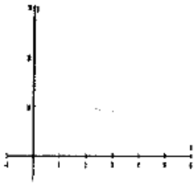
Quadratic Functions - Edexcel Past Exam Questions MARK SCHEME

Question 1: Jan 05 Q3

Question number	Scheme	Marks
	Attempt to use discriminant $b^2 - 4ac$ (Need not be equated to zero)	M1
	$144 - 4 \times k \times k = 0$	A1
	Attempt to solve for k	M1
	$k = 6$	A1 (4)
		4
	<u>Alternative for first 2 marks</u> Attempt to complete square $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$ M1 $1 - \frac{36}{k^2} = 0$ or equiv. A1 <u>Other alternatives</u> (i) $x^2 + \frac{12}{k}x + 1$ must be equivalent to $(x+1)^2$ M1 A1 Compare coefficients and attempt to solve for k: $\frac{12}{k} = 2$ $k = 6$ M1 A1 (ii) Finding the root first, e.g. $(\sqrt{k}x + \sqrt{k})^2 = 0$, so $x = -1$ M1 A1 Substitute the root to find k, $k = 6$ M1 A1 <u>Answer only</u> Scores 2 marks: M0 A0 M1 A1 The first two marks would only be scored if solution then justifies that $k = 6$ gives equal roots.	

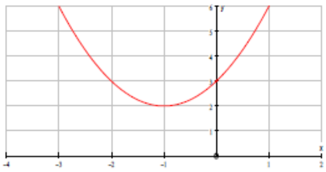
Question 2: Jan 05 Q10

Question number	Scheme	Marks
	(a) $x^2 - 6x + 18 = (x-3)^2,\ +9$	B1, M1 A1 (3)
	(b)  "U"-shaped parabola	M1
	Vertex in correct quadrant	A1ft
	P: (0, 18) (or 18 on y-axis)	B1
	Q: (3, 9)	B1ft (4)
	(c) $x^2 - 6x + 18 = 41$ or $(x-3)^2 + 9 = 41$	M1
	Attempt to solve 3 term quadratic $x = \ldots$	M1
	$x = \dfrac{6 \pm \sqrt{36 - (4 \times -23)}}{2}$ (or equiv.)	A1
	$\sqrt{128} = \sqrt{64} \times \sqrt{2}$ (or equiv. surd manipulation)	M1
	$3 + 4\sqrt{2}$ (Ignore other value)	A1 (5)
		12
	(a) M1 requires $(x \pm a)^2 \pm b \pm 18,\ a \neq 0,\ b \neq 0$ Answer only: full marks.	

Question 3: June 05 Q3

Question Number	Scheme	Marks
(a)	$x^2 - 8x - 29 \equiv (x-4)^2 - 45$ $\quad (x \pm 4)^2$ $(x-4)^2 - 16 + (-29)$ $(x \pm 4)^2 - 45$	M1 A1 A1 (3)
ALT	Compare coefficients $\quad -8 = 2a$ $\quad$ equation for a $a = -4$ <u>AND</u> $a^2 + b = -29$ $b = -45$	M1 A1 A1 (3)
(b)	$(x-4)^2 = 45$ $\quad$ (follow through their a and b from (a)) $\Rightarrow x - 4 = \pm\sqrt{45}$ $\quad c = 4$ $x = 4 \pm 3\sqrt{5}$ $\quad d = 3$	M1 A1 A1 (3) **(6)**
(a) (b)	M1 for $(x \pm 4)^2$ or an equation for a. M1 for a full method leading to $x - 4 = \ldots$ or $x = \ldots$ A1 for c and A1 for d <u>Note</u> Use of formula that ends with $\frac{8 \pm 6\sqrt{5}}{2}$ scores M1 A1 A0 (but must be $\sqrt{5}$) i.e. only penalise non-integers by one mark.	

