Integration : Area and Definite Integrals - Edexcel Past Exam Questions MARK SCHEME

Question 1 : Jan 05 Q8

Question Number	Scheme	Marks
	(a) $x^2+6x+10=3x+20$ $\Rightarrow\ x^2+3x-10=0$ $(x+5)(x-2)=0$ so $x=,-5$ or 2 sub for y in $y=3x+20, y=5$ or 26	M1 M1, A1 M1, A1 (5)
	(b) line – curve =, $10-3x-x^2$ $\int(10-3x-x^2)dx=10x-\frac{3}{2}x^2-\frac{x^3}{3}$ $\left[10x-\frac{3}{2}x^2-\frac{x^3}{3}\right]_{-5}^{2}=(20-\frac{3}{2}\times4-\frac{8}{3})-(-50-\frac{3}{2}\times25+\frac{125}{3})$ $=11\frac{1}{3}--45\frac{5}{6}=57\frac{1}{6}$	M1, A1 M1 A2/1/0√ M1 A1 (7) (12)
ALT (b)	$\int(x^2+6x+10)dx=\frac{x^3}{3}+3x^2+10x$ use of limits $=(8/3+12+20)-(-\frac{125}{3}+75-50)=(108\frac{1}{2})$ Area of Trapezium$=\frac{1}{2}(5+26)(2--5)=(51\frac{1}{3})$ Shaded area = Trapezium - $\int=108\frac{1}{2}-51\frac{1}{3}=57\frac{1}{6}$	M1 A2 M1 B1 √ M1 A1 (7)
	(a) 1st M1 for putting curve = line 3rd M1 for obtaining at least one y value. Don't need A and B identified. (b) 1st M1 for $\pm(10-3x-x^2)$ 3rd M1 (2nd on ALT) for using their limits, √ their x values from (a)	

Question 2 : June 05 Q10

Question number	Scheme	Marks
	(a) $\int(2x+8x^{-2}-5)\mathrm{d}x = x^2+\frac{8x^{-1}}{-1}-5x$	M1 A1 A1
	$\left[x^2+\frac{8x^{-1}}{-1}-5x\right]_1^4 = (16-2-20)-(1-8-5)$ (= 6)	M1
	$x=1$: $y=5$ and $x=4$: $y=3.5$	B1
	Area of trapezium = $\frac{1}{2}(5+3.5)(4-1)$ (= 12.75)	M1
	Shaded area = 12.75 – 6 = 6.75 (M: Subtract either way round)	M1 A1 (8)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2-16x^{-3}$	M1 A1
	(Increasing where) $\frac{\mathrm{d}y}{\mathrm{d}x}>0$; For $x>2$, $\frac{16}{x^3}<2$, $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}>0$ (Allow ≥)	dM1; A1 (4)
		12
	(a) Integration: One term wrong M1 A1 A0; two terms wrong M1 A0 A0. Limits: M1 for substituting limits 4 and 1 into a changed function, and subtracting the right way round.	
	Alternative:	
	$x=1$: $y=5$ and $x=4$: $y=3.5$	B1
	Equation of line: $y-5=-\frac{1}{2}(x-1)$ $y=\frac{11}{2}-\frac{1}{2}x$, subsequently used in integration with limits.	3rd M1
	$\left(\frac{11}{2}-\frac{1}{2}x\right)-\left(2x+\frac{8}{x^2}-5\right)$ (M: Subtract either way round)	4th M1
	$\int\left(\frac{21}{2}-\frac{5x}{2}-8x^{-2}\right)\mathrm{d}x = \frac{21x}{2}-\frac{5x^2}{4}-\frac{8x^{-1}}{-1}$	1st M1 A1ft A1ft
	(Penalise integration mistakes, not algebra for the ft marks)	
	$\left[\frac{21x}{2}-\frac{5x^2}{4}-\frac{8x^{-1}}{-1}\right]_1^4 = (42-20+2)-\left(\frac{21}{2}-\frac{5}{4}+8\right)$ (M: Right way round)	2nd M1
	Shaded area = 6.75	A1
	(The follow through marks are for the subtracted version, and again deduct an accuracy mark for a wrong term: One wrong M1 A1 A0; two wrong M1 A0 A0.)	
	Alternative for the last 2 marks in (b): M1: Show that $x=2$ is a minimum, using, e.g., 2nd derivative. A1: Conclusion showing understanding of "increasing", with accurate working.	

