

1 Sketch and label each pair of graphs on the same set of axes showing the coordinates of any points where the graphs intersect. Write down the equations of any asymptotes.

a $y = x^2$ and $y = x^3$

b $y = x^2$ and $y = x^4$

c $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$

d $y = x$ and $y = \sqrt{x}$

e $y = x^2$ and $y = 3x^2$

f $y = \frac{1}{x}$ and $y = \frac{2}{x}$

2 $f(x) = (x - 1)(x - 3)(x - 4)$.

a Find $f(0)$.

b Write down the solutions of the equation $f(x) = 0$.

c Sketch the curve $y = f(x)$.

3 Sketch each graph showing the coordinates of any points of intersection with the coordinate axes.

a $y = (x + 1)(x - 1)(x - 3)$

b $y = 2x(x - 1)(x - 5)$

c $y = -(x + 2)(x + 1)(x - 2)$

d $y = x^2(x - 4)$

e $y = 3x(2 + x)(1 - x)$

f $y = (x + 2)(x - 1)^2$

4 **a** Factorise fully $x^3 + 6x^2 + 9x$.

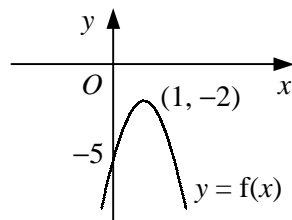
b Hence, sketch the curve $y = x^3 + 6x^2 + 9x$, showing the coordinates of any points where the curve meets the coordinate axes.

5 Given that the constants p and q are such that $p > q > 0$, sketch each of the following graphs showing the coordinates of any points of intersection with the coordinate axes.

a $y = (x - p)(x - q)^2$

b $y = (x - p)(x^2 - q^2)$

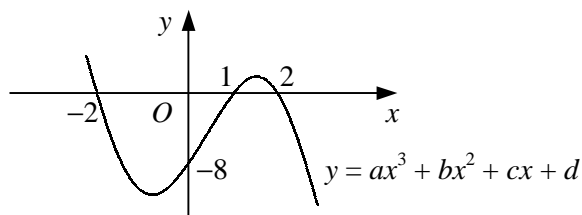
6



The diagram shows the curve with equation $y = f(x)$ which has a turning point at $(1, -2)$ and crosses the y -axis at the point $(0, -5)$.

Given that $f(x)$ is a quadratic function, find an expression for $f(x)$.

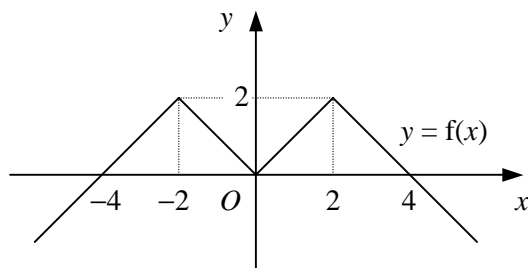
7



The diagram shows the curve with equation $y = ax^3 + bx^2 + cx + d$.

Given that the curve crosses the y -axis at the point $(0, -8)$ and crosses the x -axis at the points $(-2, 0)$, $(1, 0)$ and $(2, 0)$, find the values of the constants a , b , c and d .

8



The diagram shows the graph of $y = f(x)$.

Use the graph to write down the number of solutions that exist to each of the following equations.

- a** $f(x) = 1$ **b** $f(x) = 3$ **c** $f(x) = -1$ **d** $f(x) = 0$

9

a Sketch on the same set of axes the graphs of $y = x^2$ and $y = 1 - 2x$.

b Hence state the number of roots that the equation $x^2 + 2x - 1 = 0$ has and give a reason for your answer.

10

a Find the coordinates of the turning point of the curve $y = x^2 + 2x - 3$.

b By sketching two suitable graphs on the same set of axes, show that the equation

$$x^2 + 2x - 3 - \frac{1}{x} = 0$$

has one positive and two negative real roots.

11

Show that the line $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$.

12

a Solve the simultaneous equations

$$y = 3x + 7$$

$$y = x^2 + 5x + 8$$

b Hence, describe the geometrical relationship between the straight line $y = 3x + 7$ and the curve $y = x^2 + 5x + 8$.

13

a Find the coordinates of the points where the straight line $y = x + 6$ meets the curve $y = x^3 - 4x^2 + x + 6$.

b Given that

$$x^3 - 4x^2 + x + 6 \equiv (x + 1)(x - 2)(x - 3),$$

sketch the straight line $y = x + 6$ and the curve $y = x^3 - 4x^2 + x + 6$ on the same diagram, showing the coordinates of the points where the curve crosses the coordinate axes.

14

Find the value of the constant k such that the straight line with equation $y = 3x + k$ is a tangent to the curve with equation $y = 2x^2 - 5x + 1$.

15

Find the set of values of the constant a for which the line $y = 2 - 5x$ intersects the curve $y = x^2 + ax + 18$ at two points.

16

The curve C has the equation $y = x^2 - 2x + 6$.

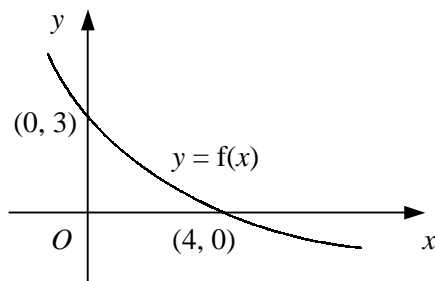
a Find the values of p for which the line $y = px + p$ is a tangent to the curve C .

b Prove that there are no real values of q for which the line $y = qx + 7$ is a tangent to the curve C .

