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C3 RATIONAL EXPRESSIONS — Worksheet A

1 Simplify

a $\dfrac{3x-1}{18x-6}$ **b** $\dfrac{6x+15}{8x+20}$ **c** $\dfrac{3y+3}{y^2+7y+6}$ **d** $\dfrac{x^2-25}{x^2-7x+10}$

e $\dfrac{a^2-a-6}{a^2+3a-18}$ **f** $\dfrac{x^2+3x}{2x^2+5x-3}$ **g** $\dfrac{3t^2-11t-4}{t^2-16}$ **h** $\dfrac{6x^2-13x+6}{12x^2+x-6}$

2 Express as simply as possible

a $\dfrac{3x^2}{9x-9}\times\dfrac{4x-4}{2x}$ **b** $\dfrac{x^2-36}{x^2+7x+10}\div\dfrac{x-6}{x+2}$

c $\dfrac{n^2+2n}{n^2+6n+8}\times\dfrac{n+4}{n^2}$ **d** $\dfrac{4x-12}{x^2-4}\times\dfrac{x^2+2x}{x^2-2x-3}$

e $\dfrac{4y^2}{2y^2+y}\div\dfrac{y^2+2y-15}{2y^2+11y+5}$ **f** $\dfrac{x^2-1}{2x^2+7x-4}\times\dfrac{6x^2-5x+1}{3x^2-4x+1}$

g $\dfrac{10x-10}{5x+15}\div\dfrac{4-3x-x^2}{x^2+7x+12}$ **h** $\dfrac{a^3-3a^2}{8a^2-4a}\div\dfrac{a^2-9}{2a^2+5a-3}$

3 Express as a single fraction in its simplest form

a $\dfrac{2}{y}+\dfrac{7}{y+4}$ **b** $\dfrac{2x}{x-5}-\dfrac{1}{x+3}$ **c** $\dfrac{7}{x(x+2)}-\dfrac{3x}{x+2}$

d $\dfrac{x}{(x-3)(x-1)}+\dfrac{5}{2(x-1)}$ **e** $\dfrac{2}{q^2+3q}+\dfrac{5q}{4q+12}$ **f** $\dfrac{4}{3x-3}+\dfrac{x+2}{x^2-x}$

g $\dfrac{4}{x+5}+\dfrac{x}{x^2+8x+15}$ **h** $\dfrac{6x}{x^2-4}-\dfrac{3}{x+2}$ **i** $\dfrac{5t+12}{2t^2+7t+3}-\dfrac{4}{2t+1}$

4 Simplify

a $\dfrac{x^2-5x}{6x-30}$ **b** $\dfrac{16-x^2}{x^2+2x-8}$ **c** $\dfrac{2x^2-4x-6}{3x^2-12x+9}$ **d** $\dfrac{x^3-x}{2x^2-x-1}$

e $\dfrac{3x-x^2}{2x^2-18}$ **f** $\dfrac{x^3+x^2-2x}{3x^2+4x-4}$ **g** $\dfrac{2+5x-3x^2}{2x^2+x-10}$ **h** $\dfrac{x^4-5x^2+4}{x^2-x-2}$

5 Express as simply as possible

a $\dfrac{10x^2-10}{5x+10}\times\dfrac{x^2+6x+8}{x^2+5x+4}$ **b** $\dfrac{t^2-2t}{2t^2-t-6}\div\dfrac{9t^2-4}{6t^2+13t+6}$

c $\dfrac{2x^2+12x+10}{4x^2-7x+3}\div\dfrac{4x^2+20x}{4x^2-3x}$ **d** $\dfrac{8x^2+6x-9}{4x^2+12x+9}\times\dfrac{2x^2+3x}{6-8x}$

e $\dfrac{x^4+6x^2+5}{x^2-9}\times\dfrac{2x^2-6x}{4x^2+4}$ **f** $\dfrac{y^4-16}{5y^2+9y-2}\div\dfrac{y^2+4}{25y^2-10y+1}$

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6 Express as a single fraction in its simplest form

a $\dfrac{5}{x^2-1} - \dfrac{1}{2x+2}$

b $\dfrac{3x}{x^2-4} - \dfrac{4}{2x^2+3x-2}$

c $\dfrac{4}{x^2+2x-3} + \dfrac{1}{x^2-3x+2}$

d $\dfrac{x+1}{x^2-25} + \dfrac{2}{x^2+5x}$

e $\dfrac{2x-1}{x^2+4x+4} + \dfrac{x}{3x+6}$

f $\dfrac{1}{x-3} + \dfrac{3}{x^2-3x} + \dfrac{x}{x^2-6x+9}$

g $\dfrac{x}{x^2-x-6} + \dfrac{2}{2x^2+3x-2}$

h $\dfrac{1}{x^2} - \dfrac{1}{3x^2-2x} + \dfrac{3}{6x-4}$

7 Solve

a $1 - \dfrac{2}{x} = \dfrac{3}{2x-5}$

b $\dfrac{2}{x^2-1} + \dfrac{3}{x+1} = 1$

c $\dfrac{20}{2x^2+5x+2} + 1 = \dfrac{10}{2x+1}$

d $\dfrac{y+3}{y+5} - \dfrac{1}{2} = \dfrac{2y-1}{y}$

e $5 + \dfrac{1}{x^2+5x+6} = \dfrac{11}{x+3}$

f $\dfrac{3}{1-4x+4x^2} - \dfrac{10}{1-4x^2} = \dfrac{5}{1+2x}$

8 $$\text{f}(x) \equiv \frac{7x-15}{x^2-5x} - \frac{4}{x-5}, \quad 0 < x < 5.$$

Show that $\text{f}(x) = \dfrac{k}{x}$, where k is an integer to be found.

