Question 1 : Jan 05 Q9

Question Number	Scheme	Marks
	(a) Perimeter $\Rightarrow 2x+2y+\pi x=80$	B1
	Area $\rightarrow A=2xy+\frac{1}{2}\pi x^2$	B1
	$y=\frac{80-2x-\pi x}{2}$ and sub in to A	M1
	$\Rightarrow A=80x-2x^2-\pi x^2+\frac{1}{2}\pi x^2$	
	i.e. $A=80x-(2+\frac{\pi}{2})x^2$ *	A1 c.s.o (4)
	(b) $\frac{dA}{dx}=80-2(2+\frac{\pi}{2})x$	M1, A1
	$\frac{dA}{dx}=0\Rightarrow 40=(2+\frac{\pi}{2})x$ so $x=,\frac{40}{2+\frac{\pi}{2}}$ or $\frac{80}{4+\pi}$ or Awrt 11.2	M1, A1 (4)
	(c) $\frac{d^2A}{dx^2}=-4-\pi$	M1
	<0 $\therefore A$ is Max	A1 (2)
	(d) Max Area $=80(b)-(2+\frac{\pi}{2})(b)^2$	M1
	$=\underline{\underline{448(m^2)}}$	A1 cao (2)
		(12)
	(b) 2^{nd} M1 for putting $\frac{dA}{dx}=0$ and attempting $x=\cdots$ (c) M1 for attempting $\frac{d^2A}{dx^2}$ (or equivalent method) A1 for a correct second derivative, <0 and comment	

Question 2 : June 05 Q1

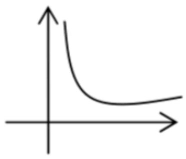
Question number	Scheme	Marks
	$\frac{dy}{dx} = 4x - 12$	B1
	$4x - 12 = 0 \qquad x = 3$	M1 A1ft
	$y = -18$	A1 (4)
		4
	M1: Equate $\frac{dy}{dx}$ (not just y) to zero and proceed to $x = \ldots$ A1ft: Follow through only from a linear equation in x. <u>Alternative:</u> $y = 2x(x-6) \Rightarrow$ Curve crosses x-axis at 0 and 6 — B1 (By symmetry) $x = 3$ — M1 A1ft $y = -18$ — A1 <u>Alternative:</u> $(x-3)^2$ — B1 for $(x-3)^2$ seen somewhere $y = 2(x^2 - 6x) = 2\{(x-3)^2 - 9\}$ $\quad x = 3$ M1 for attempt to complete square and deduce $x = \ldots$ A1ft $[(x-a)^2 \Rightarrow x = a]$ $y = -18$ — A1	

Question 3 : Jan 06 Q7

Question number	Scheme	Marks
	(a) $\frac{dy}{dx} = 6x^2 - 10x - 4$	M1 A1 (2)
	(b) $6x^2 - 10x - 4 = 0$	M1
	$2(3x + 1)(x - 2)$ [=0]	M1
	$x = 2$ or $-\frac{1}{3}$ (both x values)	A1
	Points are (2, - 10) and ($-\frac{1}{3}$, $2\frac{19}{27}$ or $\frac{73}{27}$ or 2.70 or better) (both y values)	A1 (4)
	(c) $\frac{d^2y}{dx^2} = 12x - 10$	M1 A1 (2)
	(d) $x = 2 \Rightarrow \frac{d^2y}{dx^2}(=14) \underline{> 0} \therefore$ [(2, - 10)] is a <u>Min</u>	M1
	$x = -\frac{1}{3} \Rightarrow \frac{d^2y}{dx^2}(= -14) \underline{< 0} \therefore$ [$(-\frac{1}{3}, \frac{73}{27})$] is a <u>Max</u>	A1 (2)
		10
	(a) M1 for some correct attempt to differentiate $x^n \to x^{n-1}$	
	(b) 1st M1 for setting their $\frac{dy}{dx} = 0$	
	2nd M1 for attempting to solve 3TQ but it must be based on their $\frac{dy}{dx}$.	
	NO marks for answers only in part (b)	
	(c) M1 for attempting to differentiate their $\frac{dy}{dx}$	
	(d) M1 for one correct use of their second derivative or a full method to determine the nature of one of their stationary points	
	A1 both correct (=14 and = - 14) are not required	

Question 4 : Jan 07 Q8

8(a) Trial and improvement $\quad f(v) = \frac{1400}{v} + \frac{2v}{7}$	
Attempts to evaluate f(v) for 3 values a, b, c where (i) $a < 70$, $b = 70$ and $c > 70$ or (ii) a, $b < 70$ and $c > 70$ or (iii) $a < 70$ and b, $c > 70$.	M1
All 3 correct and states $v = 70$ (exact)	A1
Then 2nd M0, 3rd M0, 2nd A0.	