Question 4: Jan 06 Q10

Question number	Scheme		Marks	
	(a) $x^2+2x+3=(x+1)^2,\ +2$	$(a=1,\quad b=2)$	B1, B1	(2)
	(b) 	"U"-shaped parabola	M1	
		Vertex in correct quadrant (ft from $(-a, b)$)	A1ft	
		(0, 3) (or 3 on y-axis)	B1	(3)
	(c) $b^2-4ac=4-12=-8$		B1	
	Negative, so curve does not cross x-axis		B1	(2)
	(d) $b^2-4ac=k^2-12$	(May be within the quadratic formula)	M1	
	$k^2-12<0$	(Correct inequality expression in any form)	A1	
	$-\sqrt{12}<k<\sqrt{12}$	(or $-2\sqrt{3}<k<2\sqrt{3}$)	M1 A1	(4)
			Total 11 marks	

(b) The B mark can be scored independently of the sketch.
(3, 0) shown on the y-axis scores the B1, but if not shown on the axis, it is B0.

(c) ".... no real roots" is insufficient for the 2nd B mark.
".... curve does not touch x-axis" is insufficient for the 2nd B mark.

(d) 2nd M1: correct solution method for <u>their</u> quadratic inequality,
e.g. $k^2-12<0$ gives k <u>between</u> the 2 critical values $\alpha<k<\beta$,
whereas $k^2-12>0$ gives $k<\alpha,\ k>\beta$.
"$k>-\sqrt{12}$ and $k<\sqrt{12}$" scores the final M1 A1, but
"$k>-\sqrt{12}$ or $k<\sqrt{12}$" scores M1 A0,
"$k>-\sqrt{12}$, $k<\sqrt{12}$" scores M1 A0.

N.B. $k<\pm\sqrt{12}$ does not score the 2nd M mark.
$k<\sqrt{12}$ does not score the 2nd M mark.

$\le$ instead of $<$: Penalise only once, on first occurrence.

Question 5: June 06 Q8

Question number	Scheme	Marks
(a)	$b^2-4ac=4p^2-4(3p+4)=4p^2-12p-16\ (=0)$	M1, A1
	or $(x+p)^2-p^2+(3p+4)=0 \Rightarrow p^2-3p-4(=0)$	
	$(p-4)(p+1)=0$	M1
	$p=$ (-1 or) 4	A1c.s.o. (4)
(b)	$x=\dfrac{-b}{2a}$ or $(x+p)(x+p)=0 \Rightarrow x=\ldots$	M1
	$x\ (=-p)=$ <u>- 4</u>	A1f.t. (2)
		6

(a)	1st M1	For use of b^2-4ac or a full attempt to complete the square leading to a 3TQ in p. May use $b^2=4ac$. One of b or c must be correct.
	1st A1	For a correct 3TQ in p. Condone missing "=0" but all 3 terms must be on one side.
	2nd M1	For attempt to solve their 3TQ leading to $p=\ldots$
	2nd A1	For $p=4$ (ignore $p=-1$).
		$b^2=4ac$ leading to $p^2=4(3p+4)$ and then "spotting" $p=4$ scores 4/4.
(b)	M1	For a full method leading to a repeated root $x=\ldots$
	A1f.t.	For $x=-4$ (- their p)
	<u>Trial and Improvement</u>	
	M2	For substituting values of p into the equation and attempting to factorize. (Really need to get to $p=4$ or -1)
	A2c.s.o.	Achieve $p=4$. Don't give without valid method being seen.

