = 1 - 2x$.

The straight line m is perpendicular to l and passes through the point with coordinates $(6, -1)$.

a Find the equation of m in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

b Find the coordinates of the point where l and m intersect. **(3)**

2 The straight line l passes through the point A $(1, -3)$ and the point B $(7, 5)$.

a Find an equation of line l. **(3)**

The line m has the equation $4x + y - 17 = 0$ and intersects l at the point C.

b Show that C is the mid-point of AB. **(4)**

c Show that the straight line perpendicular to m which passes through the point C also passes through the origin. **(4)**

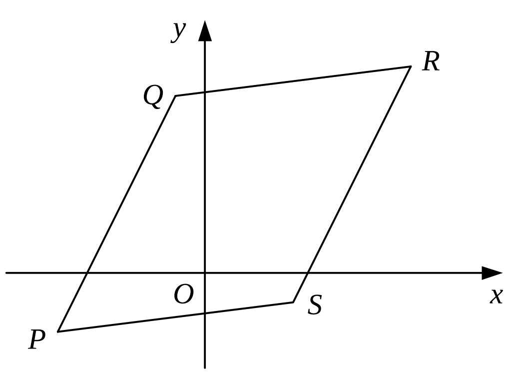
3 The point A has coordinates $(-2, 7)$ and the point B has coordinates $(4, p)$.

The point M is the mid-point of AB and has coordinates $(q, \frac{9}{2})$.

a Find the values of the constants p and q. **(3)**

b Find the equation of the straight line perpendicular to AB which passes through the point A. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

4



The points P $(-5, -2)$, Q $(-1, 6)$, R $(7, 7)$ and S $(3, -1)$ are the vertices of a parallelogram as shown in the diagram above.

a Find the length of PQ in the form $k\sqrt{5}$, where k is an integer to be found. **(3)**

b Find the coordinates of the point M, the mid-point of PQ. **(2)**

c Show that MS is perpendicular to PQ. **(4)**

d Find the area of parallelogram $PQRS$. **(4)**

5 The straight line l is parallel to the line $2x - y + 4 = 0$ and passes through the point with coordinates $(-1, -3)$.

a Find an equation of line l. **(3)**

The straight line m is perpendicular to the line $6x + 5y - 2 = 0$ and passes through the point with coordinates $(4, 4)$.

b Find the equation of line m in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

c Find, as exact fractions, the coordinates of the point where lines l and m intersect. **(3)**

6 The straight line l has gradient $\frac{1}{2}$ and passes through the point with coordinates (2, 4).

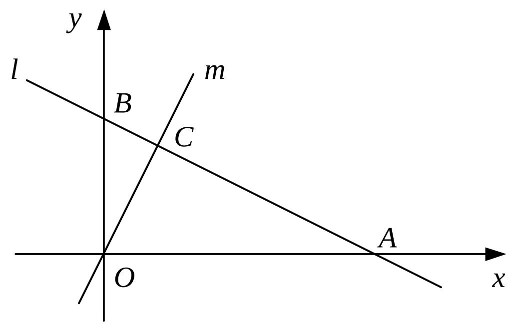
a Find the equation of l in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

The straight line m has the equation $y = 2x - 6$.

b Find the coordinates of the point where line m intersects line l. **(3)**

c Show that the quadrilateral enclosed by line l, line m and the positive coordinate axes is a kite. **(4)**

7



The diagram shows the straight line l with equation $x + 2y - 20 = 0$ and the straight line m which is perpendicular to l and passes through the origin O.

a Find the coordinates of the points A and B where l meets the x-axis and y-axis respectively. **(2)**

Given that l and m intersect at the point C,

b find the ratio of the area of triangle OAC to the area of triangle OBC. **(5)**

8 The straight line p has the equation $6x + 8y + 3 = 0$.

The straight line q is parallel to p and crosses the y-axis at the point with coordinates (0, 7).

a Find the equation of q in the form $y = mx + c$. **(2)**

The straight line r is perpendicular to p and crosses the x-axis at the point with coordinates (1, 0).

b Find the equation of r in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

c Show that the point where lines q and r intersect lies on the line $y = x$. **(4)**

9 The vertices of a triangle are the points P (3, c), Q (9, 2) and R (3c, 11) where c is a constant.

Given that $\angle PQR = 90°$,

a find the value of c, **(5)**

b show that the length of PQ is $k\sqrt{10}$, where k is an integer to be found, **(3)**

c find the area of triangle PQR. **(4)**

10 The straight line l_1 passes through the point P (1, 3) and the point Q (13, 12).

a Find the length of PQ. **(2)**

b Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

The straight line l_2 is perpendicular to l_1 and passes through the point R (2, 10).