8(a) Graph	
 Correct shape (ignore anything drawn for $v < 0$).	M1
$v = 70$ (exact)	A1
Then 2nd M0, 3rd M0, 2nd A0.	

8(b)	
Attempt to differentiate their $\frac{dC}{dv}$; $v^n \to v^{n-1}$ (including $v^0 \to 0$).	M1
$\frac{d^2C}{dv^2}$ must be correct. Ft only from their value of v and provided their value of v is +ve. Must be some (minimal) indication that their value of v is being used. Statement: "When v = their value of v, $\frac{d^2C}{dv^2} > 0$" is sufficient provided $2800v^{-3} > 0$ for their value of v. If substitution of their v seen: correct substitution of their v into $2800v^{-3}$, but, provided evaluation is +ve, ignore incorrect evaluation. N.B. Parts in mark scheme in { } do not need to be seen.	A1ft

8(c)	
Substitute their value of v that they think will give C_{min} (independent of the method of obtaining this value of v and independent of which part of the question it comes from).	M1
40 or £40 Must have part (a) completely correct (i.e. all 5 marks) to gain this A1.	A1
Answer only gains M1A1 provided part (a) is completely correct..	

Question 5 : June 07 Q10

Question number	Scheme	Marks
	(a) $4x^2 + 6xy = 600$	M1 A1
	$V = 2x^2y = 2x^2\left(\frac{600-4x^2}{6x}\right)$ $\quad V = 200x - \frac{4x^3}{3}$ $\quad$ (*)	M1 A1cso (4)
	(b) $\frac{dV}{dx} = 200 - 4x^2$	B1
	Equate their $\frac{dV}{dx}$ to 0 and solve for x^2 or x : $x^2 = 50$ or $x = \sqrt{50}$ (7.07…)	M1 A1
	Evaluate V: $\quad V = 200(\sqrt{50}) - \frac{4}{3}(50\sqrt{50}) = 943$ cm^3 $\quad$ Allow awrt	M1 A1 (5)
	(c) $\frac{d^2V}{dx^2} = -8x$ Negative, $\quad \therefore$ Maximum	M1, A1ft (2)
		11
	(a) 1st M: Attempting an expression in terms of x and y for the total surface area (the expression should be dimensionally correct). 1st A: Correct expression (not necessarily simplified), equated to 600. 2nd M: Substituting their y into $2x^2y$ to form an expression in terms of x only. (Or substituting y from $2x^2y$ into their area equation). (b) 1st A: Ignore $x = -\sqrt{50}$, if seen. The 2nd M mark (for substituting their x value into the given expression for V) is dependent on the 1st M. Final A: Allow also exact value $\frac{400\sqrt{50}}{3}$ or $\frac{2000\sqrt{2}}{3}$ or equiv. <u>single term</u>. (c) Allow marks if the work for (c) is seen in (b) (or vice-versa). M: Find second derivative <u>and consider its sign</u>. A: Second derivative following through correctly from their $\frac{dV}{dx}$, and correct reason/conclusion (it must be a maximum, not a minimum). An actual value of x does not have to be used… this mark can still be awarded if no x value has been found or if a wrong x value is used. <u>Alternative</u>: M: Find <u>value</u> of $\frac{dV}{dx}$ on each side of "$x = \sqrt{50}$" and consider sign. A: Indicate sign change of positive to negative for $\frac{dV}{dx}$, and conclude max. <u>Alternative</u>: M: Find <u>value</u> of V on each side of "$x = \sqrt{50}$" and compare with "943". A: Indicate that both values are less than 943, and conclude max.	