Question 3 : Jan 06 Q9

Question number	Scheme	Marks
	(a) $\frac{3}{2} = -2x^2 + 4x$	M1
	$4x^2 - 8x + 3 (= 0)$	A1
	$(2x-1)(2x-3) = 0$	M1
	$\underline{x = \frac{1}{2},\ \frac{3}{2}}$	A1 (4)
	(b) Area of $R = \int_{\frac{1}{2}}^{\frac{3}{2}} (-2x^2 + 4x)\ dx\ -\frac{3}{2}$ (for $-\frac{3}{2}$)	B1
	$\int (-2x^2 + 4x)\ dx = \left[-\frac{2}{3}x^3 + 2x^2\right]$ (Allow $\pm$[ ], accept $\frac{4}{2}x^2$)	M1 [A1]
	$\int_{\frac{1}{2}}^{\frac{3}{2}} (-2x^2 + 4x)\ dx = \left(-\frac{2}{3} \times \frac{3^3}{2^3} + 2 \times \frac{3^2}{2^2}\right) -, \left(-\frac{2}{3} \times \frac{1}{2^3} + 2 \times \frac{1}{2^2}\right)$	M1 M1
	$\left(= \frac{11}{6}\right)$	
	Area of $R = \frac{11}{6} - \frac{3}{2} = \underline{\frac{1}{3}}$ (Accept exact equivalent but not 0.33…)	A1cao (6)
		10

Question 4 : June 06 Q2

Question number	Scheme	Marks
	$\int (3x^2+5+4x^{-2})\,dx = \frac{3x^3}{3}+5x+\frac{4x^{-1}}{-1} \quad (= x^3+5x-4x^{-1})$	M1 A1 A1
	$[x^3+5x-4x^{-1}]_1^2 = (8+10-2)-(1+5-4), \quad = 14$	M1, A1 (5)
		5
	Integration: Accept any correct version, simplified or not. All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0. The given function must be integrated to score M1, and not e.g. $3x^4+5x^2+4$. Limits: M1: Substituting 2 and 1 into a 'changed function' and subtracting, either way round.	

Question 5 : Jan 07 Q1

Question Number	Scheme	Marks
(a)	$f'(x) = 3x^2+6x$	B1
	$f''(x) = 6x+6$	M1, A1cao (3)

Notes cao = correct answer only

1(a)	
Acceptable alternatives include $3x^2+6x^1$; $3x^2+3\times 2x$; $3x^2+6x+0$ Ignore LHS (e.g. use [whether correct or not] of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$) $3x^2+6x+c$ or $3x^2+6x+$ constant (i.e. the written word constant) is B0	B1
M1 Attempt to differentiate their $f'(x)$; $x^n \to x^{n-1}$. $x^n \to x^{n-1}$ seen in at least one of the terms. Coefficient of $x^{\cdots}$ ignored for the method mark. $x^2 \to x^1$ and $x \to x^0$ are acceptable.	M1
Acceptable alternatives include $6x^1+6x^0$; $3\times 2x+3\times 2$ $6x+6+c$ or $6x+6+$ constant is A0	A1 cao