c Find an equation of line l_2. **(3)**

d Find the coordinates of the point where lines l_1 and l_2 intersect. **(3)**

e Find the area of triangle PQR. **(3)**

1 **a** $= \frac{5-1}{5-3} = 2$ **b** $= \frac{9-7}{10-4} = \frac{1}{3}$ **c** $= \frac{5-1}{2-6} = -1$ **d** $= \frac{8-2}{2+2} = \frac{3}{2}$

e $= \frac{-1-3}{7-1} = -\frac{2}{3}$ **f** $= \frac{-7-5}{-5-4} = \frac{4}{3}$ **g** $= \frac{-8-0}{0+2} = -4$ **h** $= \frac{-2-6}{-7-8} = \frac{8}{15}$

2 **a** grad = 4, y-int = –1 **b** grad = $\frac{1}{3}$, y-int = 3 **c** grad = –1, y-int = 6 **d** grad = –2, y-int = $-\frac{3}{5}$

3 **a** $y = -x - 3$
grad = –1
y-int = –3

b $2y = x - 6$
$y = \frac{1}{2}x - 3$
grad = $\frac{1}{2}$
y-int = –3

c $3y = -3x + 2$
$y = -x + \frac{2}{3}$
grad = –1
y-int = $\frac{2}{3}$

d $5y = 4x + 1$
$y = \frac{4}{5}x + \frac{1}{5}$
grad = $\frac{4}{5}$
y-int = $\frac{1}{5}$

4 **a** $y - 1 = 2(x - 4)$ **b** $y + 5 = 5(x - 2)$

c $y - 1 = -3(x + 1)$ **d** $y - 6 = \frac{1}{2}(x - 1)$

e $y + \frac{1}{4} = -2(x - \frac{3}{4})$ **f** $y + 7 = -\frac{1}{5}(x + 3)$

5 **a** $y - 2 = 3(x - 1)$
$y = 3x - 1$

b $y - 3 = -(x - 5)$
$y = -x + 8$

c $y + 3 = 4(x + 2)$
$y = 4x + 5$

d $y - 1 = -2(x + 4)$
$y = -2x - 7$

e $y - 1 = \frac{1}{3}(x + 3)$
$y = \frac{1}{3}x + 2$

f $y + 2 = -\frac{5}{6}(x - 9)$
$y = -\frac{5}{6}x + \frac{11}{2}$

6 **a** $y + 4 = x - 2$
$x - y - 6 = 0$

b $y - 1 = \frac{1}{2}(x - 6)$
$2y - 2 = x - 6$
$x - 2y - 4 = 0$

c $y - 8 = -4(x + 1)$
$y - 8 = -4x - 4$
$4x + y - 4 = 0$

d $y - 5 = \frac{2}{5}(x + 3)$
$5y - 25 = 2x + 6$
$2x - 5y + 31 = 0$

e $y + \frac{1}{8} = -3(x - \frac{3}{2})$
$8y + 1 = -24x + 36$
$24x + 8y - 35 = 0$

f $y + 7 = -\frac{3}{4}(x - \frac{2}{3})$
$4y + 28 = -3x + 2$
$3x + 4y + 26 = 0$

7 **a** grad $= \frac{13-1}{4-0} = 3$
$y = 3x + 1$

b grad $= \frac{-1-9}{7-2} = -2$
$y - 9 = -2(x - 2)$
$y = -2x + 13$

c grad $= \frac{7-3}{2+4} = \frac{2}{3}$
$y - 3 = \frac{2}{3}(x + 4)$
$y = \frac{2}{3}x + \frac{17}{3}$

d grad $= \frac{8+2}{2+\frac{1}{2}} = 4$
$y - 8 = 4(x - 2)$
$y = 4x$

e grad $= \frac{-5+2}{18-3} = -\frac{1}{5}$
$y + 2 = -\frac{1}{5}(x - 3)$
$y = -\frac{1}{5}x - \frac{7}{5}$