1 Describe how the graph of $y = f(x)$ is transformed to give the graph of

- a** $y = f(x - 1)$ **b** $y = f(x) - 3$ **c** $y = 2f(x)$ **d** $y = f(4x)$
e $y = -f(x)$ **f** $y = \frac{1}{5}f(x)$ **g** $y = f(-x)$ **h** $y = f(\frac{2}{3}x)$

2



The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points $(0, 3)$ and $(4, 0)$.

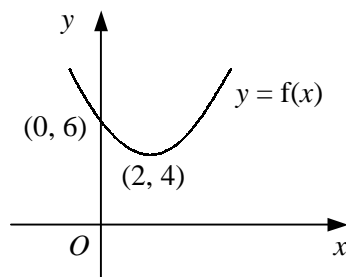
Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

- a** $y = 3f(x)$ **b** $y = f(x + 4)$ **c** $y = -f(x)$ **d** $y = f(\frac{1}{2}x)$

3 Find and simplify an equation of the graph obtained when

- a** the graph of $y = 2x + 5$ is translated by 1 unit in the positive y -direction,
b the graph of $y = 1 - 4x$ is stretched by a factor of 3 in the y -direction, about the x -axis,
c the graph of $y = 3x + 1$ is translated by 4 units in the negative x -direction,
d the graph of $y = 4x - 7$ is reflected in the x -axis.

4



The diagram shows the curve with equation $y = f(x)$ which has a turning point at $(2, 4)$ and crosses the y -axis at the point $(0, 6)$.

Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

- a** $y = f(x) - 3$ **b** $y = f(x + 2)$ **c** $y = f(2x)$ **d** $y = \frac{1}{2}f(x)$

5 Describe a single transformation that would map the graph of $y = x^3$ onto the graph of

- a** $y = 4x^3$ **b** $y = (x - 2)^3$ **c** $y = -x^3$ **d** $y = x^3 + 5$

6 Describe a single transformation that would map the graph of $y = x^2 + 2$ onto the graph of

- a** $y = 2x^2 + 4$ **b** $y = x^2 - 5$ **c** $y = \frac{1}{9}x^2 + 2$ **d** $y = x^2 + 4x + 6$

- 7 Find and simplify an equation of the graph obtained when
- the graph of $y = x^2 + 2x$ is translated by 1 unit in the positive x -direction,
 - the graph of $y = x^2 - 4x + 5$ is stretched by a factor of $\frac{1}{3}$ in the x -direction, about the y -axis.
 - the graph of $y = x^2 + x - 6$ is reflected in the y -axis,
 - the graph of $y = 2x^2 - 3x$ is stretched by a factor of 2 in the x -direction, about the y -axis.

8 $f(x) \equiv x^2 - 4x.$

- Find the coordinates of the turning point of the graph $y = f(x).$
- Sketch each pair of graphs on the same set of axes showing the coordinates of the turning point of each graph.
 - $y = f(x)$ and $y = 3 + f(x)$
 - $y = f(x)$ and $y = f(x - 2)$
 - $y = f(x)$ and $y = f(2x)$

- 9 Sketch each pair of graphs on the same set of axes.

- $y = x^2$ and $y = (x + 3)^2$
- $y = x^3$ and $y = x^3 + 4$
- $y = \frac{1}{x}$ and $y = \frac{1}{x-2}$
- $y = \sqrt{x}$ and $y = \sqrt{2x}$

- 10
 - Describe two different transformations, each of which would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{1}{3x}$.
 - Describe two different transformations, each of which would map the graph of $y = x^2$ onto the graph of $y = 4x^2$.

11 $f(x) \equiv (x + 4)(x + 2)(x - 1).$

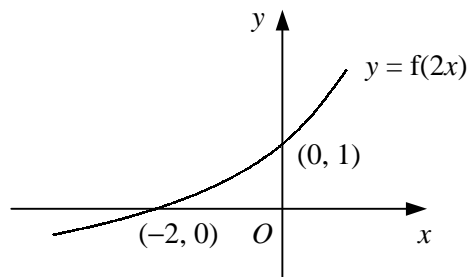
Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

- $y = f(x)$
- $y = f(x - 4)$
- $y = f(-x)$
- $y = f(2x)$

- 12 The curve $y = f(x)$ is a parabola and the coordinates of its turning point are (a, b) . Write down, in terms of a and b , the coordinates of the turning point of the graph

- $y = 3f(x)$
- $y = 4 + f(x)$
- $y = f(x + 1)$
- $y = f(\frac{1}{3}x)$

13



The diagram shows the curve with equation $y = f(2x)$ which crosses the coordinate axes at the points $(-2, 0)$ and $(0, 1)$.