9 $$\text{f}(x) \equiv \frac{x-5}{3x^2+5x-2} + \frac{2}{3x-1}, \quad x > 1.$$

Show that $\text{f}(x) = \dfrac{1}{x+2}$.

10 Given that $\text{f}(x) \equiv \dfrac{x+2}{x-2}$, $x \neq \pm 2$, show that $\text{f}(x) - \dfrac{1}{\text{f}(x)} = \dfrac{8x}{x^2-4}$.

11 **a** Express $\dfrac{2}{x+5} + \dfrac{3}{(x+2)(x+5)}$ as a single fraction in its simplest form.

b Hence solve the equation

$$\frac{2}{x+5} + \frac{3}{(x+2)(x+5)} = \frac{1}{3},$$

giving your answers to 2 decimal places.

12 Show that the equation $\dfrac{3}{4x+2} - \dfrac{5}{4x^2+4x+1} = 2$ has no real roots.

13 Express $\left(\dfrac{6}{x+5} - \dfrac{1}{x}\right) \div \dfrac{x-1}{x^2-25}$ as a single fraction in its simplest form.

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C3 RATIONAL EXPRESSIONS Worksheet B

1 Find the quotient and remainder obtained in dividing

a $(3x^3 - 10x^2 - 9x + 15)$ by $(x - 4)$ **b** $(2x^3 - 11x^2 - x + 3)$ by $(2x - 1)$

c $(4x^3 + 8x^2 + 7x + 32)$ by $(2x + 5)$ **d** $(1 - 22x^2 - 6x^3)$ by $(3x + 2)$

2 **a** Show that $(x + 2)$ is a factor of $(x^3 + 4x^2 + x - 6)$.

b Fully factorise $x^3 + 4x^2 + x - 6$.

c Simplify $\dfrac{x^3 + 4x^2 + x - 6}{x^2 - 9}$.

3 **a** Show that $(2x - 3)$ is a factor of $(2x^3 - 5x^2 + 13x - 15)$.

b Simplify $\dfrac{2x^3 - 5x^2 + 13x - 15}{2x^2 - 7x + 6}$.

4 **a** State a linear factor of $x^3 - 1$.

b Simplify $\dfrac{x^3 - 1}{x^2 + x - 2}$.

5 Find the integers A and B such that

$$\frac{2x+5}{x+3} \equiv A + \frac{B}{x+3}.$$

6 Express each of the following in the form $A + \dfrac{B}{f(x)}$, where $f(x)$ is linear.

a $\dfrac{x+2}{x+1}$ **b** $\dfrac{x+3}{x-2}$ **c** $\dfrac{x}{1-x}$ **d** $\dfrac{2x+1}{x+2}$ **e** $\dfrac{x-1}{2x-1}$ **f** $\dfrac{1-4x}{3+2x}$

7 Find the quotient and remainder obtained in dividing

a $(x^2 + 3x + 5)$ by $(x^2 + x + 2)$ **b** $(2x^2 + 3x - 8)$ by $(x^2 - x - 4)$

c $(x^2 + 7)$ by $(x^2 + 3x - 1)$ **d** $(3x^2 - x - 4)$ by $(x^2 + 2)$

e $(x^3 - 2x^2 - 5x + 8)$ by $(x^2 + x - 2)$ **f** $(2x^3 - 7x^2 + 1)$ by $(x^2 - 5x + 1)$

g $(3x^3 + 6x^2 - 2x + 5)$ by $(3x^2 + 4)$ **h** $(6x^3 - x^2 - 44x - 6)$ by $(2x^2 - 5x - 2)$

8 **a** Divide $(x^3 + 5x^2 + 7x - 13)$ by $(x^2 + 3x - 4)$.

b Hence show that

$$\frac{x^3 + 5x^2 + 7x - 13}{x^2 + 3x - 4} \equiv x + 2 + \frac{5}{x+4}.$$

9

$$f(x) = \frac{x^3 - 2x^2 - 21x + 70}{x^2 + 2x - 15}, \quad x \neq 3.$$

a Express $f(x)$ in the form $Ax + B + \dfrac{C}{g(x)}$, where $g(x)$ is linear.

b Hence, or otherwise, solve the equation $f(x) = \dfrac{3x-7}{x-3}$.