Question 6 : June 06 Q10

Question number	Scheme	Marks
	(a) $\frac{dy}{dx} = 3x^2 - 16x + 20$	M1 A1
	$3x^2 - 16x + 20 = 0$ $(3x-10)(x-2) = 0$ $x = \ldots,$ $\frac{10}{3}$ and 2	dM1, A1 (4)
	(b) $\frac{d^2y}{dx^2} = 6x - 16$ At $x = 2$, $\frac{d^2y}{dx^2} = \ldots$	M1
	-4 (or < 0, or both), therefore maximum	A1ft (2)
	(c) $\int (x^3 - 8x^2 + 20x)dx = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{20x^2}{2}$ $(+C)$	M1 A1 A1 (3)
	(d) $4 - \frac{64}{3} + 40$ $\left(= \frac{68}{3}\right)$	M1
	A: $x = 2$: $y = 8 - 32 + 40 = 16$ (May be scored elsewhere)	B1
	Area of $\Delta = \frac{1}{2}\left(\frac{10}{3} - 2\right) \times 16$ $\left(\frac{1}{2}(x_B - x_A) \times y_A\right)$ $\left(= \frac{32}{3}\right)$	M1
	Shaded area $= \frac{68}{3} + \frac{32}{3} = \frac{100}{3}$ $\left(= 33\frac{1}{3}\right)$	M1 A1 (5)
		14
	(a) The second M is dependent on the first, and requires an attempt to solve a 3 term quadratic. (b) M1: Attempt second differentiation and substitution of one of the x values. A1ft: Requires correct second derivative and negative value of the second derivative, but ft from their x value. (c) All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0. (d) Limits M1: Substituting their lower x value into a 'changed' expression. Area of triangle M1: Fully correct method. Alternative for the triangle (finding an equation for the straight line then integrating) requires a fully correct method to score the M mark. Final M1: Fully correct method (beware valid alternatives!)	

Question 7 : Jan 07 Q7

Question Number	Scheme	Marks
	$y = x(x^2 - 6x + 5)$ $= x^3 - 6x^2 + 5x$	M1, A1
	$\int (x^3 - 6x^2 + 5x)\,\mathrm{d}x = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$	M1, A1ft
	$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_0^1 = \left(\frac{1}{4} - 2 + \frac{5}{2}\right) - 0 = \frac{3}{4}$	M1
	$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_1^2 = (4 - 16 + 10) - \frac{3}{4} = -\frac{11}{4}$	M1, A1(both)
	$\therefore$ total area $= \frac{3}{4} + \frac{11}{4}$	M1
	$= \frac{7}{2}$ o.e.	A1cso **(9)**

Question 8 : June 07 Q1

Question number	Scheme	Marks
	$\int x^{-\frac{1}{2}} \mathrm{d}x = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (Or equivalent, such as $2x^{\frac{1}{2}}$, or $2\sqrt{x}$)	M1 A1
	$\left[\dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_1^8 = 2\sqrt{8} - 2 = -2 + 4\sqrt{2}$ [or $4\sqrt{2} - 2$, or $2(2\sqrt{2} - 1)$, or $2(-1 + 2\sqrt{2})$]	M1 A1
		(4) **4**
	1st M: $x^{-\frac{1}{2}} \to kx^{\frac{1}{2}}$, $k \neq 0$. 2nd M: Substituting limits 8 and 1 into a 'changed' function (i.e. not $\dfrac{1}{\sqrt{x}}$ or $x^{-\frac{1}{2}}$), and subtracting, either way round. 2nd A: This final mark is still scored if $-2 + 4\sqrt{2}$ is reached via a decimal. N.B. Integration constant $+C$ may appear, e.g. $\left[\dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + C\right]_1^8 = (2\sqrt{8} + C) - (2 + C) = -2 + 4\sqrt{2}$ (Still full marks) But… a final answer such as $-2 + 4\sqrt{2} + C$ is A0. N.B. It will sometimes be necessary to 'ignore subsequent working' (isw) after a correct form is seen, e.g. $\int x^{-\frac{1}{2}} \mathrm{d}x = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (M1 A1), followed by incorrect simplification $\int x^{-\frac{1}{2}} \mathrm{d}x = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = \dfrac{1}{2}x^{\frac{1}{2}}$ (still M1 A1)…. The second M mark is still available for substituting 8 and 1 into $\dfrac{1}{2}x^{\frac{1}{2}}$ and subtracting.	

Question 9 : Jan 08 Q7

Question Number	Scheme	Marks
(a)	Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$)	B1 (1)
	or showing (6,0) (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing x = 6]	
(b)	Solving $2x = 6x - x^2$ $(x^2 = 4x)$ to x = ...	M1
	$x = 4$ (and $x = 0$)	A1
	Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,	A1 (3)
(c)	(Area =) $\int_{(0)}^{(4)}(6x - x^2)\,dx$ Limits not required	M1
	Correct integration $3x^2 - \frac{x^3}{3}$ (+ c)	A1
	Correct use of correct limits on their result above (see notes on limits)	M1
	["$3x^2 - \frac{x^3}{3}$"]^4 – ["$3x^2 - \frac{x^3}{3}$"]$_0$ with limits substituted [= 48 - $21\frac{1}{3} = 26\frac{2}{3}$]	
	Area of triangle = 2 × 8 =16 (Can be awarded even if no M scored, i.e. B1)	A1
	Shaded area = ± (area under curve – area of triangle) applied correctly	M1
	$(= 26\frac{2}{3} - 16)$ $= 10\frac{2}{3}$ (awrt 10.7)	A1 (6)**[10]**