Showing the coordinates of any points of intersection with the coordinate axes, sketch on separate diagrams the curves

- $y = 3f(2x)$
- $y = f(x)$

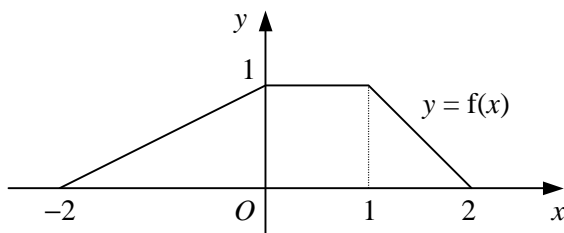
- 1 a Solve the simultaneous equations

$$y = 3x - 4$$

$$y = 4x^2 - 9x + 5 \quad (4)$$

- b Hence, describe the geometrical relationship between the straight line $y = 3x - 4$ and the curve $y = 4x^2 - 9x + 5$. (1)

2

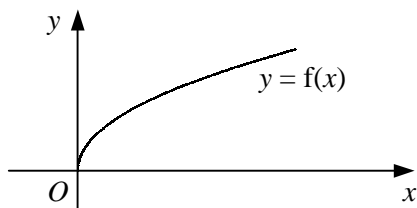


The diagram shows the graph of $y = f(x)$ which is defined for $-2 \leq x \leq 2$.

Labelling the axes in a similar way, sketch on separate diagrams the graphs of

- a $y = 3f(x)$, (2)
 b $y = f(x + 1)$, (2)
 c $y = f(-x)$. (2)
- 3 a Show that the line $y = 4x + 1$ does not intersect the curve $y = x^2 + 5x + 2$. (4)
 b Find the values of m such that the line $y = mx + 1$ meets the curve $y = x^2 + 5x + 2$ at exactly one point. (4)

4



The diagram shows the curve with the equation $y = f(x)$ where

$$f(x) \equiv \sqrt{x}, \quad x \geq 0.$$

- a Sketch on the same set of axes the graphs of $y = 1 + f(x)$ and $y = f(x + 3)$. (4)
 b Find the coordinates of the point of intersection of the two graphs drawn in part a. (4)
- 5 The curve C has the equation $y = x^2 + kx - 3$ and the line l has the equation $y = k - x$, where k is a constant.
 Prove that for all real values of k , the line l will intersect the curve C at exactly two points. (7)

6

$$f(x) \equiv 2x^2 - 4x + 5.$$

- a Express $f(x)$ in the form $a(x + b)^2 + c$. (3)
 b Showing the coordinates of the turning point in each case, sketch on the same set of axes the curves
 i $y = f(x)$,
 ii $y = f(x + 3)$. (4)

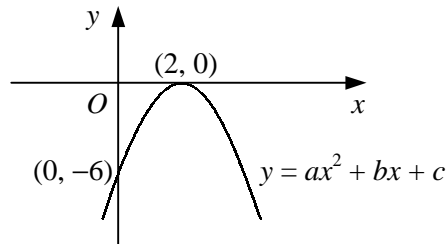
- 7 a Sketch on the same diagram the straight line $y = 2x - 5$ and the curve $y = x^3 - 3x^2$, showing the coordinates of any points where each graph meets the coordinate axes. (4)

- b Hence, state the number of real roots that exist for the equation

$$x^3 - 3x^2 - 2x + 5 = 0,$$

- giving a reason for your answer. (2)

8



The diagram shows the curve with the equation $y = ax^2 + bx + c$.

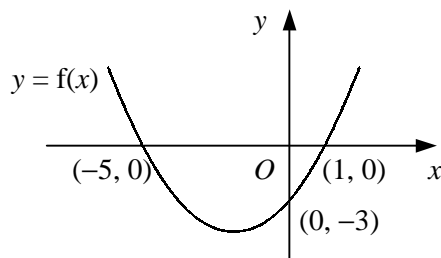
Given that the curve crosses the y -axis at the point $(0, -6)$ and touches the x -axis at the point $(2, 0)$, find the values of the constants a , b and c . (6)

- 9 a Show that

$$(1 - x)(2 + x)^2 \equiv 4 - 3x^2 - x^3. \quad (3)$$

- b Hence, sketch the curve $y = 4 - 3x^2 - x^3$, showing the coordinates of any points of intersection with the coordinate axes. (3)

10



The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points $(-5, 0)$, $(1, 0)$ and $(0, -3)$.

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the curves

a $y = -f(x)$, (2)

b $y = f(x - 5)$, (2)

c $y = f(2x)$. (2)

- 11 a Describe fully the transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x + 1)$. (2)

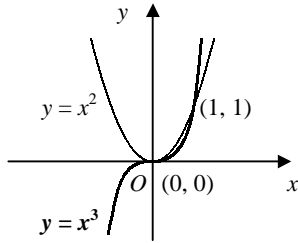
- b Sketch the graph of $y = \frac{1}{x+1}$, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes. (3)

- c By sketching another suitable curve on your diagram in part b, show that the equation

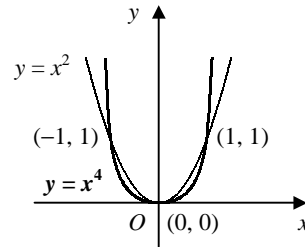
$$x^3 - \frac{1}{x+1} = 2$$

- has one positive and one negative real root. (4)