C3 RATIONAL EXPRESSIONS Worksheet C

1 Express $\dfrac{6}{x^2-9} - \dfrac{7}{2x^2-5x-3}$ as a single fraction in its simplest form. **(6)**

2
$$f(x) \equiv \frac{3}{2x+3} - \frac{x+9}{2x^2+11x+12}, \quad x > 0.$$

Show that $f(x) = \dfrac{1}{x+4}$. **(5)**

3 **a** Express $\dfrac{1}{x-6} - \dfrac{2}{x^2-36}$ as a single fraction in its simplest form. **(3)**

b Hence solve the equation
$$\frac{1}{x-6} - \frac{2}{x^2-36} = \frac{1}{2},$$
giving your answers in the form $a + b\sqrt{5}$, where $a, b \in \mathbb{Z}$. **(4)**

4
$$f(x) \equiv 2x^3 - 5x^2 - 23x - 10.$$

a Show that $(x - 5)$ is a factor of $f(x)$. **(2)**

b Express $\dfrac{f(x)}{2x^2-9x-5}$ in its simplest form. **(5)**

5 Given that the equation
$$\frac{x+6}{x^2+9x+18} + \frac{x-p}{x+7} = 0$$
has real, equal roots, find the possible values of the constant p. **(7)**

6 Express $\dfrac{1}{3x-1} - \dfrac{3x}{9x^2-6x+1} - \dfrac{1}{3x^2-x}$ as a single fraction in its simplest form. **(5)**

7 **a** Simplify

i $\dfrac{7x+14}{4-x^2}$,

ii $\dfrac{2x^2+x-28}{3x^2+12x}$. **(4)**

b Hence show that the equation $\dfrac{7x+14}{4-x^2} = \dfrac{2x^2+x-28}{3x^2+12x}$ has no real roots. **(4)**

8 The first three terms of an arithmetic series are $\dfrac{1}{t-2}$, $\dfrac{1}{2}$ and $\dfrac{4}{t^2-2t}$ respectively.

a Show that $\dfrac{4}{t^2-2t} + \dfrac{1}{t-2} = 1$. **(2)**

b Given that the common difference of the series is not zero, find the value of t and the first term of the series. **(5)**

C3 RATIONAL EXPRESSIONS — Answers - Worksheet A

1

a $= \dfrac{3x-1}{6(3x-1)} = \dfrac{1}{6}$

b $= \dfrac{3(2x+5)}{4(2x+5)} = \dfrac{3}{4}$

c $= \dfrac{3(y+1)}{(y+6)(y+1)}$
$= \dfrac{3}{y+6}$

d $= \dfrac{(x+5)(x-5)}{(x-2)(x-5)}$
$= \dfrac{x+5}{x-2}$

e $= \dfrac{(a+2)(a-3)}{(a+6)(a-3)}$
$= \dfrac{a+2}{a+6}$

f $= \dfrac{x(x+3)}{(2x-1)(x+3)}$
$= \dfrac{x}{2x-1}$

g $= \dfrac{(3t+1)(t-4)}{(t+4)(t-4)}$
$= \dfrac{3t+1}{t+4}$

h $= \dfrac{(3x-2)(2x-3)}{(4x+3)(3x-2)}$
$= \dfrac{2x-3}{4x+3}$

2

a $= \dfrac{3x^2}{9(x-1)} \times \dfrac{4(x-1)}{2x} = \dfrac{2x}{3}$

b $= \dfrac{(x+6)(x-6)}{(x+2)(x+5)} \times \dfrac{x+2}{x-6} = \dfrac{x+6}{x+5}$

c $= \dfrac{n(n+2)}{(n+4)(n+2)} \times \dfrac{n+4}{n^2} = \dfrac{1}{n}$

d $= \dfrac{4(x-3)}{(x+2)(x-2)} \times \dfrac{x(x+2)}{(x+1)(x-3)} = \dfrac{4x}{(x-2)(x+1)}$

e $= \dfrac{4y^2}{y(2y+1)} \times \dfrac{(2y+1)(y+5)}{(y+5)(y-3)} = \dfrac{4y}{y-3}$

f $= \dfrac{(x+1)(x-1)}{(2x-1)(x+4)} \times \dfrac{(3x-1)(2x-1)}{(3x-1)(x-1)} = \dfrac{x+1}{x+4}$

g $= \dfrac{10(x-1)}{5(x+3)} \times \dfrac{(x+3)(x+4)}{(4+x)(1-x)} = -2$

h $= \dfrac{a^2(a-3)}{4a(2a-1)} \times \dfrac{(2a-1)(a+3)}{(a+3)(a-3)} = \dfrac{a}{4}$