f grad $= \frac{0.4-4}{-2+3.2} = -3$
$y - 4 = -3(x + 3.2)$
$y = -3x - 5.6$

8 **a** grad = $\frac{2-0}{5-3} = 1$

$y = x - 3$

$x - y - 3 = 0$

b grad = $\frac{-4-8}{5+1} = -2$

$y - 8 = -2(x + 1)$

$y - 8 = -2x - 2$

$2x + y - 6 = 0$

c grad = $\frac{5-3}{7+5} = \frac{1}{6}$

$y - 3 = \frac{1}{6}(x + 5)$

$6y - 18 = x + 5$

$x - 6y + 23 = 0$

d grad = $\frac{-17+1}{8+4} = -\frac{4}{3}$

$y + 1 = -\frac{4}{3}(x + 4)$

$3y + 3 = -4x - 16$

$4x + 3y + 19 = 0$

e grad = $\frac{0+1.5}{7-2} = 0.3$

$y = 0.3(x - 7)$

$10y = 3x - 21$

$3x - 10y - 21 = 0$

f grad = $\frac{1-\frac{1}{10}}{3+\frac{3}{5}} = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x - 3)$

$4y - 4 = x - 3$

$x - 4y + 1 = 0$

9 **a** grad = $\frac{2-8}{3+6} = -\frac{2}{3}$

$\therefore\ y - 8 = -\frac{2}{3}(x + 6)$

[$2x + 3y - 12 = 0$]

b sub.

$2(9) + 3(-2) - 12 = 18 - 6 - 12 = 0$

$\therefore$ C lies on l

10 $k - 3(2k) + 15 = 0$

$15 = 5k$

$k = 3$

11 $2(4p) - 4(p^2) + 5 = 0$

$4p^2 - 8p - 5 = 0$

$(2p + 1)(2p - 5) = 0$

$p = -\frac{1}{2}$ or $\frac{5}{2}$

12 **a** $x = 0$: $y = 5$

$y = 0$: $x = -\frac{5}{2}$

$(-\frac{5}{2}, 0)$ and $(0, 5)$

b $x = 0$: $y = 2$

$y = 0$: $x = -6$

$(-6, 0)$ and $(0, 2)$

c $x = 0$: $y = \frac{3}{4}$

$y = 0$: $x = \frac{3}{2}$

$(0, \frac{3}{4})$ and $(\frac{3}{2}, 0)$

d $x = 0$: $y = -\frac{10}{3}$

$y = 0$: $x = 2$

$(0, -\frac{10}{3})$ and $(2, 0)$

13 **a** $x = 0 \Rightarrow y = -\frac{5}{3}$

$y = 0 \Rightarrow x = 6$ $\therefore (0, -\frac{5}{3})$ and $(6, 0)$

b area = $\frac{1}{2} \times 6 \times \frac{5}{3} = 5$

14 **a** $= \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5$

b $= \sqrt{3^2 + 1^2}$

$= \sqrt{10}$

c $= \sqrt{8^2 + 15^2}$

$= \sqrt{289} = 17$

d $= \sqrt{16^2 + 12^2}$

$= \sqrt{400} = 20$

e $= \sqrt{2^2 + 5^2}$

$= \sqrt{29}$

f $= \sqrt{8^2 + 4^2}$

$= \sqrt{80} = 4\sqrt{5}$

15 let centre be C $\therefore$ radius = $CP = \sqrt{20^2 + 15^2} = \sqrt{625} = 25$

$\therefore\ CQ^2 = 15^2 + c^2 = 25^2$

$c^2 = 625 - 225 = 400$

$c = \pm 20$

$CR^2 = (k - 2)^2 + 24^2 = 25^2$

$(k - 2)^2 = 625 - 576 = 49$

$k - 2 = \pm 7$

$k = -5$ or 9

16 $AB^2 = 8^2 + 10^2 = 164$

$AB = \sqrt{164} = 2\sqrt{41}$

radius $= \frac{1}{2}AB = \sqrt{41}$

area $= \pi \times (\sqrt{41})^2 = 41\pi$

17 **a** $PQ = \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10}$

$PR = \sqrt{1^2 + 17^2} = \sqrt{290}$

$QR = \sqrt{5^2 + 15^2} = \sqrt{250} = 5\sqrt{10}$

b $PQ^2 + QR^2 = 40 + 250 = 290 = PR^2$

$\therefore$ by converse of Pythagoras'