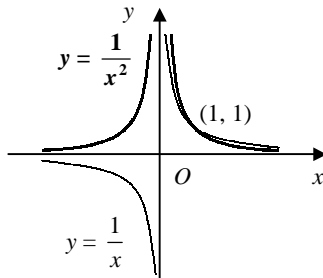
1 a



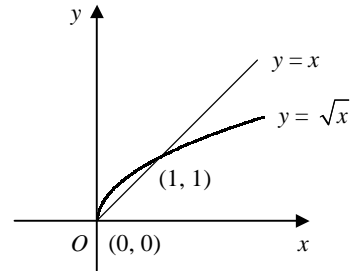
b



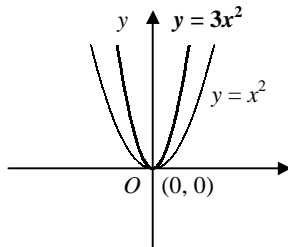
c

asymptotes: $y = 0$ and $x = 0$

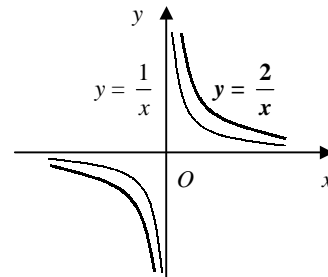
d



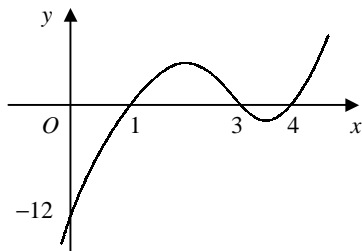
e



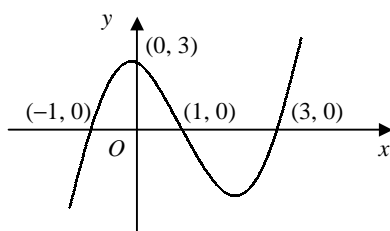
f

asymptotes: $y = 0$ and $x = 0$ 2 a $= (-1) \times (-3) \times (-4) = -12$ b $x = 1, 3, 4$

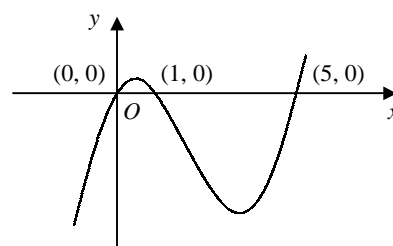
c

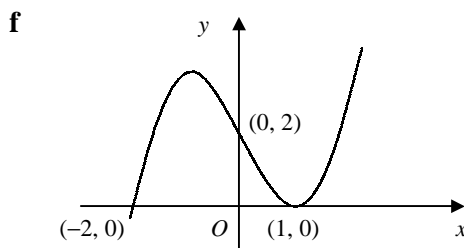
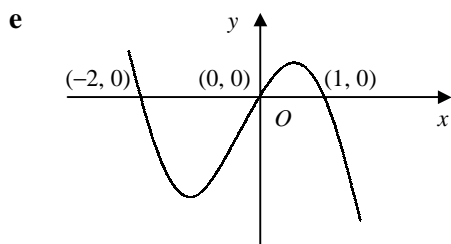
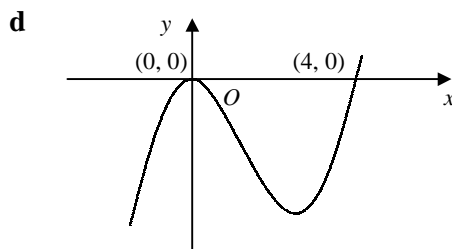
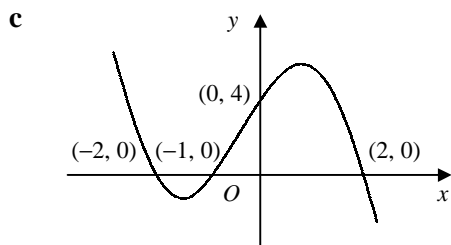


3 a

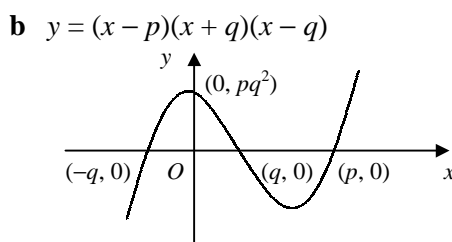
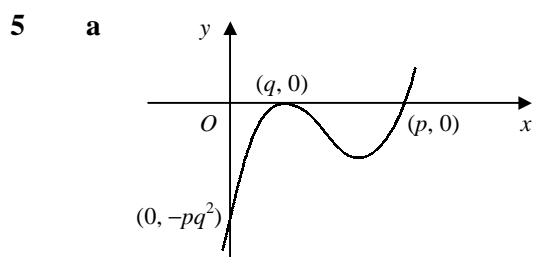
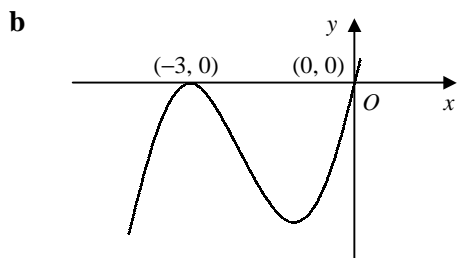


b





4 a $= x(x^2 + 6x + 9) = x(x + 3)^2$



6 TP at $(1, -2)$
 $\therefore f(x) = k(x - 1)^2 - 2$
 crosses y-axis at $(0, -5)$
 $\therefore -5 = k - 2$
 $k = -3$
 $\therefore f(x) = -3(x - 1)^2 - 2$
 $[f(x) = -3x^2 + 6x - 5]$