. **Question 10 : June 08 Q8**

Question Number	Scheme	Marks
(*a*)	$\left(\frac{dy}{dx} =\right) 8 + 2x - 3x^2$	M1 A1
	$3x^2 - 2x - 8 = 0$ $(3x + 4)(x - 2) = 0$ $x = 2$	A1 cso (3)
(*b*)	Area of triangle = $\frac{1}{2} \times 2 \times 22$	M1 A1
	$\int 10 + 8x + x^2 - x^3\,dx = 10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$	M1 A1 A1
	$\left[10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 = ...\left(= 20 + 16 + \frac{8}{3} - 4\right)$	M1
	Area of $R = 34\frac{2}{3} - 22 = \frac{38}{3}\left(= 12\frac{2}{3}\right)$ (Or $12.\dot{6}$)	M1 A1 (8)
		(11 marks)

Question 11 : Jan 09 Q2

Question Number	Scheme	Marks
	$y=(1+x)(4-x)=4+3x-x^2$ M: Expand, giving 3 (or 4) terms	M1
	$\int(4+3x-x^2)dx = 4x+\frac{3x^2}{2}-\frac{x^3}{3}$ M: Attempt to integrate	M1 A1
	$=[\ldots\ldots\ldots\ldots]_{-1}^{4}=\left(16+24-\frac{64}{3}\right)-\left(-4+\frac{3}{2}+\frac{1}{3}\right)=\frac{125}{6}$ $\left(=20\frac{5}{6}\right)$	M1 A1 (5) [5]
Notes	**M1** needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. $1\times4=5$, but there needs to be a 'constant' an 'x term' and an 'x^2 term'. The x terms do not need to be collected. (Need not be seen if next line correct) Attempt to integrate means that $x^n\to x^{n+1}$ for at least one of the terms, then **M1** is awarded (even 4 becoming $4x$ is sufficient) – one correct power sufficient. **A1** is for correct answer only, not follow through. But allow $2x^2-\frac{1}{2}x^2$ or any correct equivalent. Allow + c, and even allow an evaluated extra constant term. **M1**: Substitute limit 4 and limit −1 into a changed function (must be −1) and indicate subtraction (either way round). **A1** must be exact, not 20.83 or similar. If recurring indicated can have the mark. Negative area, even if subsequently positive loses the A mark.	
Special cases	(i) Uses calculator method: **M1** for expansion (if seen) **M1** for limits if answer correct, so 0 , 1 or 2 marks out of 5 is possible (Most likely **M0 M0 A0 M1 A0**) (ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark **M1** gained. (iii) Using original method, but then change all signs after expansion is likely to lead to: **M1 M1 A0, M1 A0 i.e. 3/5**	

Question 12 : June 09 Q1

Question Number	Scheme	Marks
Q	$\int\left(2x+3x^{\frac{1}{2}}\right)\,dx=\frac{2x^2}{2}+\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$	M1 A1A1
	$\int_1^4\left(2x+3x^{\frac{1}{2}}\right)\,dx=\left[x^2+2x^{\frac{3}{2}}\right]_1^4=(16+2\times8)-(1+2)$	M1
	$=29$ (29 + C scores A0)	A1 (5)
		[5]

1[st] M1 for attempt to integrate $x\to kx^2$ or $x^{\frac{1}{2}}\to kx^{\frac{3}{2}}$.

1[st] A1 for $\frac{2x^2}{2}$ or a simplified version.

2[nd] A1 for $\frac{3x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ or $\frac{3x\sqrt{x}}{\left(\frac{3}{2}\right)}$ or a simplified version.

Ignore + C, if seen, but two correct terms and an extra non-constant term scores M1A1A0.

2[nd] M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation).

Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear.