$\angle PQR$ is a right-angle

c area $= \frac{1}{2} \times PQ \times QR = 50$

18 **a** $(\frac{0+8}{2}, \frac{2+4}{2}) = (4, 3)$ **b** $(\frac{1+7}{2}, \frac{9+5}{2}) = (4, 7)$ **c** $(\frac{-5+3}{2}, \frac{1-7}{2}) = (-1, -3)$

d $(\frac{-5+7}{2}, \frac{-7-5}{2}) = (1, -6)$ **e** $(\frac{1+2}{2}, \frac{0+9}{2}) = (\frac{3}{2}, \frac{9}{2})$ **f** $(\frac{-1+4}{2}, \frac{-2-5}{2}) = (\frac{3}{2}, -\frac{7}{2})$

g $(\frac{2.4+0.6}{2}, \frac{3.1+4.5}{2}) = (1.5, 3.8)$ **h** $(\frac{0+\frac{1}{2}}{2}, \frac{3+\frac{3}{2}}{2}) = (\frac{1}{4}, \frac{9}{4})$ **i** $(\frac{-\frac{5}{4}-1}{2}, \frac{2-\frac{3}{5}}{2}) = (-\frac{9}{8}, \frac{7}{10})$

19 **a** grad $= \frac{-1-1}{4+2} = -\frac{1}{3}$

$y - 1 = -\frac{1}{3}(x + 2)$

$3y - 3 = -x - 2$

$x + 3y - 1 = 0$

b mid-point of $PQ = (\frac{-2+4}{2}, \frac{1-1}{2}) = (1, 0)$

grad of $l_2 = \frac{0-4}{1-2} = 4$

$y = 4(x - 1)$

$y = 4x - 4$

20 **a** $2x + 1 = 3x - 1$
$x = 2$
$\therefore$ (2, 5)

b $x + 7 = 4 - 2x$
$3x = -3$
$x = -1$
$\therefore$ (−1, 6)

c $5x - 4 = 3x - 1$
$2x = 3$
$x = \frac{3}{2}$
$\therefore$ $(\frac{3}{2}, \frac{7}{2})$

d adding
$4x = 0$
$x = 0$
$\therefore$ (0, 2)

e $6x + 3y - 6 = 0$
$x + 3y + 9 = 0$
subtracting
$5x - 15 = 0$
$x = 3$
$\therefore$ (3, −4)

f $6x + 4y = 0$
$x + 4y - 2 = 0$
subtracting
$5x + 2 = 0$
$x = -\frac{2}{5}$
$\therefore$ $(-\frac{2}{5}, \frac{3}{5})$