Question 6 : Jan 08 Q9

Question Number	Scheme	Marks
(a)	(Total area) = $3xy + 2x^2$	B1
	(Vol:) $x^2y = 100$ $\quad (y = \frac{100}{x^2}, xy = \frac{100}{x})$	B1
(b)	Deriving expression for area in terms of x only	M1
	(Substitution, or clear use of, y or xy into expression for area)	
	(Area =) $\frac{300}{x} + 2x^2$ **AG**	A1 cso (4)
(c)	$\frac{dA}{dx} = -\frac{300}{x^2} + 4x$	M1A1
	Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1	
	[$x^3 = 75$]	
	$x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$)	A1 (4)
	$\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4$ = positive , > 0; therefore minimum	M1;A1 (2)
(d)	Substituting found value of x into (a)	M1
	(Or finding y for found x and substituting both in $3xy + 2x^2$)	
	[$y = \frac{100}{4.2172^2} = 5.6228$]	
	Area = 106.707 awrt 107	A1 (2)
Notes	(a) First B1: Earned for correct unsimplified expression, isw. (b) First M1: At least one power of x decreased by 1, and no "c" term. (c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or "positive" A1: Candidate's $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion "so minimum", (allow QED, √). (may be wrong x, or even no value of x found) Alternative: M1: Find value of $\frac{dA}{dx}$ on either side of "$x = \sqrt[3]{75}$" and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum. OR M1: Consider values of A on either side of "$x = \sqrt[3]{75}$" and compare with"107" A1: Both values greater than "$x = 107$" and conclude minimum. Allow marks for (c) and (d) where seen; even if part labelling confused. Throughout , allow confused notation, such as dy/dx for dA/dx.	

Question 7 : Jan 09 Q10

Question Number	Scheme	Marks
(a)	$2\pi rh + 2\pi r^2 = 800$	B1
	$h = \dfrac{400 - \pi r^2}{\pi r}$, $\quad V = \pi r^2\left(\dfrac{400 - \pi r^2}{\pi r}\right) = 400r - \pi r^3$ (*)	M1, M1 A1 (4)
(b)	$\dfrac{dV}{dr} = 400 - 3\pi r^2$	M1 A1
	$400 - 3\pi \ \ r^2 = 0 \quad r^2 = ..., \quad r = \sqrt{\dfrac{400}{3\pi}} \quad (= 6.5\ (2\text{ s.f.}))$	M1 A1
	$V = 400r - \pi r^3 = 1737 = \dfrac{800}{3}\sqrt{\dfrac{400}{3\pi}}\ (\text{cm}^3)$ (accept awrt 1737 or exact answer)	M1 A1 (6)
(c)	$\dfrac{d^2V}{dr^2} = -6\pi r$, Negative, $\therefore$ maximum (Parts (b) and (c) should be considered together when marking)	M1 A1 (2) [12]
Other methods for part (c):	Either: M: Find value of $\dfrac{dV}{dr}$ on each side of "$r = \sqrt{\dfrac{400}{3\pi}}$" and consider sign. A: Indicate sign change of positive to negative for $\dfrac{dV}{dr}$, and conclude max. Or: M: Find value of V on each side of "$r = \sqrt{\dfrac{400}{3\pi}}$" and compare with "1737". A: Indicate that both values are less than 1737 or 1737.25, and conclude max.	
Notes (a)	**B1**: For any correct form of this equation (may be unsimplified, may be implied by 1st M1) **M1** : Making h the subject of their three or four term formula **M1**: Substituting expression for h into $\pi r^2 h$ (independent mark) Must now be expression in r only. **A1**: cso	
(b)	**M1**: At least one power of r decreased by 1 **A1**: cao **M1**: Setting $\dfrac{dV}{dr}$ =0 and finding a value for correct power of r for candidate **A1** : This mark may be credited if the value of V is correct. Otherwise answers should round to 6.5 (allow ± 6.5)or be exact answer **M1**: Substitute a positive value of r to give V **A1**: 1737 or 1737.25..... or exact answer	

(c)	**M1**: needs complete method e.g.attempts differentiation (power reduced) of their first derivative and considers its sign **A1(first method)** should be $-6\pi r$ (do not need to substitute r and can condone wrong r if found in (b)) Need to conclude maximum or indicate by a tick that it is maximum. Throughout allow confused notation such as dy/dx for dV/dr	
Alternative for (a)	$A = 2\pi r^2 + 2\pi rh$, $\frac{A}{2} \times r = \pi r^3 + \pi r^2 h$ is **M1** Equate to $400r$ **B1** Then $V = 400r - \pi r^3$ is **M1 A1**	