7 crosses x-axis at $(-2, 0)$, $(1, 0)$ and $(2, 0)$
 $\therefore y = k(x + 2)(x - 1)(x - 2)$
 crosses y-axis at $(0, -8)$
 $\therefore -8 = 4k$
 $k = -2$
 $\therefore y = -2(x + 2)(x - 1)(x - 2)$
 $= -2(x + 2)(x^2 - 3x + 2)$
 $= -2(x^3 - 3x^2 + 2x + 2x^2 - 6x + 4)$
 $= -2x^3 + 2x^2 + 8x - 8$
 $\therefore a = -2, b = 2, c = 8, d = -8$

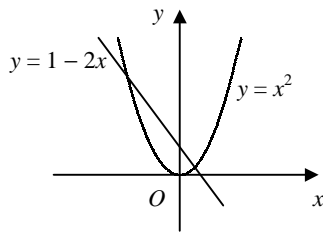
8 a 4

b 0

c 2

d 3

9 a



b 2 roots as $x^2 + 2x - 1 = 0 \Rightarrow x^2 = 1 - 2x$ and the graphs of $y = x^2$ and $y = 1 - 2x$ intersect at 2 points

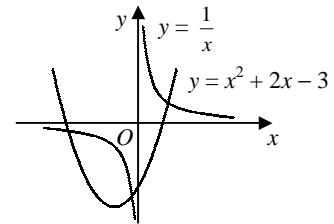
10 a $x^2 + 2x - 3 = (x + 1)^2 - 1 - 3 = (x + 1)^2 - 4 \therefore$ turning point is $(-1, -4)$