No working:
The answer 29 with no working scores M0A0A0M1A0 (1 mark).

Question 13 : Jan 10 Q7

Question Number	Scheme	Marks
(a)	Puts $y=0$ and attempts to solve quadratic e.g. $(x-4)(x-1)=0$ Points are $(1,0)$ and $(4, 0)$	M1 A1 (2)
(b)	$x=5$ gives $y=25-25+4$ and so $(5, 4)$ lies on the curve	B1cso (1)
(c)	$\int (x^2-5x+4)\,dx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \quad (+c)$	M1A1 (2)
(d)	Area of triangle $= \frac{1}{2}\times 4\times 4 = 8$ or $\int (x-1)\,dx = \frac{1}{2}x^2 - x$ with limits 1 and 5 to give 8	B1
	Area under the curve $= \int_4^5 \qquad \frac{1}{3}\times 5^3 - \frac{5}{2}\times 5^2 + 4\times 5 \quad \left[= -\frac{5}{6}\right]$	M1
	$\frac{1}{3}\times 4^3 - \frac{5}{2}\times 4^2 + 4\times 4 \quad \left[= -\frac{8}{3}\right]$	M1
	$\int_4^5 = -\frac{5}{6} - -\frac{8}{3} = \frac{11}{6}$ or equivalent (allow 1.83 or 1.8 here)	A1 cao
	Area of $R = 8 - \frac{11}{6} = 6\frac{1}{6}$ or $\frac{37}{6}$ or $6.1\dot{6}$ (not 6.17)	A1 cao (5) [10]
(a)	M1 for attempt to find L and M A1 Accept $x=1$ and $x=4$, then isw or accept $L=(1,0)$, $M=(4,0)$ Do not accept $L=1$, $M=4$ nor (0 , 1), (0 , 4) (unless subsequent work) Do not need to distinguish L and M. Answers imply M1A1.	
(b)	See substitution, working should be shown, need conclusion which could be just $y=4$ or a tick. Allow $y=25-25+4=4$ But not $25-25+4=4$. ($y=4$ may appear at start) Usually $0=0$ or $4=4$ is B0	
(c)	M1 for attempt to integrate $x^2 \to kx^3, x \to kx^2$ or $4 \to 4x$ A1 for correct integration of all three terms (do not need constant) isw. Mark correct work when seen. So e.g. $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x$ is A1 then $2x^3 - 15x^2 + 24x$ would be ignored as subsequent work.	
(d)	B1 for this triangle only (not triangle LMN) 1st M1 for substituting 5 into their changed function 2nd M1 for substituting 4 into their changed function	
(d)	Alternative method: $\int_1^5 (x-1)-(x^2-5x+4)dx + \int_1^4 x^2-5x+4dx$ can lead to correct answer Constructs $\int_1^5 (x-1)-(x^2-5x+4)dx$ is B1 M1 for substituting 5 and 1 and subtracting in first integral M1 for substituting 4 and 1 and subtracting in second integral A1 for answer to first integral i.e. $\frac{32}{3}$ (allow 10.7) and A1 for final answer as before..	
(d)	Another alternative $\int_4^5 (x-1)-(x^2-5x+4)dx +$ *area of triangle LMP* Constructs $\int_4^5 (x-1)-(x^2-5x+4)dx$ is B1 M1 for substituting 5 and 4 and subtracting in first integral M1 for complete method to find area of triangle (4.5) A1 for answer to first integral i.e. $\frac{5}{3}$ and A1 for final answer as before.	
(d)	Could also use $\int_4^5 (4x-16)-(x^2-5x+4)dx +$ *area of triangle LMN* Similar scheme to previous one. Triangle has area 6 A1 for finding Integral has value $\frac{1}{6}$ and A1 for final answer as before.	