Question 8 : Jan 10 Q9

Question Number	Scheme	Marks
(a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10\right]$	
	$[y' =] \quad 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$	M1 A1
	Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$	M1
	So $x = \quad ,\frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)	M1, A1
	$x = 4, \Rightarrow y = 12 \times 2 - 4^{\frac{3}{2}} - 10$, so $y = 6$	dM1,A1 (7)
(b)	$y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since $x > 0$] It is a maximum	B1 (1) [10]
(a)	1st M1 for an attempt to differentiate a fractional power $x^n \to x^{n-1}$ A1 a.e.f – can be unsimplified 2nd M1 for forming a suitable equation using their $y'=0$ 3rd M1 for correct processing of fractional powers leading to $x = \ldots$ (Can be implied by $x = 4$) A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only. 4th M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1	
(b)	M1 for differentiating their y' again A1 should be simplified	
(c)	B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier). (Treat parts (a),(b) and (c) together for award of marks)	

Question 9 : June 10 Q3

Question Number	Scheme	Marks
	(a) $\left(\frac{dy}{dx}=\right)\ 2x-\frac{1}{2}kx^{-\frac{1}{2}}$ (Having an extra term, e.g. $+C$, is A0)	M1 A1 (2)
	(b) Substituting $x = 4$ into their $\frac{dy}{dx}$ and 'compare with zero' (The mark is allowed for : $<, >, =, \leq, \geq$)	M1
	$8-\frac{k}{4}<0$ $k>32$ (or $32<k$) Correct inequality needed	A1 (2) 4
	(a) M: $x^2 \to cx$ or $k\sqrt{x} \to cx^{-\frac{1}{2}}$ (c constant, $c \neq 0$) (b) Substitution of $x = 4$ into y scores M0. However, $\frac{dy}{dx}$ is sometimes called y, and in this case the M mark can be given. $\frac{dy}{dx}=0$ may be 'implied' for M1, when, for example, a value of k or an inequality solution for k is found. Working must be seen to justify marks in (b), i.e. $k > 32$ alone is M0 A0.	

Question 10 : June 10 Q8

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required)	M1 A1
	At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*)	A1 cso
	N.B. The '= 0' must be seen at some stage to score the final mark.	
	Alternatively: (using $k = 28$)	
	$\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1)	
	'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at $x = 2$ represents the maximum turning point.	(3)

Question 11 : Jan 11 Q10

Question Number	Scheme		Marks
(a)	$V = 4x(5-x)^2 = 4x(25-10x+x^2)$		
	So, $V = 100x - 40x^2 + 4x^3$	$\pm\alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$	M1
		$V = 100x - 40x^2 + 4x^3$	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$	At least two of their expanded terms differentiated correctly.	M1
		$100 - 80x + 12x^2$	A1 cao
			(4)
(b)	$100 - 80x + 12x^2 = 0$	Sets their $\frac{dV}{dx}$ from part (a) $= 0$	M1
	$\{\Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x-5)(x-5) = 0\}$		
	$\{\text{As } 0 < x < 5\}$ $x = \frac{5}{3}$	$x = \frac{5}{3}$ or $x =$ awrt 1.67	A1
	$x = \frac{5}{3},\ V = 4(\frac{5}{3})(5 - \frac{5}{3})^2$	Substitute candidate's value of x **where** $0 < x < 5$ into a formula for V.	dM1
	So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074...$	Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	A1
			(4)
(c)	$\frac{d^2V}{dx^2} = -80 + 24x$	Differentiates their $\frac{dV}{dx}$ **correctly** to give $\frac{d^2V}{dx^2}$.	M1
	When $x = \frac{5}{3}$, $\frac{d^2V}{dx^2} = -80 + 24\left(\frac{5}{3}\right)$		
	$\frac{d^2V}{dx^2} = -40 < 0 \Rightarrow V$ is a maximum	$\underline{\frac{d^2V}{dx^2} = -40}$ and $\underline{< 0 \text{ or negative}}$ and $\underline{\text{maximum}}$.	A1 cso
			(2) [10]
	Notes		
(a)	1st M1 for a three term cubic in the form $\pm\alpha x \pm \beta x^2 \pm \gamma x^3$. Note that an un-combined $\pm\alpha x \pm \lambda x^2 \pm \mu x^2 \pm \gamma x^3$, $\alpha, \lambda, \mu, \gamma \neq 0$ is fine for the 1st M1. 1st A1 for either $100x - 40x^2 + 4x^3$ or $100x - 20x^2 - 20x^2 + 4x^3$. 2nd M1 for any two of their expanded terms differentiated correctly. NB: If expanded expression is divided by a constant, then the 2nd M1 can be awarded for at least two terms are correct. Note for un-combined $\pm\lambda x^2 \pm \mu x^2$, $\pm 2\lambda x \pm 2\mu x$ counts as one term differentiated correctly. 2nd A1 for $100 - 80x + 12x^2$, **cao**. **Note:** See appendix for those candidates who apply the product rule of differentiation.		