b $x^2 + 2x - 3 - \frac{1}{x} = 0 \Rightarrow x^2 + 2x - 3 = \frac{1}{x}$

\therefore roots where $y = x^2 + 2x - 3$ and $y = \frac{1}{x}$ intersect

graphs intersect at 1 point for $x > 0$ and 2 points for $x < 0$

\therefore one positive and two negative real roots



11 $x - 3 = x^2 - 5x + 6$

$$x^2 - 6x + 9 = 0$$

$$(x - 3)^2 = 0$$

repeated root

$\therefore y = x - 3$ is tangent to $y = x^2 - 5x + 6$

12 a $x^2 + 5x + 8 = 3x + 7$

$$x^2 + 2x + 1 = 0$$

$$(x + 1)^2 = 0$$

$$x = -1 \therefore x = -1, y = 4$$

b repeated root

$\therefore y = 3x + 7$ is tangent to $y = x^2 + 5x + 8$ at the point $(-1, 4)$

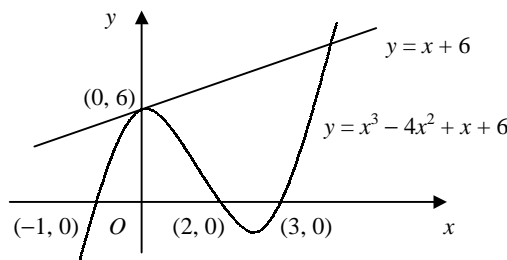
13 a $x^3 - 4x^2 + x + 6 = x + 6$

$$x^3 - 4x^2 = 0$$

$$x^2(x - 4) = 0$$

$$x = 0, 4 \therefore (0, 6) \text{ and } (4, 10)$$

b



14 $2x^2 - 5x + 1 = 3x + k$

$$2x^2 - 8x + 1 - k = 0$$

for tangent, repeated root $\therefore b^2 - 4ac = 0$

$$\therefore 64 - 8(1 - k) = 0$$

$$k = -7$$

15 $x^2 + ax + 18 = 2 - 5x$

$$x^2 + (a + 5)x + 16 = 0$$

intersect at 2 points $\therefore b^2 - 4ac > 0$

$$\therefore (a + 5)^2 - 64 > 0$$

$$a^2 + 10a - 39 > 0$$

$$(a + 13)(a - 3) > 0$$

$$a < -13 \text{ or } a > 3$$

16 a $x^2 - 2x + 6 = px + p$

$$x^2 - (p + 2)x + 6 - p = 0$$

for tangent, repeated root $\therefore b^2 - 4ac = 0$

$$\therefore (p + 2)^2 - 4(6 - p) = 0$$

$$p^2 + 8p - 20 = 0$$

$$(p + 10)(p - 2) = 0$$

$$p = -10, 2$$

b $x^2 - 2x + 6 = qx + 7$

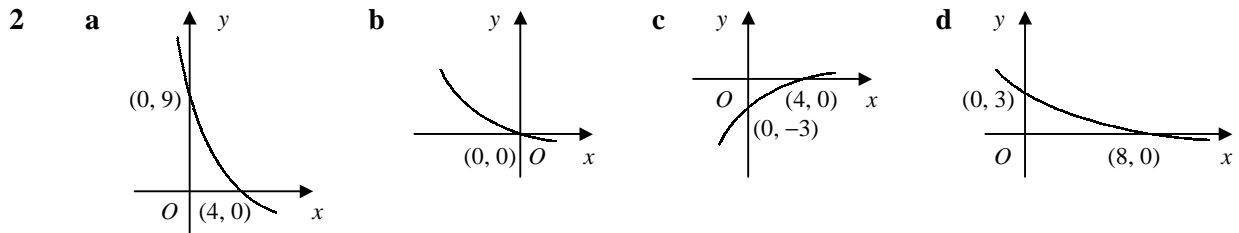
$$x^2 - (q + 2)x - 1 = 0$$

for tangent, repeated root $\therefore b^2 - 4ac = 0$

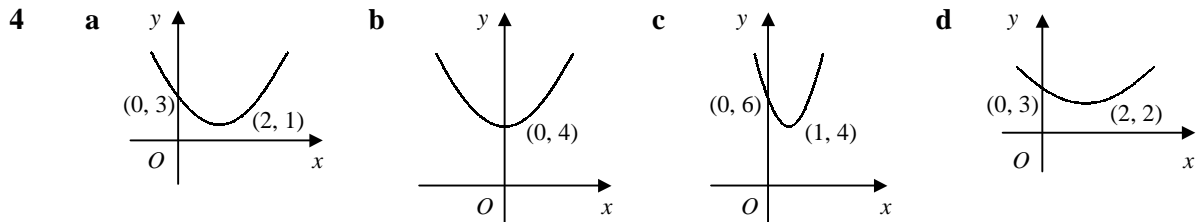
$$\Rightarrow (q + 2)^2 + 4 = 0$$

but for real q , $(q + 2)^2 \geq 0 \therefore$ no solutions

- 1 a translated 1 unit in positive x -direction
 c stretched by a factor of 2 in y -direction
 e reflected in the x -axis
 g reflected in the y -axis
- b translated 3 units in negative y -direction
 d stretched by a factor of $\frac{1}{4}$ in x -direction
 f stretched by a factor of $\frac{1}{5}$ in y -direction
 h stretched by a factor of $\frac{3}{2}$ in x -direction



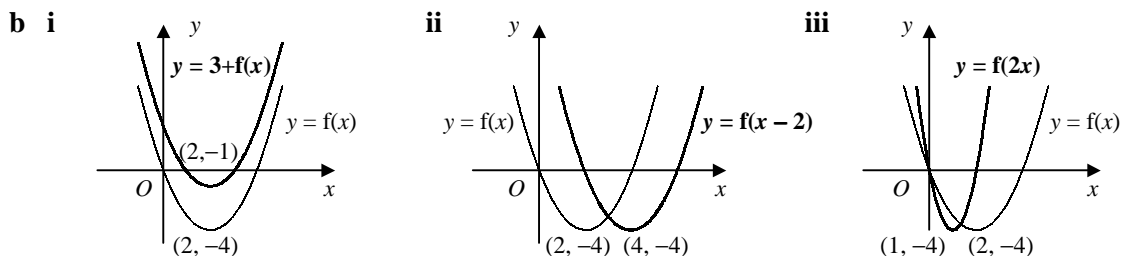
- 3 a $y = 2x + 5 + 1 \Rightarrow y = 2x + 6$
 c $y = 3(x + 4) + 1 \Rightarrow y = 3x + 13$
- b $y = 3(1 - 4x) \Rightarrow y = 3 - 12x$
 d $y = -(4x - 7) \Rightarrow y = 7 - 4x$



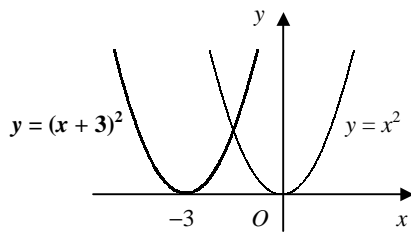
- 5 a stretch by a factor of 4 in y -direction
 c reflection in the x -axis
- b translation by 2 units in positive x -direction
 d translation by 5 units in positive y -direction
- 6 a $y = 2(x^2 + 2)$
 stretch by a factor of 2 in y -direction
- b $y = (x^2 + 2) - 7$
 translation by 7 units in negative y -direction
- c $y = (\frac{1}{3}x)^2 + 2$
 stretch by a factor of 3 in x -direction
- d $y = (x + 2)^2 + 2$
 translation by 2 units in negative x -direction

- 7 a $y = (x - 1)^2 + 2(x - 1) \Rightarrow y = x^2 - 1$
 b $y = (3x)^2 - 4(3x) + 5 \Rightarrow y = 9x^2 - 12x + 5$
 c $y = (-x)^2 + (-x) - 6 \Rightarrow y = x^2 - x - 6$
 d $y = 2(\frac{1}{2}x)^2 - 3(\frac{1}{2}x) \Rightarrow y = \frac{1}{2}x^2 - \frac{3}{2}x$

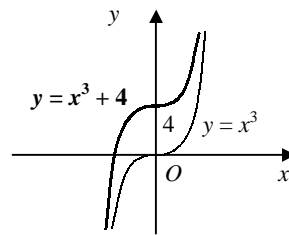
- 8 a $f(x) = (x - 2)^2 - 4 \therefore$ turning point $(2, -4)$