Question 6: Jan 07 Q5

Question number	Scheme	Marks
	Use of $b^2 - 4ac$, perhaps implicit (e.g. in quadratic formula)	M1
	$(-3)^2 - 4 \times 2 \times -(k+1) < 0$ $\quad (9 + 8(k+1) < 0)$	A1
	$8k < -17$ $\quad$ (Manipulate to get $pk < q$, or $pk > q$, or $pk = q$)	M1
	$k < -\frac{17}{8}$ $\quad \left(\text{Or equiv} : k < -2\frac{1}{8} \text{ or } k < -2.125\right)$	A1cso (4)
		4
	1^{st} M: Could also be, for example, comparing or equating b^2 and $4ac$. Must be considering the given quadratic equation. There must not be x terms in the expression, but there must be a k term. 1^{st} A: Correct expression (need not be simplified) and correct inequality sign. Allow also $-3^2 - 4 \times 2 \times -(k+1) < 0$. 2^{nd} M: Condone sign or bracketing mistakes in manipulation. Not dependent on 1^{st} M, but should not be given for irrelevant work. M0 M1 could be scored: e.g. where $b^2 + 4ac$ is used instead of $b^2 - 4ac$. Special cases: 1. Where there are x terms in the discriminant expression, but then division by x^2 gives an inequality/equation in k. (This could score M0 A0 M1 A1). 2. Use of $\leq$ instead of $<$ loses one A mark only, at first occurrence, so an otherwise correct solution leading to $k \leq -\frac{17}{8}$ scores M1 A0 M1 A1. N.B. Use of $b = 3$ instead of $b = -3$ implies no A marks.	

Question 7: June 07 Q7

Question number	Scheme	Marks
	(a) Attempt to use discriminant b^2-4ac	M1
	$k^2-4(k+3)>0 \Rightarrow k^2-4k-12>0$ (*)	A1cso (2)
	(b) $k^2-4k-12=0 \Rightarrow$	
	$(k\pm a)(k\pm b)$, with $ab=12$ or $(k=)\dfrac{4\pm\sqrt{4^2--4\times12}}{2}$ or $(k-2)^2\pm2^2-12$	M1
	$k=-2$ and 6 (both)	A1
	$\underline{k<-2,\ \ k>6}$ or $\underline{(-\infty,-2);(6,\infty)}$ M: choosing "outside"	M1 A1ft (4)
		6
(a)	M1 for use of b^2-4ac, one of b or c must be correct. Or full attempt using completing the square that leads to a 3TQ in k e.g. $\left(\left[x+\frac{k}{2}\right]^2=\right)\frac{k^2}{4}-(k+3)$ A1cso Correct argument to printed result. Need to state (or imply) that $b^2-4ac>0$ and no incorrect working seen. Must have >0. If >0 just appears with $k^2-4(k+3)>0$ that is OK. If >0 appears on last line only with no explanation give A0. b^2-4ac followed by $k^2-4k-12>0$ only is insufficient so M0A0 e.g. $k^2-4\times1\times k+3$ (missing brackets) can get M1A0 but $k^2+4(k+3)$ is M0A0 (wrong formula) Using $\sqrt{b^2-4ac}>0$ is M0.	
(b)	1st M1 for attempting to find critical regions. Factors, formula or completing the square. 1st A1 for $k=6$ and -2 only 2nd M1 for choosing the outside regions 2nd A1f.t. as printed or f.t. their (non identical) critical values $6<k<-2$ is M1A0 but ignore if it follows a correct version $-2<k<6$ is M0A0 whatever their diagram looks like Condone use of x instead of k for critical values and final answers in (b). Treat this question as 3 two mark parts. If part (a) is seen in (b) or vice versa marks can be awarded.	