3

a $= \dfrac{2(y+4)+7y}{y(y+4)}$
$= \dfrac{9y+8}{y(y+4)}$

b $= \dfrac{2x(x+3)-(x-5)}{(x-5)(x+3)}$
$= \dfrac{2x^2+5x+5}{(x-5)(x+3)}$

c $= \dfrac{7-3x^2}{x(x+2)}$

d $= \dfrac{2x+5(x-3)}{2(x-3)(x-1)}$
$= \dfrac{7x-15}{2(x-3)(x-1)}$

e $= \dfrac{2}{q(q+3)} + \dfrac{5q}{4(q+3)}$
$= \dfrac{8+5q^2}{4q(q+3)}$

f $= \dfrac{4}{3(x-1)} + \dfrac{x+2}{x(x-1)}$
$= \dfrac{4x+3(x+2)}{3x(x-1)} = \dfrac{7x+6}{3x(x-1)}$

g $= \dfrac{4}{x+5} + \dfrac{x}{(x+3)(x+5)}$
$= \dfrac{4(x+3)+x}{(x+3)(x+5)}$
$= \dfrac{5x+12}{(x+3)(x+5)}$

h $= \dfrac{6x}{(x+2)(x-2)} - \dfrac{3}{x+2}$
$= \dfrac{6x-3(x-2)}{(x+2)(x-2)}$
$= \dfrac{3x+6}{(x+2)(x-2)}$
$= \dfrac{3(x+2)}{(x+2)(x-2)} = \dfrac{3}{x-2}$

i $= \dfrac{5t+12}{(2t+1)(t+3)} - \dfrac{4}{2t+1}$
$= \dfrac{5t+12-4(t+3)}{(2t+1)(t+3)}$
$= \dfrac{t}{(2t+1)(t+3)}$

4

a $= \dfrac{x(x-5)}{6(x-5)} = \dfrac{x}{6}$

b $= \dfrac{(4+x)(4-x)}{(x+4)(x-2)}$
$= \dfrac{4-x}{x-2}$

c $= \dfrac{2(x-3)(x+1)}{3(x-1)(x-3)}$
$= \dfrac{2(x+1)}{3(x-1)}$

d $= \dfrac{x(x+1)(x-1)}{(2x+1)(x-1)}$
$= \dfrac{x(x+1)}{2x+1}$

e $= \dfrac{x(3-x)}{2(x+3)(x-3)}$
$= -\dfrac{x}{2(x+3)}$

f $= \dfrac{x(x+2)(x-1)}{(3x-2)(x+2)}$
$= \dfrac{x(x-1)}{3x-2}$

g $= \dfrac{(2-x)(1+3x)}{(2x+5)(x-2)}$
$= -\dfrac{3x+1}{2x+5}$

h $= \dfrac{(x^2-1)(x^2-4)}{(x+1)(x-2)}$
$= \dfrac{(x+1)(x-1)(x+2)(x-2)}{(x+1)(x-2)}$
$= (x-1)(x+2)$

5 **a** $= \frac{10(x+1)(x-1)}{5(x+2)} \times \frac{(x+2)(x+4)}{(x+1)(x+4)} = 2(x-1)$

b $= \frac{t(t-2)}{(2t+3)(t-2)} \times \frac{(3t+2)(2t+3)}{(3t+2)(3t-2)} = \frac{t}{3t-2}$

c $= \frac{2(x+1)(x+5)}{(4x-3)(x-1)} \times \frac{x(4x-3)}{4x(x+5)} = \frac{x+1}{2(x-1)}$

d $= \frac{(4x-3)(2x+3)}{(2x+3)^2} \times \frac{x(2x+3)}{2(3-4x)} = -\frac{x}{2}$

e $= \frac{(x^2+1)(x^2+5)}{(x+3)(x-3)} \times \frac{2x(x-3)}{4(x^2+1)} = \frac{x(x^2+5)}{2(x+3)}$

f $= \frac{(y^2+4)(y+2)(y-2)}{(5y-1)(y+2)} \times \frac{(5y-1)^2}{y^2+4} = (y-2)(5y-1)$