21 l: $x = 0 \Rightarrow y = 1 \quad \therefore P(0, 1)$

m: $x = 0 \Rightarrow y = 15 \quad \therefore Q(0, 15)$

$l \qquad x - 2y + 2 = 0$

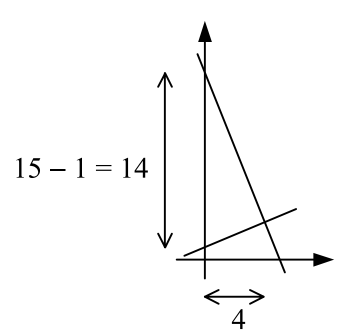
$m \Rightarrow \quad 6x + 2y - 30 = 0$

adding, $\quad 7x - 28 = 0$

$x = 4$

sub. $\quad y = 3 \quad \therefore R(4, 3)$

area $= \frac{1}{2} \times 14 \times 4 = 28$



1 **a** grad of $y = 3 - 2x$ is -2

parallel grad $= -2$

b $2x - 5y + 1 = 0 \Rightarrow y = \frac{2}{5}x + \frac{1}{5}$

grad of $y = \frac{2}{5}x + \frac{1}{5}$ is $\frac{2}{5}$

parallel grad $= \frac{2}{5}$

c grad of $y = 3x + 4$ is 3

perp grad $= \frac{-1}{3} = -\frac{1}{3}$

d $x + 2y - 3 = 0 \Rightarrow y = \frac{3}{2} - \frac{1}{2}x$

grad of $y = \frac{3}{2} - \frac{1}{2}x$ is $-\frac{1}{2}$

perp grad $= \frac{-1}{-\frac{1}{2}} = 2$

2 **a** grad of $y = 4x - 1$ is 4

parallel grad = 4

$\therefore\ y - 7 = 4(x - 1)$

$y = 4x + 3$

b grad of $y = 6 - x$ is -1

perp grad = 1

$\therefore\ y - 3 = x + 4$

$y = x + 7$

c grad of $x - 3y = 0$ is $\frac{1}{3}$

perp grad $= -3$

$\therefore\ y + 2 = -3(x + 2)$

$y = -3x - 8$

3 **a** grad of $2x - 3y + 5 = 0$ is $\frac{2}{3}$

parallel grad $= \frac{2}{3}$

$\therefore\ y + 1 = \frac{2}{3}(x - 3)$

$3y + 3 = 2x - 6$

$2x - 3y - 9 = 0$

b grad of $3x + 4y = 1$ is $-\frac{3}{4}$

perp grad $= \frac{4}{3}$

$\therefore\ y - 5 = \frac{4}{3}(x - 2)$

$3y - 15 = 4x - 8$

$4x - 3y + 7 = 0$

c grad of $3x + 5y = 6$ is $-\frac{3}{5}$

parallel grad $= -\frac{3}{5}$

$\therefore\ y + 7 = -\frac{3}{5}(x + 4)$

$5y + 35 = -3x - 12$

$3x + 5y + 47 = 0$

4 **a** mid-point $= (\frac{0+8}{2}, \frac{4+0}{2})$

$= (4, 2)$

grad $= \frac{0-4}{8-0} = -\frac{1}{2}$

perp grad = 2

$\therefore\ y - 2 = 2(x - 4)$

$y - 2 = 2x - 8$

$2x - y - 6 = 0$

b mid-point $= (\frac{2+4}{2}, \frac{7+1}{2})$

$= (3, 4)$

grad $= \frac{1-7}{4-2} = -3$

perp grad $= \frac{1}{3}$

$\therefore\ y - 4 = \frac{1}{3}(x - 3)$

$3y - 12 = x - 3$

$x - 3y + 9 = 0$

c mid-point $= (\frac{-3+9}{2}, \frac{-2+1}{2})$

$= (3, -\frac{1}{2})$

grad $= \frac{1+2}{9+3} = \frac{1}{4}$

perp grad $= -4$

$\therefore\ y + \frac{1}{2} = -4(x - 3)$

$2y + 1 = -8x + 24$

$8x + 2y - 23 = 0$

5 **a** grad $AB = \frac{-1+3}{4+6} = \frac{1}{5}$

grad $BC = \frac{4+1}{3-4} = -5$

b grad $AB \times$ grad $BC = \frac{1}{5} \times -5 = -1$

$\therefore$ AB is perpendicular to BC

$\therefore\ \angle ABC = 90°$

6 $2x - 3y + 5 = 0 \Rightarrow y = \frac{2}{3}x + \frac{5}{3}\quad \therefore$ grad $= \frac{2}{3}$

$3x + ky - 1 = 0 \Rightarrow y = -\frac{3}{k}x + \frac{1}{k}\quad \therefore$ grad $= -\frac{3}{k}$

perp $\therefore\ \frac{2}{3} \times -\frac{3}{k} = -1$

$k = 2$

7 **a** grad = $\frac{7-5}{1+5}$ = $\frac{1}{3}$

$\therefore\ y - 5 = \frac{1}{3}(x + 5)$

$3y - 15 = x + 5$

$x - 3y + 20 = 0$

b $M = (\frac{-5+1}{2}, \frac{5+7}{2}) = (-2, 6)$

grad $OM = \frac{6-0}{-2-0} = -3$

grad $l \times$ grad $OM = \frac{1}{3} \times (-3) = -1$

$\therefore$ OM is perpendicular to l

8 **a** $p \Rightarrow y = \frac{3}{4}x + 2 \ \therefore$ grad = $\frac{3}{4}$

parallel grad = $\frac{3}{4}$

$\therefore\ y - 5 = \frac{3}{4}(x - 8)$

$y = \frac{3}{4}x - 1$

b perp grad = $-\frac{4}{3}$

$\therefore\ y - 6 = -\frac{4}{3}(x + 4)$

$3y - 18 = -4x - 16$

$4x + 3y - 2 = 0$

c $q \Rightarrow 3x - 4y - 4 = 0$

$\Rightarrow 9x - 12y - 12 = 0$

$r \Rightarrow 16x + 12y - 8 = 0$

adding, $25x - 20 = 0$

$x = \frac{4}{5}$

$\therefore\ (\frac{4}{5}, -\frac{2}{5})$

9 **a** grad = $\frac{-5-7}{1+3} = -3$

$\therefore\ y - 7 = -3(x + 3)$

$3x + y + 2 = 0$

b perp grad = $\frac{1}{3}$

$\therefore\ l_2: \quad y - 6 = \frac{1}{3}(x - 4)$

$3y - 18 = x - 4$

$x - 3y + 14 = 0$

$l_1 \Rightarrow 9x + 3y + 6 = 0$

adding, $10x + 20 = 0$

$x = -2$

$\therefore$ pt of intersection $(-2, 4)$

$\therefore$ dist from origin = $\sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