Question 8: Jan 08 Q8

Question number	Scheme	Marks
	(a) $x^2 + kx + (8-k)$ $(=0)$ $8-k$ need not be bracketed	M1
	$b^2 - 4ac = k^2 - 4(8-k)$	M1
	$b^2 - 4ac < 0 \Rightarrow k^2 + 4k - 32 < 0$ (*)	A1cso (3)
	(b) $(k+8)(k-4) = 0$ $k = \ldots$	M1
	$k = -8$ $k = 4$	A1
	Choosing 'inside' region (between the two k values)	M1
	$-8 < k < 4$ or $4 > k > -8$	A1 (4)
		7
	(a) 1st M: Using the k from the right hand side to form 3-term quadratic in x ('= 0' can be implied), or… attempting to complete the square $\left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 8 - k\ (=0)$ or equiv., using the k from the right hand side. For either approach, condone sign errors. 1st M may be implied when candidate moves straight to the discriminant. 2nd M: Dependent on the 1st M. Forming expressions in k (with no x's) by using b^2 and $4ac$. (Usually seen as the discriminant $b^2 - 4ac$, but separate expressions are fine, and also allow the use of $b^2 + 4ac$. (For 'completing the square' approach, the expression must be clearly separated from the equation in x). If b^2 and $4ac$ are used in the quadratic formula, they must be clearly separated from the formula to score this mark. For any approach, condone sign errors. If the wrong statement $\sqrt{b^2 - 4ac} < 0$ is seen, maximum score is M1 M1 A0. (b) Condone the use of x (instead of k) in part (b). 1st M: Attempt to solve a 3-term quadratic equation in k. It might be different from the given quadratic in part (a). Ignore the use of < in solving the equation. The 1st M1 A1 can be scored if -8 and 4 are achieved, even if stated as $k < -8$, $k < 4$. Allow the first M1 A1 to be scored in part (a). N.B. '$k > -8$, $k < 4$' scores 2nd M1 A0 '$k > -8$ or $k < 4$' scores 2nd M1 A0 '$k > -8$ and $k < 4$' scores 2nd M1 A1 '$k = -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$' scores 2nd M0 A0 Use of ≤ (in the answer) loses the final mark.	

Question 9: June 08 Q8

Question Number	Scheme	Marks
(a)	[No real roots implies $b^2-4ac<0$.] $b^2-4ac=q^2-4\times 2q\times(-1)$	M1
	So $q^2-4\times 2q\times(-1)<0$ i.e. $q^2+8q<0$ (*)	A1 cso (2)
(b)	$q(q+8)=0$ or $(q\pm 4)^2\pm 16=0$	M1
	(q) $=0$ or -8 (2 cvs)	A1
	$-8<q<0$ <u>or</u> $q\in(-8, 0)$ <u>or</u> $q<0$ and $q>-8$	A1 ft (3)
		(5 marks)

Question 10: Jan 09 Q7

Question Number	Scheme	Marks
(a)	$b^2 - 4ac > 0 \Rightarrow \quad 16 - 4k(5-k) > 0$ or equiv., e.g. $16 > 4k(5-k)$	M1A1
	So $k^2 - 5k + 4 > 0$ (Allow any order of terms, e.g. $4 - 5k + k^2 > 0$) (*)	A1cso (3)
(b)	Critical Values $(k-4)(k-1) = 0 \quad k = ...$	M1
	$k = 1$ or 4	A1
	Choosing "outside" region	M1
	$k < 1$ or $k > 4$	A1 (4) [7]

For this question, ignore (a) and (b) labels and award marks wherever correct work is seen.

(a) M1 for attempting to use the discriminant of the initial equation (> 0 not required, but substitution of a, b and c in the correct formula is required).

If the formula $b^2 - 4ac$ is seen, at least 2 of a, b and c must be correct.

If the formula $b^2 - 4ac$ is not seen, all 3 (a, b and c) must be correct.

This mark can still be scored if substitution in $b^2 - 4ac$ is within the quadratic formula.

This mark can also be scored by comparing b^2 and $4ac$ (with substitution).

However, use of $b^2 + 4ac$ is M0.

1st A1 for fully correct expression, possibly unsimplified, with $>$ symbol. NB must appear before the last line, even if this is simply in a statement such as $b^2 - 4ac > 0$ or 'discriminant positive'. Condone a bracketing slip, e.g. $16 - 4 \times k \times 5 - k$ if subsequent work is correct and convincing.

2nd A1 for a fully correct derivation with no incorrect working seen. Condone a bracketing slip if otherwise correct and convincing.