Question 14 : June 10 Q8

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required)	M1 A1
	At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*)	A1 cso
	N.B. The '= 0' must be seen at some stage to score the final mark.	
	Alternatively: (using $k = 28$) $\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1)	
	'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at $x = 2$ represents the maximum turning point.	(3)
	(b) $\int (x^3 - 10x^2 + 28x)\,dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$	M1 A1
	$\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = ...$ $\left(= 4 - \frac{80}{3} + 56 = \frac{100}{3}\right)$	M1
	(With limits 0 to 2, substitute the limit 2 into a 'changed function')	
	y-coordinate of $P = 8 - 40 + 56 = 24$ Allow if seen in part (a)	B1
	(The B1 for 24 may be scored by implication from later working)	
	Area of rectangle = $2 \times$ (their y-coordinate of P)	
	Area of R = (their 48) $-$ $\left(\text{their } \frac{100}{3}\right)$ $= \frac{44}{3}$ $\left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$	M1 A1
	If the subtraction is the 'wrong way round', the final A mark is lost.	(6) 9
	(a) M: $x^n \to cx^{n-1}$ (c constant, $c \neq 0$) for one term, seen in part (a). (b) 1st M: $x^n \to cx^{n+1}$ (c constant, $c \neq 0$) for one term. Integrating the gradient function loses this M mark. 2ndM: Requires use of limits 0 and 2, with 2 substituted into a 'changed function'. (It may, for example, have been differentiated). Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle. A1: Must be exact, not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$. Alternative: (effectively finding area of rectangle by integration) $\int \{24 - (x^3 - 10x^2 + 28x)\}\,dx = 24x - \left(\frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}\right)$, etc. This can be marked equivalently, with the 1st A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2nd M. If the subtraction is the 'wrong way round', the final A mark is lost.	

Question 15 : Jan 11 Q4

Question Number	Scheme		Marks
(a)	Seeing -1 and 5. (See note below.)		B1 **(1)**
(b)	$(x+1)(x-5) = \underline{x^2 - 4x - 5}$ or $\underline{x^2 - 5x + x - 5}$		B1
	$\int (x^2 - 4x - 5)\,dx = \frac{x^3}{3} - \frac{4x^2}{2} - 5x\ \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms correctly ft.	M1A1ft A1
	$\left[\frac{x^3}{3} - \frac{4x^2}{2} - 5x\right]_{-1}^{5} = (\ldots\ldots) - (\ldots\ldots)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \left(\frac{125}{3} - \frac{100}{2} - 25\right) - \left(-\frac{1}{3} - 2 + 5\right) = \left(-\frac{100}{3}\right) - \left(\frac{8}{3}\right) = -36 \right\}$		
	Hence, Area $= 36$	Final answer must be 36, not -36	A1 **(6)** **[7]**

Notes

(a) B1: for -1 and 5. Note that $(-1, 0)$ and $(5, 0)$ are acceptable for B1. Also allow $(0, -1)$ and $(0, 5)$ generously for B1. Note that if a candidate writes down that $A: (5,0)$, $B: (-1,0)$, (ie A and B interchanged,) then B0. Also allow values inserted in the correct position on the x-axis of the graph.

(b) B1 for $x^2 - 4x - 5$ or $x^2 - 5x + x - 5$. If you believe that the candidate is applying the Way 2 method then $-x^2 + 4x + 5$ or $-x^2 + 5x - x + 5$ would then be fine for B1.
1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms.
Note that $-5 \to 5x$ is sufficient for M1.
1st A1 at least two out of three terms correctly ft from their multiplied out brackets.
2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'.
Allow 2nd A1 also for $\frac{x^3}{3} - \frac{5x^2}{2} + \frac{x^2}{2} - 5x$. Note that $-\frac{5x^2}{2} + \frac{x^2}{2}$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark.
2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -1 (and not 1 if the candidate has stated $x = -1$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round.
3rd A1: For a final answer of 36, not -36.

Note: An alternative method exists where the candidate states from the outset that Area $(R) = -\int_{-1}^{5}(x^2 - 4x + 5)\,dx$ is detailed in the Appendix.