1 **a** $y + 5 = -3(x - 3)$ $[y = 4 - 3x]$

b grad $= \frac{1+2}{4+1} = \frac{3}{5}$

$\therefore\ y + 2 = \frac{3}{5}(x + 1)$

$5y + 10 = 3x + 3$

$3x - 5y - 7 = 0$

c $3x - 5(4 - 3x) - 7 = 0$

$18x - 27 = 0$

$x = \frac{3}{2}$

$\therefore\ P\ (\frac{3}{2}, -\frac{1}{2})$

2 **a** $\frac{k+3}{7-2} = \frac{3}{2}$

$2(k + 3) = 15$

$k = \frac{9}{2}$

b mid-point $= (\frac{2+7}{2}, \frac{-3+\frac{9}{2}}{2}) = (\frac{9}{2}, \frac{3}{4})$

perp grad $= -\frac{2}{3}$

$\therefore\ y - \frac{3}{4} = -\frac{2}{3}(x - \frac{9}{2})$

$12y - 9 = -8x + 36$

$8x + 12y - 45 = 0$

3 **a** grad $= \frac{8-4}{-5-5} = -\frac{2}{5}$

$\therefore\ y - 4 = -\frac{2}{5}(x - 5)$

$5y - 20 = -2x + 10$

$2x + 5y - 30 = 0$

b $M = (\frac{5+1}{2}, \frac{4+11}{2}) = (3, 7\frac{1}{2})$

c grad $OM = 7\frac{1}{2} \div 3 = \frac{5}{2}$

grad $OM \times$ grad $AB = \frac{5}{2} \times -\frac{2}{5} = -1$

$\therefore$ OM is perpendicular to AB

4 **a** $l \Rightarrow$ $9x + 3y - 27 = 0$

subtracting, $7x - 15 = 0$

$x = \frac{15}{7}$

$\therefore\ A\ (\frac{15}{7}, \frac{18}{7})$

b l meeets y-axis: $x = 0 \Rightarrow y = 9$

m meeets y-axis: $x = 0 \Rightarrow y = 4$

area of $R_1 = \frac{1}{2} \times 5 \times \frac{15}{7} = \frac{75}{14}$

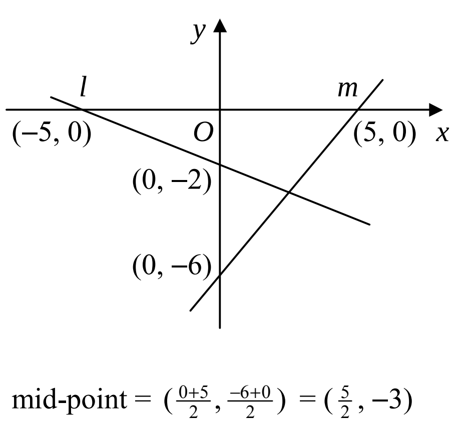
l meeets x-axis: $y = 0 \Rightarrow x = 3$