Using $\sqrt{b^2 - 4ac} > 0$:
Only available mark is the first M1 (unless recovery is seen).

(b) 1st M1 for attempt to solve an appropriate 3TQ

1st A1 for both $k = 1$ and 4 (only the critical values are required, so accept, e.g. $k > 1$ and $k > 4$). **

2nd M1 for choosing the "outside" region. A diagram or table alone is not sufficient.
Follow through their values of k.
The set of values must be 'narrowed down' to score this M mark… listing everything $k < 1$, $1 < k < 4$, $k > 4$ is M0.

2nd A1 for correct answer only, condone "$k < 1$, $k > 4$" and even "$k < 1$ and $k > 4$", but "$1 > k > 4$" is A0.

** Often the statement $k > 1$ and $k > 4$ is followed by the correct final answer. Allow full marks.

Seeing 1 and 4 used as critical values gives the first M1 A1 by implication.

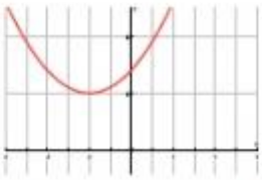
In part (b), condone working with x's except for the final mark, where the set of values must be a set of values of k (i.e. 3 marks out of 4).

Use of $\leq$ (or $\geq$) in the final answer loses the final mark.

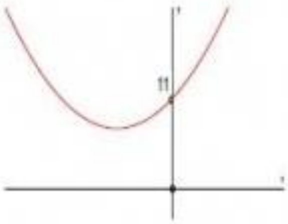
Question 11: June 09 Q6

Question Number	Scheme	Marks
Q	$b^2 - 4ac$ attempted, in terms of p. $(3p)^2 - 4p = 0$ o.e. Attempt to solve for p e.g. $p(9p-4)=0$ Must potentially lead to $p = k,\ k \neq 0$ $p = \frac{4}{9}$ (Ignore $p = 0$, if seen)	M1 A1 M1 A1cso **[4]**
	1st M1 for an attempt to substitute into $b^2 - 4ac$ or $b^2 = 4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2 - 4\times1\times p = 0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p = \frac{4}{9}$. Accept factors or use of quadratic formula or $\left(p \pm \frac{2}{9}\right)^2 = k^2$ (o.e. eg) $\left(3p \pm \frac{2}{3}\right)^2 = k^2$ or equivalent work on <u>their</u> eqn. $9p^2 = 4p \Rightarrow \frac{9p^{\not 2}}{\not p} = 4$ which would lead to $9p = 4$ is OK for this 2nd M1 ALT <u>Comparing coefficients</u> M1 for $(x+\alpha)^2 = x^2 + \alpha^2 + 2\alpha x$ and A1 for a correct equation eg $3p = 2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p} = \frac{2}{3}$ or better <u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