Question 12 : June 11 Q8

Question Number	Scheme		Marks
(a)	$\{V=\}\ 2x^2y=81$	$2x^2y=81$	B1 oe
	$\{L = 2(2x+x+2x+x)+4y \Rightarrow L = 12x+4y\}$		
	$y=\frac{81}{2x^2} \Rightarrow L=12x+4\left(\frac{81}{2x^2}\right)$	Making y the subject of their expression and substitute this into the correct L formula.	M1
	So, $L=12x+\frac{162}{x^2}$ **AG**	Correct solution only. **AG.**	A1 cso
			[3]
(b)	$\frac{dL}{dx}=12-\frac{324}{x^3}\ \{=12-324x^{-3}\}$	Either $12x\to 12$ or $\frac{162}{x^2}\to\frac{\pm\lambda}{x^3}$	M1
		Correct differentiation (need not be simplified).	A1 aef
	$\left\{\frac{dL}{dx}=\right\}12-\frac{324}{x^3}=0 \Rightarrow x^3=\frac{324}{12};=27 \Rightarrow x=3$	$L'=0$ and "their $x^3=\pm$ value" or "their $x^{-3}=\pm$ value"	M1;
		$x=\sqrt[3]{27}$ or $x=3$	A1 cso
	$\{x=3,\}\ L=12(3)+\frac{162}{3^2}=54$ (cm)	Substitute candidate's value of $x(\neq 0)$ into a formula for L.	ddM1
		54	A1 cao
			[6]
(c)	$\{\text{For } x=3\},\ \frac{d^2L}{dx^2}=\frac{972}{x^4}>0 \Rightarrow$ Minimum	Correct ft L'' and considering sign.	M1
		$\frac{972}{x^4}$ and >0 and conclusion.	A1 [2]
			11

(a)	B1: For any correct form of $2x^2y=81$. (may be unsimplified). Note that $2x^3=81$ is B0. Otherwise, candidates can use any symbol or letter in place of y. M1: Making y the subject of their formula and substituting this into a correct expression for L. A1: Correct solution only. Note that the answer is given.
(b)	**Note you can mark parts (b) and (c) together.** 2nd M1: Setting their $\frac{dL}{dx}=0$ and "candidate's ft *correct* power of x = a value". The power of x must be consistent with their differentiation. If inequalities are used this mark cannot be gained until candidate states value of x or L from their x without inequalities. $L'=0$ can be implied by $12=\frac{324}{x^3}$. 2nd A1: $x^3=27 \Rightarrow x=\pm 3$ scores A0. 2nd A1: can be given for no value of x given but followed through by correct working leading to $L=54$. 3rd M1: Note that this method mark is dependent upon the two previous method marks being awarded.
(c)	M1: for attempting correct ft second derivative and considering its sign. A1: Correct second derivative of $\frac{972}{x^4}$ (need not be simplified) and a valid reason (e.g. >0), and conclusion. Need to conclude minimum (allow x and not L is a minimum) or indicate by a tick that it is a minimum. The actual value of the second derivative, if found, can be ignored, although substituting their L and not x into L'' is A0. **Note:** 2 marks can be scored from a wrong value of x, no value of x found or from not substituting in the value of their x into L''. Gradient test or testing values either side of their x scores M0A0 in part (c). **Throughout this question allow confused notation such as $\frac{dy}{dx}$ for $\frac{dL}{dx}$.**