m meeets x-axis: $y = 0 \Rightarrow x = 6$

area of $R_2 = \frac{1}{2} \times 3 \times \frac{18}{7} = \frac{54}{14}$

area R_1 : area of $R_2 = \frac{75}{14} : \frac{54}{14} = 25 : 18$

5 **a**



b mid-point $= (\frac{0+5}{2}, \frac{-6+0}{2}) = (\frac{5}{2}, -3)$

sub. in l: $2(\frac{5}{2}) + 5(-3) + 10$

$= 5 - 15 + 10 = 0$

$\therefore$ l passes through mid-point of AB

6 **a** grad $= \frac{4+4}{5+10} = \frac{8}{15}$

$\therefore\ y - 4 = \frac{8}{15}(x - 5)$

$15y - 60 = 8x - 40$

$8x - 15y + 20 = 0$

b $x = 0 \Rightarrow y = \frac{4}{3}$

$y = 0 \Rightarrow x = -\frac{5}{2}$

area $= \frac{1}{2} \times \frac{5}{2} \times \frac{4}{3} = \frac{5}{3}$

c $PQ^2 = (\frac{5}{2})^2 + (\frac{4}{3})^2$

$= \frac{25}{4} + \frac{16}{9}$

$= \frac{289}{36}$

$PQ = \sqrt{\frac{289}{36}} = \frac{17}{6} = 2\frac{5}{6}$

7 **a** grad = $\frac{-5-1}{-4+8} = -\frac{3}{2}$

$\therefore\ y - 1 = -\frac{3}{2}(x + 8)$

$2y - 2 = -3x - 24$

$3x + 2y + 22 = 0$

b mid-point = $(\frac{-8-4}{2}, \frac{1-5}{2}) = (-6, -2)$

distance = $\sqrt{6^2 + 2^2} = \sqrt{40}$

$= 2\sqrt{10}$ $\quad [k = 2]$

8 **a** $y - 4 = \frac{1}{3}(x + 3)$

$3y - 12 = x + 3$

$x - 3y + 15 = 0$

b $(q, 7) \Rightarrow q - (3 \times 7) + 15 = 0$

$\therefore\ q = 6$

$(6, 7) \Rightarrow (5 \times 6) + 7p - 2 = 0$

$\therefore\ p = -4$

9 **a** grad = $\frac{6-2}{6+4} = \frac{2}{5}$

$\therefore\ y - 2 = \frac{2}{5}(x + 4)$

$5y - 10 = 2x + 8$

$2x - 5y + 18 = 0$

b $y - 6 = -(x - 6)$ $\quad [y = 12 - x]$

c grad DC = grad AB = $\frac{2}{5}$

$\therefore$ eqn DC is $\quad y - 7 = \frac{2}{5}(x + 2)$

$y = \frac{2}{5}x + 7\frac{4}{5}$

at C: $\quad 12 - x = \frac{2}{5}x + 7\frac{4}{5}$

$60 - 5x = 2x + 39$

$x = 3$

$\therefore\ C\ (3, 9)$

d grad $AC = \frac{9-2}{3+4} = 1$

grad $AC \times$ grad $BC = 1 \times -1 = -1$

$\therefore\ AC$ is perpendicular to BC

$\therefore\ \angle ACB = 90°$

10 **a** grad = $\frac{6-2\sqrt{3}}{\sqrt{3}-1} = \frac{6-2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{6\sqrt{3}+6-6-2\sqrt{3}}{3-1} = \frac{4\sqrt{3}}{2}$

$= 2\sqrt{3}$

b l : $y - 2\sqrt{3} = 2\sqrt{3}(x - 1)$

$y = 2\sqrt{3}\,x$

when $x = 0,\ y = 0$

$\therefore$ passes through origin

c perp grad = $-\frac{1}{2\sqrt{3}}$

$\therefore\ y - 2\sqrt{3} = -\frac{1}{2\sqrt{3}}(x - 1)$

$2\sqrt{3}\,y - 12 = -x + 1$

$x + 2\sqrt{3}\,y - 13 = 0$

1 **a** grad $l = -2$

$\therefore$ grad $m = \frac{1}{2}$

$y + 1 = \frac{1}{2}(x - 6)$

$2y + 2 = x - 6$

$x - 2y - 8 = 0$

b $x - 2(1 - 2x) - 8 = 0$

$5x - 10 = 0$

$x = 2 \;\; \therefore (2, -3)$

2 **a** grad $= \frac{5+3}{7-1} = \frac{4}{3}$

$\therefore \; y + 3 = \frac{4}{3}(x - 1) \quad [4x - 3y - 13 = 0]$

b subtracting, $4y - 4 = 0$

$y = 1 \;\; \therefore C(4, 1)$

mid-point $= (\frac{1+7}{2}, \frac{-3+5}{2}) = (4, 1)$

$\therefore$ C is the mid-point of AB

c grad $m = -4$

$\therefore$ grad perp to $m = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x - 4)$

$\therefore \; y = \frac{1}{4}x$ which passes through (0, 0)