6 **a** $= \frac{5}{(x+1)(x-1)} - \frac{1}{2(x+1)}$

$= \frac{10-(x-1)}{2(x+1)(x-1)}$

$= \frac{11-x}{2(x+1)(x-1)}$

b $= \frac{3x}{(x+2)(x-2)} - \frac{4}{(2x-1)(x+2)}$

$= \frac{3x(2x-1)-4(x-2)}{(x+2)(x-2)(2x-1)}$

$= \frac{6x^2-7x+8}{(x+2)(x-2)(2x-1)}$

c $= \frac{4}{(x+3)(x-1)} + \frac{1}{(x-1)(x-2)}$

$= \frac{4(x-2)+(x+3)}{(x+3)(x-1)(x-2)}$

$= \frac{5x-5}{(x+3)(x-1)(x-2)}$

$= \frac{5}{(x+3)(x-2)}$

d $= \frac{x+1}{(x+5)(x-5)} + \frac{2}{x(x+5)}$

$= \frac{x(x+1)+2(x-5)}{x(x+5)(x-5)}$

$= \frac{x^2+3x-10}{x(x+5)(x-5)}$

$= \frac{(x+5)(x-2)}{x(x+5)(x-5)}$

$= \frac{x-2}{x(x-5)}$

e $= \frac{2x-1}{(x+2)^2} + \frac{x}{3(x+2)}$

$= \frac{3(2x-1)+x(x+2)}{3(x+2)^2}$

$= \frac{x^2+8x-3}{3(x+2)^2}$

f $= \frac{1}{x-3} + \frac{3}{x(x-3)} + \frac{x}{(x-3)^2}$

$= \frac{x(x-3)+3(x-3)+x^2}{x(x-3)^2}$

$= \frac{2x^2-9}{x(x-3)^2}$

g $= \frac{x}{(x+2)(x-3)} + \frac{2}{(2x-1)(x+2)}$

$= \frac{x(2x-1)+2(x-3)}{(x+2)(x-3)(2x-1)}$

$= \frac{2x^2+x-6}{(x+2)(x-3)(2x-1)}$

$= \frac{(2x-3)(x+2)}{(x+2)(x-3)(2x-1)}$

$= \frac{2x-3}{(x-3)(2x-1)}$

h $= \frac{1}{x^2} - \frac{1}{x(3x-2)} + \frac{3}{2(3x-2)}$

$= \frac{2(3x-2)-2x+3x^2}{2x^2(3x-2)}$

$= \frac{3x^2+4x-4}{2x^2(3x-2)}$

$= \frac{(3x-2)(x+2)}{2x^2(3x-2)}$

$= \frac{x+2}{2x^2}$

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7 **a** $x(2x-5)-2(2x-5)=3x$

$x^2-6x+5=0$

$(x-1)(x-5)=0$

$x=1, 5$

b $\frac{2}{(x+1)(x-1)}+\frac{3}{x+1}=1$

$2+3(x-1)=x^2-1$

$x^2-3x=0$

$x(x-3)=0$

$x=0, 3$

c $\frac{20}{(2x+1)(x+2)}+1=\frac{10}{2x+1}$

$20+2x^2+5x+2=10(x+2)$

$2x^2-5x+2=0$

$(2x-1)(x-2)=0$

$x=\frac{1}{2}, 2$

d $2y(y+3)-y(y+5)=2(y+5)(2y-1)$

$2y^2+6y-y^2-5y=4y^2+18y-10$

$3y^2+17y-10=0$

$y=\frac{-17\pm\sqrt{289+120}}{6}=\frac{-17\pm\sqrt{409}}{6}$

$y=-6.20, 0.537$ (3sf)

e $5+\frac{1}{(x+3)(x+2)}=\frac{11}{x+3}$

$5(x^2+5x+6)+1=11(x+2)$

$5x^2+14x+9=0$

$(5x+9)(x+1)=0$

$x=-\frac{9}{5}, -1$

f $\frac{3}{(1-2x)^2}-\frac{10}{(1+2x)(1-2x)}=\frac{5}{1+2x}$

$3(1+2x)-10(1-2x)=5(1-4x+4x^2)$

$10x^2-23x+6=0$

$(10x-3)(x-2)=0$

$x=\frac{3}{10}, 2$

8 $f(x)=\frac{7x-15}{x(x-5)}-\frac{4}{x-5}$

$=\frac{7x-15-4x}{x(x-5)}=\frac{3x-15}{x(x-5)}$

$=\frac{3(x-5)}{x(x-5)}=\frac{3}{x}$ [$k=3$]

9 $f(x)=\frac{x-5}{(3x-1)(x+2)}+\frac{2}{3x-1}$

$=\frac{x-5+2(x+2)}{(3x-1)(x+2)}=\frac{3x-1}{(3x-1)(x+2)}$

$=\frac{1}{x+2}$

10 $f(x)-\frac{1}{f(x)}=\frac{x+2}{x-2}-\frac{x-2}{x+2}$

$=\frac{(x+2)^2-(x-2)^2}{(x-2)(x+2)}$

$=\frac{x^2+4x+4-(x^2-4x+4)}{x^2-4}$

$=\frac{8x}{x^2-4}$

11 **a** $=\frac{2(x+2)+3}{(x+2)(x+5)}=\frac{2x+7}{(x+2)(x+5)}$

b $\frac{2x+7}{(x+2)(x+5)}=\frac{1}{3}$

$3(2x+7)=(x+2)(x+5)$

$x^2+x-11=0$

$x=\frac{-1\pm\sqrt{1+44}}{2}=\frac{-1\pm\sqrt{45}}{2}=-3.85, 2.85$

12 $\frac{3}{2(2x+1)}-\frac{5}{(2x+1)^2}=2$

$3(2x+1)-10=4(4x^2+4x+1)$

$16x^2+10x+11=0$

$b^2-4ac=100-704=-604$

$b^2-4ac<0$ $\therefore$ no real roots

13 $=\frac{6x-(x+5)}{x(x+5)}\div\frac{x-1}{(x+5)(x-5)}$

$=\frac{5x-5}{x(x+5)}\times\frac{(x+5)(x-5)}{x-1}$

$=\frac{5(x-1)}{x(x+5)}\times\frac{(x+5)(x-5)}{x-1}$

$=\frac{5(x-5)}{x}$

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1 **a**

$$\begin{array}{r|rrrr} & 3x^2 & +\ 2x & -\ 1 & \\ \hline x-4 & 3x^3 & -\ 10x^2 & -\ 9x & +\ 15 \\ & 3x^3 & -\ 12x^2 & & \\ \hline & & 2x^2 & -\ 9x & \\ & & 2x^2 & -\ 8x & \\ \hline & & & -\ x & +\ 15 \\ & & & -\ x & +\ 4 \\ \hline & & & & 11 \end{array}$$