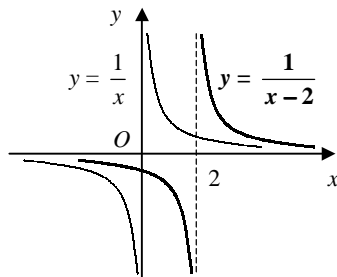
9 a



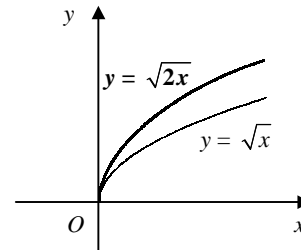
b



c

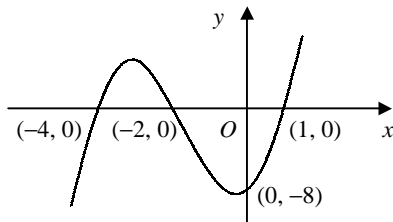


d

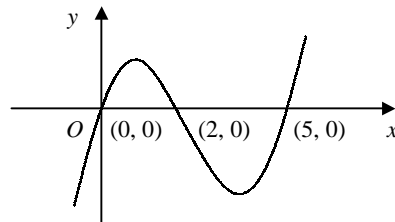


- 10 a let $f(x) = \frac{1}{x} \therefore \frac{1}{3x} = \frac{1}{3}f(x)$ or $f(3x)$
 \therefore stretch by a factor of $\frac{1}{3}$ in y -direction
 or stretch by a factor of $\frac{1}{3}$ in x -direction
- b let $g(x) = x^2 \therefore 4x^2 = 4g(x)$ or $g(2x)$
 \therefore stretch by a factor of 4 in y -direction
 or stretch by a factor of $\frac{1}{2}$ in x -direction

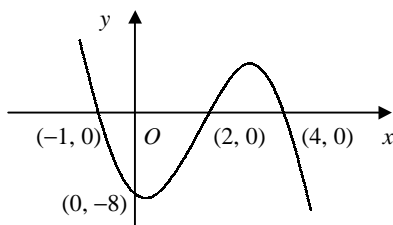
11 a



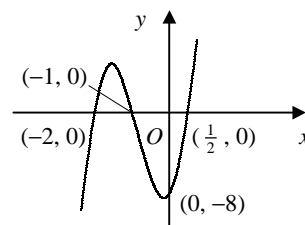
b



c

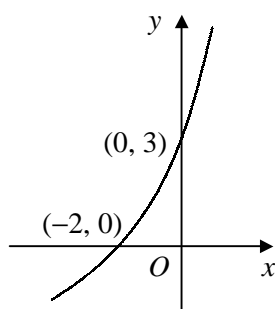


d

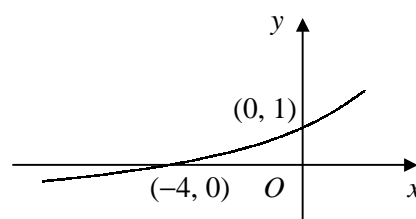


- 12 a $(a, 3b)$ b $(a, b + 4)$ c $(a - 1, b)$ d $(3a, b)$

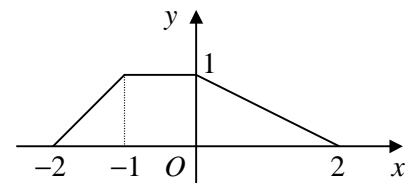
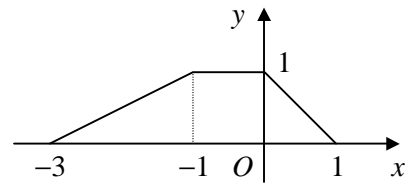
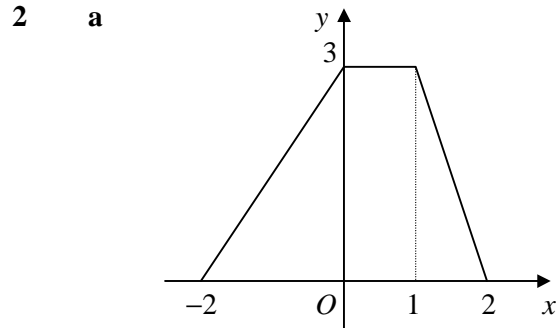
13 a



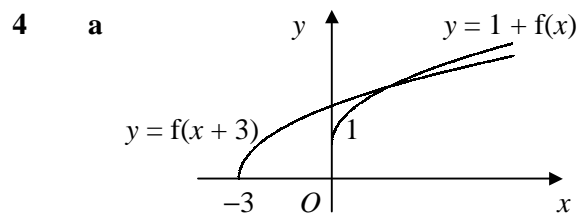
b



- 1 a** $4x^2 - 9x + 5 = 3x - 4$
 $4x^2 - 12x + 9 = 0$
 $(2x - 3)^2 = 0$
 $x = \frac{3}{2}$
 $\therefore x = \frac{3}{2}, y = \frac{1}{2}$
- b** $y = 3x - 4$ is a tangent to the curve
 $y = 4x^2 - 9x + 5$ at the point $(\frac{3}{2}, \frac{1}{2})$