Question Number	Scheme		Marks
Aliter **(b)** Way 2	$(x+1)(x-5) = \underline{x^2 - 4x - 5}$ or $\underline{x^2 - 5x + x - 5}$	Can be implied by later working.	B1
	$-\int (x^2 - 4x - 5)\,dx = -\frac{x^3}{3} + \frac{4x^2}{2} + 5x\ \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 any two out of three terms correctly ft.	M1A1ft A1
	$\left[-\frac{x^3}{3} + \frac{4x^2}{2} + 5x\right]_{-1}^{5} = (\ldots\ldots) - (\ldots\ldots)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \left(-\frac{125}{3} + \frac{100}{2} + 25\right) - \left(\frac{1}{3} + 2 - 5\right) = \left(\frac{100}{3}\right) - \left(-\frac{8}{3}\right) \right\}$		
	Hence, Area $= 36$		A1 **(6)**

Question 16 : June 11 Q9

Question Number	Scheme		Marks
(a)	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	$\{\text{Curve} = \text{Line}\} \Rightarrow -x^2 + 2x + 24 = x + 4$	Eliminating y correctly.	B1
	$x^2 - x - 20\,\{= 0\} \Rightarrow (x-5)(x+4)\,\{= 0\} \Rightarrow x = ...$	Attempt to solve a ***resulting*** quadratic to give $x =$ their values.	M1
	So, $x = 5, -4$	Both $x = 5$ and $x = -4$.	A1
	So corresponding y-values are $y = 9$ and $y = 0$.	See notes below.	B1ft [4]
(b)	$\left\{\int(-x^2 + 2x + 24)\,dx\right\} = \underline{-\frac{x^3}{3} + \frac{2x^2}{2} + 24x}\ \{+c$	M1: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms. 2nd A1 for <u>correct answer</u>.	M1A1A1
	$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{5} = (......) - (......)$	Substitutes 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + 25 + 120\right) - \left(\frac{64}{3} + 16 - 96\right) = \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162\right\}$		
	Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$	Uses correct method for finding area of triangle.	M1
	So area of R is $162 - 40.5 = 121.5$	Area under curve – Area of triangle.	M1
		121.5	A1 oe cao
			[7]
			11

Question Number	Scheme	Marks
(a)	1st B1: For correctly eliminating either x or y. Candidates will usually write $-x^2 + 2x + 24 = x + 4$. This mark can be implied by the resulting quadratic. M1: For solving their quadratic (which must be different to $-x^2 + 2x + 24$) to give $x = ...$ See introduction for Method mark for solving a 3TQ. It must result from some attempt to eliminate one of the variables. A1: For both $x = 5$ and $x = -4$. 2nd B1ft: For correctly substituting their values of x in equation of line or parabola to give ***both correct ft*** y-values. (You may have to get your calculators out if they substitute their x into $y = -x^2 + 2x + 24$). <u>Note:</u> For $x = 5, -4 \Rightarrow y = 9$ and $y = 0 \Rightarrow$ eg. $(-4, 9)$ and $(5, 0)$, award B1 isw. If the candidate gives additional answers to $(-4, 0)$ and $(5, 9)$, then withhold the final B1 mark. <u>Special Case</u>: Award SC: B0M0A0B1 for $\{A\}(-4, 0)$. You may see this point marked on the diagram. <u>Note:</u> SC: B0M0A0B1 for solving $0 = -x^2 + 2x + 24$ to give $\{A\}(-4, 0)$ and/or $(6, 10)$. Note: Do not give marks for working in part (b) which would be creditable in part (a).	
(b)	1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $24 \to 24x$ is sufficient for M1. 1st A1 at least two out of three terms correctly integrated. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Area of triangle $= \frac{1}{2}(\text{their } x_2 - \text{their } x_1)(\text{their } y_2)$ or Area of triangle $= \int_{x_1}^{x_2} x + 4\,\{dx\}$. Where $x_1 =$ their -4, $x_2 =$ their 5 and $y_2 =$ their y usually found in part (a). 4th M1: Area under curve – Area under triangle, where both Area under curve > 0 and Area under triangle > 0 and Area under curve > Area under triangle. 3rd A1: 121.5 or $\frac{243}{2}$ oe **cao.**	