quotient: $3x^2 + 2x - 1$, remainder: 11

b

$$\begin{array}{r|rrrr} & x^2 & -\ 5x & -\ 3 & \\ \hline 2x-1 & 2x^3 & -\ 11x^2 & -\ x & +\ 3 \\ & 2x^3 & -\ x^2 & & \\ \hline & & -\ 10x^2 & -\ x & \\ & & -\ 10x^2 & +\ 5x & \\ \hline & & & -\ 6x & +\ 3 \\ & & & -\ 6x & +\ 3 \\ \hline \end{array}$$

quotient: $x^2 - 5x - 3$, remainder: 0

c

$$\begin{array}{r|rrrr} & 2x^2 & -\ x & +\ 6 & \\ \hline 2x+5 & 4x^3 & +\ 8x^2 & +\ 7x & +\ 32 \\ & 4x^3 & +\ 10x^2 & & \\ \hline & & -\ 2x^2 & +\ 7x & \\ & & -\ 2x^2 & -\ 5x & \\ \hline & & & 12x & +\ 32 \\ & & & 12x & +\ 30 \\ \hline & & & & 2 \end{array}$$

quotient: $2x^2 - x + 6$, remainder: 2

d

$$\begin{array}{r|rrrr} & -\ 2x^2 & -\ 6x & +\ 4 & \\ \hline 3x+2 & -\ 6x^3 & -\ 22x^2 & +\ 0x & +\ 1 \\ & -\ 6x^3 & -\ 4x^2 & & \\ \hline & & -\ 18x^2 & +\ 0x & \\ & & -\ 18x^2 & -\ 12x & \\ \hline & & & 12x & +\ 1 \\ & & & 12x & +\ 8 \\ \hline & & & & -\ 7 \end{array}$$

quotient: $-2x^2 - 6x + 4$, remainder -7

2 **a** let $f(x) = x^3 + 4x^2 + x - 6$

$f(-2) = -8 + 16 - 2 - 6 = 0$

$\therefore$ $(x + 2)$ is a factor

b

$$\begin{array}{r|rrrr} & x^2 & +\ 2x & -\ 3 & \\ \hline x+2 & x^3 & +\ 4x^2 & +\ x & -\ 6 \\ & x^3 & +\ 2x^2 & & \\ \hline & & 2x^2 & +\ x & \\ & & 2x^2 & +\ 4x & \\ \hline & & & -\ 3x & -\ 6 \\ & & & -\ 3x & -\ 6 \\ \hline \end{array}$$

$\therefore$ $x^3 + 4x^2 + x - 6$

$= (x + 2)(x^2 + 2x - 3)$

$= (x + 2)(x + 3)(x - 1)$

c $= \dfrac{(x+2)(x+3)(x-1)}{(x+3)(x-3)}$

$= \dfrac{(x+2)(x-1)}{x-3}$

3 **a** let $f(x) = 2x^3 - 5x^2 + 13x - 15$

$f(\frac{3}{2}) = \frac{27}{4} - \frac{45}{4} + \frac{39}{2} - 15 = 0$

$\therefore$ $(2x - 3)$ is a factor

b

$$
\begin{array}{r|rrrr}
 & x^2 & -\ x & +\ 5 & \\
\hline
2x-3 & 2x^3 & -\ 5x^2 & +\ 13x & -\ 15 \\
 & 2x^3 & -\ 3x^2 & & \\
\hline
 & & -\ 2x^2 & +\ 13x & \\
 & & -\ 2x^2 & +\ 3x & \\
\hline
 & & & 10x & -\ 15 \\
 & & & 10x & -\ 15 \\
\hline
\end{array}
$$

$\therefore$ $2x^3 - 5x^2 + 13x - 15$

$= (2x - 3)(x^2 - x + 5)$

$\therefore$ $\dfrac{2x^3 - 5x^2 + 13x - 15}{2x^2 - 7x + 6}$

$= \dfrac{(2x-3)(x^2 - x + 5)}{(2x-3)(x-2)}$

$= \dfrac{x^2 - x + 5}{x - 2}$

4 **a** $x - 1$

b

$$
\begin{array}{r|rrrr}
 & x^2 & +\ x & +\ 1 & \\
\hline
x-1 & x^3 & +\ 0x^2 & +\ 0x & -\ 1 \\
 & x^3 & -\ x^2 & & \\
\hline
 & & x^2 & +\ 0x & \\
 & & x^2 & -\ x & \\
\hline
 & & & x & -\ 1 \\
 & & & x & -\ 1 \\
\hline
\end{array}
$$