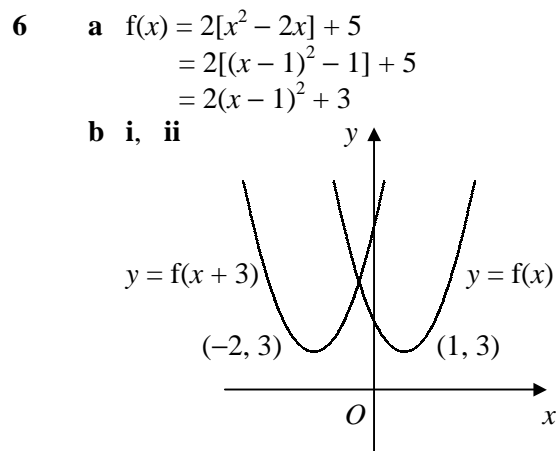


- 3 a** $x^2 + 5x + 2 = 4x + 1$
 $x^2 + x + 1 = 0$
 $b^2 - 4ac = 1 - 4 = -3$
 $b^2 - 4ac < 0 \therefore$ no real roots
 \therefore does not intersect
- b** $x^2 + 5x + 2 = mx + 1$
 $x^2 + (5 - m)x + 1 = 0$
 only one root $\therefore b^2 - 4ac = 0$
 $(5 - m)^2 - 4 = 0$
 $5 - m = \pm 2$
 $m = 3$ or 7

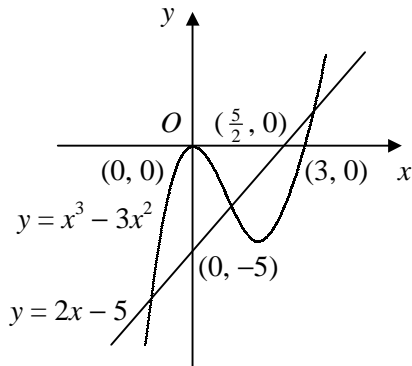


- b** $1 + \sqrt{x} = \sqrt{x+3}$
 $(1 + \sqrt{x})^2 = x + 3$
 $1 + 2\sqrt{x} + x = x + 3$
 $\sqrt{x} = 1$
 $x = 1 \therefore (1, 2)$

- 5** $x^2 + kx - 3 = k - x$
 $x^2 + (k + 1)x - (k + 3) = 0$
 $b^2 - 4ac = (k + 1)^2 + 4(k + 3)$
 $= k^2 + 6k + 13$
 $= (k + 3)^2 - 9 + 13$
 $= (k + 3)^2 + 4$
 real $k \Rightarrow (k + 3)^2 \geq 0$
 $\Rightarrow (k + 3)^2 + 4 \geq 4$
 $\therefore b^2 - 4ac > 0$
 \Rightarrow real and distinct roots
 $\therefore l$ intersects C at exactly two points



7 a $y = x^3 - 3x^2 = x^2(x - 3)$



b 3 real roots

$x^3 - 3x^2 - 2x + 5 = 0 \Rightarrow x^3 - 3x^2 = 2x - 5$
 the graphs of $y = x^3 - 3x^2$ and $y = 2x - 5$
 intersect at three points

8 touches x -axis at $(2, 0)$

$\therefore y = k(x - 2)^2$

crosses y -axis at $(0, -6)$

$\therefore -6 = 4k$

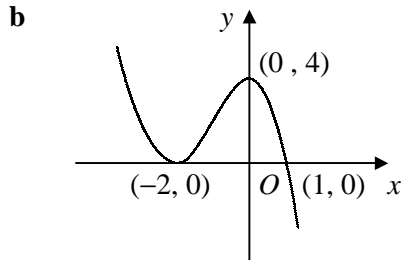
$k = -\frac{3}{2}$

$\therefore y = -\frac{3}{2}(x - 2)^2$

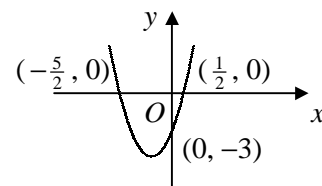
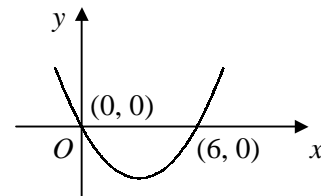
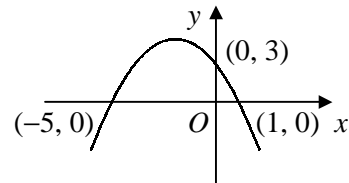
$y = -\frac{3}{2}x^2 + 6x - 6$

$\therefore a = -\frac{3}{2}, b = 6$ and $c = -6$

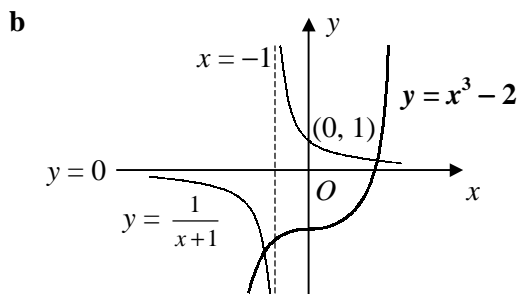
9 a LHS = $(1 - x)(2 + x)^2$
 $= (1 - x)(4 + 4x + x^2)$
 $= (4 + 4x + x^2) - x(4 + 4x + x^2)$
 $= 4 + 4x + x^2 - 4x - 4x^2 - x^3$
 $= 4 - 3x^2 - x^3$
 $=$ RHS



10 a



11 a translation by 1 unit in the negative x -direction



c $x^3 - \frac{1}{x+1} = 2 \Rightarrow x^3 - 2 = \frac{1}{x+1}$

the graphs $y = x^3 - 2$ and $y = \frac{1}{x+1}$ intersect

at one point for $x > 0$ and at one point for $x < 0$

\therefore one positive and one negative real root