Question Number	Scheme	Marks
Aliter (b) **Way 2**	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$ 3rd M1: Uses integral of $(x + 4)$ with correct ft limits. Area of $R = \int_{-4}^{5} (-x^2 + 2x + 24) - (x + 4)\,dx$ 4th M1: Uses "curve" – "line" function with correct ft limits.	
	$= -\frac{x^3}{3} + \frac{x^2}{2} + 20x \{+ c\}$ M: $x^n \to x^{n+1}$ for any one term. A1 at least two out of three terms Correct answer (Ignore $+ c$).	M1 A1ft A1
	$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x\right]_{-4}^{5} = (......) - (......)$ Substitutes 5 and -4 (or *their limits* from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + \frac{25}{2} + 100\right) - \left(\frac{64}{3} + 8 - 80\right) = \left(70\frac{5}{6}\right) - \left(-50\frac{2}{3}\right)\right\}$	
	See above working to decide to award 3rd M1 mark here: ***See above working to decide to award 4th M1 mark here:***	M1 M1
	So area of R is $= 121.5$ 121.5	A1 oe **cao** [7] **11**
(b)	1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $20 \to 20x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft. Note this accuracy mark is ft in Way 2. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. Allow 2nd A1 also for $-\frac{x^3}{3} + \frac{2x^2}{2} + 24x - \left(\frac{x^2}{2} + 4x\right)$. Note that $\frac{2x^2}{2} - \frac{x^2}{2}$ or $24x - 4x$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Uses the integral of $(x + 4)$ with correct ft limits of their x_1 and their x_2 (usually found in part (a)) {where $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (5, 9)$.} This mark is usually found in the first line of the candidate's working in part (b). 4th M1: Uses "curve" – "line" function with correct ft (usually found in part (a)) limits. Subtraction must be correct way round. This mark is usually found in the first line of the candidate's working in part (b). Allow $\int_{-4}^{5} (-x^2 + 2x + 24) - x + 4\,\{dx\}$ for this method mark. 3rd A1: 121.5 oe **cao.** **Note: SPECIAL CASE for this alternative method** Area of $R = \int_{-4}^{5} (x^2 - x - 20)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 20x\right]_{-4}^{5} = \left(\frac{125}{3} - \frac{25}{2} - 100\right) - \left(-\frac{64}{3} - 8 + 80\right)$ The working so far would score SPEICAL CASE M1A1A1M1M1M0A0. The candidate may then go on to state that $= \left(-70\frac{5}{6}\right) - \left(50\frac{2}{3}\right) = -\frac{243}{2}$ If the candidate then multiplies their answer by -1 then they would gain the 4th M1 and 121.5 would gain the final A1 mark.	

Question Number	Scheme		Marks
Aliter (a) **Way 2**	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$ $\{\text{Curve} = \text{Line}\} \Rightarrow y = -(y-4)^2 + 2(y-4) + 24$ $y^2 - 9y \{= 0\} \Rightarrow y(y-9) \{= 0\} \Rightarrow y = ...$ So, $y = 0, 9$ So corresponding y-values are $x = -4$ and $x = 5$.	Eliminating x correctly. Attempt to solve a resulting quadratic to give $y =$ their values. Both $y = 0$ and $y = 9$. See notes below.	B1 M1 A1 B1ft **[4]**
	2nd B1ft: For correctly substituting their values of y in equation of line or parabola to give ***both correct ft*** x-values.		
(b)	**<u>Alternative Methods for obtaining the M1 mark for use of limits:</u>** There are two alternative methods can candidates can apply for finding "162". <u>Alternative 1:</u> $\int_{-4}^{0}(-x^2 + 2x + 24)\,dx + \int_{0}^{5}(-x^2 + 2x + 24)\,dx$ $= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{0} + \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{0}^{5}$ $= (0) - \left(\frac{64}{3} + 16 - 96\right) + \left(-\frac{125}{3} + 25 + 120\right) - (0)$ $= \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162$ <u>Alternative 2:</u> $\int_{-4}^{6}(-x^2 + 2x + 24)\,dx - \int_{5}^{6}(-x^2 + 2x + 24)\,dx$ $= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{6} - \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{5}^{6}$ $= \left\{\left(-\frac{216}{3} + 36 + 144\right) - \left(\frac{64}{3} + 16 - 96\right)\right\} - \left\{\left(-\frac{216}{3} + 36 + 144\right) - \left(-\frac{125}{3} + 25 + 120\right)\right\}$ $= \left\{(108) - \left(-58\frac{2}{3}\right)\right\} - \left\{(108) - \left(103\frac{1}{3}\right)\right\}$ $= \left(166\frac{2}{3}\right) - \left(4\frac{2}{3}\right) = 162$		