$\therefore$ $x^3 - 1 = (x - 1)(x^2 + x + 1)$

$\therefore$ $\dfrac{x^3 - 1}{x^2 + x - 2} = \dfrac{(x-1)(x^2 + x + 1)}{(x-1)(x+2)}$

$= \dfrac{x^2 + x + 1}{x + 2}$

5 $\dfrac{2x+5}{x+3} = \dfrac{2(x+3) - 1}{x+3} = 2 - \dfrac{1}{x+3}$

$\therefore$ $A = 2, B = -1$

6 **a** $= \dfrac{(x+1)+1}{x+1} = 1 + \dfrac{1}{x+1}$

b $= \dfrac{(x-2)+5}{x-2} = 1 + \dfrac{5}{x-2}$

c $= \dfrac{-(1-x)+1}{1-x} = -1 + \dfrac{1}{1-x}$

d $= \dfrac{2(x+2)-3}{x+2} = 2 - \dfrac{3}{x+2}$

e $= \dfrac{\frac{1}{2}(2x-1) - \frac{1}{2}}{2x-1} = \frac{1}{2} - \dfrac{1}{2(2x-1)}$

f $= \dfrac{-2(3+2x)+7}{3+2x} = -2 + \dfrac{7}{3+2x}$

7 **a**

$$\begin{array}{rl} & 1 \\ x^2+x+2\,\big) & x^2 + 3x + 5 \\ & x^2 + x + 2 \\ \hline & 2x + 3 \end{array}$$

quotient: 1, remainder: $2x + 3$

b

$$\begin{array}{rl} & 2 \\ x^2-x-4\,\big) & 2x^2 + 3x - 8 \\ & 2x^2 - 2x - 8 \\ \hline & 5x \end{array}$$

quotient: 2, remainder: $5x$

c

$$\begin{array}{rl} & 1 \\ x^2+3x-1\,\big) & x^2 + 0x + 7 \\ & x^2 + 3x - 1 \\ \hline & -3x + 8 \end{array}$$

quotient: 1, remainder: $-3x + 8$

d

$$\begin{array}{rl} & 3 \\ x^2+2\,\big) & 3x^2 - x - 4 \\ & 3x^2 + 0x + 6 \\ \hline & -x - 10 \end{array}$$

quotient: 3, remainder: $-x - 10$

e

$$\begin{array}{rl} & x - 3 \\ x^2+x-2\,\big) & x^3 - 2x^2 - 5x + 8 \\ & x^3 + x^2 - 2x \\ \hline & -3x^2 - 3x + 8 \\ & -3x^2 - 3x + 6 \\ \hline & 2 \end{array}$$

quotient: $x - 3$, remainder: 2

f

$$\begin{array}{rl} & 2x + 3 \\ x^2-5x+1\,\big) & 2x^3 - 7x^2 + 0x + 1 \\ & 2x^3 - 10x^2 + 2x \\ \hline & 3x^2 - 2x + 1 \\ & 3x^2 - 15x + 3 \\ \hline & 13x - 2 \end{array}$$

quotient: $2x + 3$, remainder: $13x - 2$

g

$$\begin{array}{rl} & x + 2 \\ 3x^2+4\,\big) & 3x^3 + 6x^2 - 2x + 5 \\ & 3x^3 + 0x^2 + 4x \\ \hline & 6x^2 - 6x + 5 \\ & 6x^2 + 0x + 8 \\ \hline & -6x - 3 \end{array}$$

quotient: $x + 2$, remainder: $-6x - 3$

h

$$\begin{array}{rl} & 3x + 7 \\ 2x^2-5x-2\,\big) & 6x^3 - x^2 - 44x - 6 \\ & 6x^3 - 15x^2 - 6x \\ \hline & 14x^2 - 38x - 6 \\ & 14x^2 - 35x - 14 \\ \hline & -3x + 8 \end{array}$$

quotient: $3x + 7$, remainder: $-3x + 8$

8 **a**

$$\begin{array}{rl} & x + 2 \\ x^2+3x-4\,\big) & x^3 + 5x^2 + 7x - 13 \\ & x^3 + 3x^2 - 4x \\ \hline & 2x^2 + 11x - 13 \\ & 2x^2 + 6x - 8 \\ \hline & 5x - 5 \end{array}$$