3 **a** $M = (q, \frac{9}{2}) = (\frac{-2+4}{2}, \frac{7+p}{2})$

$\therefore \; p = 2, \; q = 1$

b grad $AB = \frac{2-7}{4+2} = -\frac{5}{6}$

$\therefore$ grad perp to $AB = \frac{6}{5}$

$y - 7 = \frac{6}{5}(x + 2)$

$5y - 35 = 6x + 12$

$6x - 5y + 47 = 0$

4 **a** $PQ^2 = 4^2 + 8^2 = 80$

$PQ = \sqrt{80} = 4\sqrt{5} \quad [k = 4]$

b $M = (\frac{-5-1}{2}, \frac{-2+6}{2}) = (-3, 2)$

c grad $MS = \frac{-1-2}{3+3} = -\frac{1}{2}$

grad $PQ = \frac{6+2}{-1+5} = 2$

grad $MS \times$ grad $PQ = -\frac{1}{2} \times 2 = -1$

$\therefore$ MS is perpendicular to PQ

d $MS = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$

area $= PQ \times MS = 60$

5 **a** grad of $2x - y + 4 = 0$ is 2

$\therefore$ grad of $l = 2$

$y + 3 = 2(x + 1) \quad [y = 2x - 1]$

b grad of $6x + 5y - 2 = 0$ is $-\frac{6}{5}$

$\therefore$ grad of $m = \frac{5}{6}$

$y - 4 = \frac{5}{6}(x - 4)$

$6y - 24 = 5x - 20$

$5x - 6y + 4 = 0$

c $5x - 6(2x - 1) + 4 = 0$

$10 - 7x = 0$

$x = \frac{10}{7} \quad \therefore (1\frac{3}{7}, 1\frac{6}{7})$

6 **a** $y - 4 = \frac{1}{2}(x - 2)$

$2y - 8 = x - 2$

$x - 2y + 6 = 0$

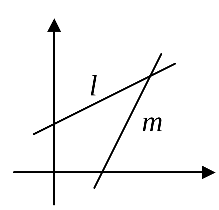
b $x - 2(2x - 6) + 6 = 0$

$18 - 3x = 0$

$x = 6 \;\; \therefore (6, 6)$

c l meets y-axis at (0, 3)

m meets x-axis at (3, 0)



(0, 0) and (6, 6) on $y = x$

(0, 3) and (3, 0) symmetrical about $y = x$

$\therefore$ quadrilateral is a kite

7 **a** at A, $y = 0 \;\therefore\; x = 20$

at B, $x = 0 \;\therefore\; y = 10$

$\therefore\; A\,(20, 0),\; B\,(0, 10)$

b $l \;\Rightarrow\; y = 10 - \frac{1}{2}x$

$\therefore$ grad of $l = -\frac{1}{2}$

$\therefore$ grad of $m = 2$

$m: \quad y = 2x$

at C, $10 - \frac{1}{2}x = 2x$

$x = 4 \;\therefore\; C\,(4, 8)$

$\therefore$ area of $\Delta\,OAC$: area of $\Delta\,OBC$

$= \frac{1}{2} \times 20 \times 8 : \frac{1}{2} \times 10 \times 4$

$= 4 : 1$

8 **a** grad q = grad $p = -\frac{3}{4}$

$\therefore\; y = -\frac{3}{4}x + 7$

b grad $r = \frac{4}{3}$

$\therefore\; y = \frac{4}{3}(x - 1)$

$3y = 4x - 4$

$4x - 3y - 4 = 0$

c $\frac{4}{3}x - \frac{4}{3} = -\frac{3}{4}x + 7$

$16x - 16 = -9x + 84$

$25x = 100$

$x = 4 \;\therefore\; (4, 4)$

$\therefore$ lies on $y = x$

9 **a** grad $PQ = \frac{2-c}{9-3} = \frac{2-c}{6}$

grad $QR = \frac{11-2}{3c-9} = \frac{3}{c-3}$

$\angle PQR = 90° \;\therefore\; PQ$ perp to QR

$\therefore\; \frac{2-c}{6} \times \frac{3}{c-3} = -1$

$3(2 - c) = -6(c - 3)$

$3c = 12$

$c = 4$

b $PQ^2 = 6^2 + 2^2 = 40$

$PQ = \sqrt{40} = 2\sqrt{10}$ $\quad [k = 2]$

c $QR = \sqrt{3^2 + 9^2} = \sqrt{90} = 3\sqrt{10}$

area $= \frac{1}{2} \times PQ \times QR = 30$

10 **a** $PQ^2 = 12^2 + 9^2 = 225$

$PQ = \sqrt{225} = 15$

b grad $= \frac{12-3}{13-1} = \frac{3}{4}$

$\therefore\; y - 3 = \frac{3}{4}(x - 1)$

$4y - 12 = 3x - 3$

$3x - 4y + 9 = 0$

c grad $l_2 = -\frac{4}{3}$

$y - 10 = -\frac{4}{3}(x - 2) \quad [4x + 3y - 38 = 0]$

d $l_1 \;\Rightarrow\; 9x - 12y + 27 = 0$

$l_2 \;\Rightarrow\; 16x + 12y - 152 = 0$

adding $\quad 25x - 125 = 0$

$x = 5 \;\therefore\; (5, 6)$

e distance R to $(5, 6) = \sqrt{3^2 + 4^2} = 5$

area $= \frac{1}{2} \times 15 \times 5 = 37\frac{1}{2}$