$\therefore\ = x + 2$ remainder $5x - 5$

b LHS $= x + 2 + \dfrac{5x-5}{x^2+3x-4}$

$$= x + 2 + \frac{5(x-1)}{(x-1)(x+4)}$$

$$= x + 2 + \frac{5}{x+4}$$

9 **a**

$$\begin{array}{rl} & x - 4 \\ x^2+2x-15\,\big) & x^3 - 2x^2 - 21x + 70 \\ & x^3 + 2x^2 - 15x \\ \hline & -4x^2 - 6x + 70 \\ & -4x^2 - 8x + 60 \\ \hline & 2x + 10 \end{array}$$

$$\therefore\ f(x) = x - 4 + \frac{2x+10}{x^2+2x-15}$$

$$= x - 4 + \frac{2(x+5)}{(x+5)(x-3)}$$

$$= x - 4 + \frac{2}{x-3}$$

b $\dfrac{3x-7}{x-3} = \dfrac{3(x-3)+2}{x-3} = 3 + \dfrac{2}{x-3}$

$$\therefore\ f(x) = \frac{3x-7}{x-3}$$

$$\Rightarrow\quad x - 4 + \frac{2}{x-3} = 3 + \frac{2}{x-3}$$

$$x - 4 = 3$$

$$x = 7$$

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C3 RATIONAL EXPRESSIONS *Answers - Worksheet C*

1
$$= \frac{6}{(x+3)(x-3)} - \frac{7}{(2x+1)(x-3)}$$
$$= \frac{6(2x+1)-7(x+3)}{(x+3)(x-3)(2x+1)} = \frac{5x-15}{(x+3)(x-3)(2x+1)}$$
$$= \frac{5(x-3)}{(x+3)(x-3)(2x+1)} = \frac{5}{(x+3)(2x+1)}$$

2
$$\text{f}(x) = \frac{3}{2x+3} - \frac{x+9}{(2x+3)(x+4)}$$
$$= \frac{3(x+4)-(x+9)}{(2x+3)(x+4)}$$
$$= \frac{2x+3}{(2x+3)(x+4)} = \frac{1}{x+4}$$

3 **a**
$$= \frac{1}{x-6} - \frac{2}{(x+6)(x-6)} = \frac{(x+6)-2}{(x-6)(x+6)}$$
$$= \frac{x+4}{(x-6)(x+6)}$$

b
$$\frac{x+4}{(x-6)(x+6)} = \frac{1}{2}$$
$$2(x+4) = x^2 - 36$$
$$x^2 - 2x - 44 = 0$$
$$x = \frac{2 \pm \sqrt{4+176}}{2} = \frac{2 \pm 6\sqrt{5}}{2} = 1 \pm 3\sqrt{5}$$

4 **a** $\text{f}(5) = 250 - 125 - 115 - 10 = 0$

$\therefore$ $(x-5)$ is a factor of $\text{f}(x)$

b
$$\begin{array}{r|rrrr} & 2x^2 & +\,5x & +\,2 & \\ \hline x-5 & 2x^3 & -\,5x^2 & -\,23x & -\,10 \\ & 2x^3 & -\,10x^2 & & \\ \hline & & 5x^2 & -\,23x & \\ & & 5x^2 & -\,25x & \\ \hline & & & 2x & -\,10 \\ & & & 2x & -\,10 \\ \hline \end{array}$$

$\therefore$ $\text{f}(x) = (x-5)(2x^2 + 5x + 2)$
$= (x-5)(2x+1)(x+2)$

$$\therefore \frac{\text{f}(x)}{2x^2-9x-5} = \frac{(x-5)(2x+1)(x+2)}{(2x+1)(x-5)}$$
$$= x + 2$$

5
$$\frac{x+6}{(x+3)(x+6)} + \frac{x-p}{x+7} = 0$$
$$(x+7) + (x-p)(x+3) = 0$$
$$x^2 + (4-p)x + 7 - 3p = 0$$

real, equal roots $\therefore$ $b^2 - 4ac = 0$

$(4-p)^2 - 4(7-3p) = 0$

$p^2 + 4p - 12 = 0$

$(p+6)(p-2) = 0$

$p = -6, 2$

6
$$= \frac{1}{3x-1} - \frac{3x}{(3x-1)^2} - \frac{1}{x(3x-1)}$$
$$= \frac{x(3x-1) - 3x^2 - (3x-1)}{x(3x-1)^2}$$
$$= \frac{1-4x}{x(3x-1)^2}$$

7 **a** **i**
$$= \frac{7(x+2)}{(2+x)(2-x)} = \frac{7}{2-x}$$

ii
$$= \frac{(2x-7)(x+4)}{3x(x+4)} = \frac{2x-7}{3x}$$

b
$$\frac{7}{2-x} = \frac{2x-7}{3x}$$
$$21x = (2-x)(2x-7)$$
$$x^2 + 5x + 7 = 0$$
$$b^2 - 4ac = 25 - 28 = -3$$

$b^2 - 4ac < 0$ $\therefore$ no real roots

8 **a**
$$\frac{1}{2} - \frac{1}{t-2} = \frac{4}{t^2-2t} - \frac{1}{2}$$
$$\therefore \frac{4}{t^2-2t} + \frac{1}{t-2} = 1$$

b
$$\frac{4}{t(t-2)} + \frac{1}{t-2} = 1$$
$$4 + t = t^2 - 2t$$
$$t^2 - 3t - 4 = 0$$
$$(t+1)(t-4) = 0$$
$$t = -1, 4$$

but if $t = 4$, common difference $= 0$

$\therefore$ $t = -1$, first term $= -\frac{1}{3}$