Question 12: Jan 10 Q10

Question number	Scheme	Marks
	(a) $(x+2k)^2$ or $\left(x+\frac{4k}{2}\right)^2$	M1
	$(x \pm F)^2 \pm G \pm 3 \pm 11k$ (where F and G are <u>any</u> functions of k, not involving x)	M1
	$(x+2k)^2 - 4k^2 + (3+11k)$ Accept unsimplified equivalents such as $\left(x+\frac{4k}{2}\right)^2 - \left(\frac{4k}{2}\right)^2 + 3 + 11k$, <u>and i.s.w. if necessary</u>.	A1 (3)
	(b) Accept part (b) solutions seen in part (a). $"4k^2 - 11k - 3" = 0$ $(4k+1)(k-3) = 0$ $k = \ldots,$	M1
	[Or, 'starting again', $b^2 - 4ac = (4k)^2 - 4(3+11k)$ and proceed to $k = \ldots$] $-\frac{1}{4}$ and 3 (Ignore any inequalities for the first 2 marks in (b)).	A1
	Using $b^2 - 4ac < 0$ for no real roots, i.e. $"4k^2 - 11k - 3" < 0$, to establish inequalities involving their <u>two</u> critical values m and n (<u>even if the inequalities are wrong</u>, e.g. $k < m, k < n$).	M1
	$-\frac{1}{4} < k < 3$ (See conditions below) Follow through their critical values. The final A1ft is still scored if the answer $m < k < n$ follows $k < m$, $k < n$. <u>Using x instead of k in the final answer</u> loses only the 2nd A mark, (condone use of x in earlier working).	A1ft (4)
	(c) Shape ∪ (seen in (c))	B1
	 Minimum in correct quadrant, <u>not</u> touching the x-axis, <u>not</u> on the y-axis, and there must be no other minimum or maximum.	B1
	(0, 14) or 14 on y-axis. Allow (14, 0) marked on y-axis. n.b. Minimum is at $(-2, 10)$, (but there is no mark for this).	B1 (3) **[10]**
	(b) 1st M: Forming and solving a 3-term quadratic in k (usual rules.. see general principles at end of scheme). The quadratic must come from $"b^2 - 4ac"$, or from the "q" in part (a). Using <u>wrong discriminant</u>, e.g. $"b^2 + 4ac"$ will score <u>no marks</u> in part (b). 2nd M: As defined in main scheme above. 2nd A1ft: $m < k < n$, where $m < n$, for their critical values m and n. Other possible forms of the answer (in each case $m < n$): (i) $n > k > m$ (ii) $k > m$ <u>and</u> $k < n$ In this case the word "and" must be seen (implying intersection). (iii) $k \in (m, n)$ (iv) $\{k : k > m\} \cap \{k : k < n\}$ <u>Not</u> just a number line. <u>Not</u> just $k > m$, $k < n$ (without the word "and"). (c) Final B1 is dependent upon a sketch having been attempted in part (c).	

Question 13: June 10 Q4

Question Number	Scheme		Marks	
(a)	$(x+3)^2+2$	or $p=3$ or $\frac{6}{2}$	B1	
		$q=2$	B1	(2)
(b)		U shape with min in 2nd quad (Must be above x-axis and not on y=axis)	B1	
		U shape crossing y-axis at (0, 11) only (Condone (11,0) marked on y-axis)	B1	(2)
(c)	$b^2-4ac=6^2-4\times11$		M1	
	$= \underline{-8}$		A1	(2)
				6

	Notes
(a)	Ignore an "= 0" so $(x+3)^2+2=0$ can score both marks
(b)	The U shape can be interpreted fairly generously. Penalise an obvious V on 1st B1 only. The U needn't have equal "arms" as long as there is a clear min that "holds water" 1st B1 for U shape with minimum in 2nd quad. Curve need not cross the y-axis but minimum should NOT touch x-axis and should be left of (not on) y-axis 2nd B1 for U shaped curve crossing at (0, 11). Just 11 marked on y-axis is fine. The point must be marked on the sketch (do not allow from a table of values) Condone stopping at (0, 11)
(c)	M1 for some correct substitution into b^2-4ac. This may be as part of the quadratic formula but must be in part (c) and must be only numbers (no x terms present). Substitution into $b^2<4ac$ or $b^2=4ac$ or $b^2>4ac$ is M0 A1 for -8 only. If they write $-8<0$ treat the <0 as ISW and award A1 If they write $-8\geq0$ then score A0 A substitution in the quadratic formula leading to -8 inside the square root is A0. So substituting into $b^2-4ac<0$ leading to $-8<0$ can score M1A1. Only award marks for use of the discriminant in part (c)

Question 14: Jan 11 Q8

(a)	$b^2-4ac=(k-3)^2-4(3-2k)$	M1
	$k^2-6k+9-4(3-2k)>0$ or $(k-3)^2-12+8k>0$ or better	M1
	$\underline{k^2+2k-3>0}$ *	A1cso
		(3)
(b)	$(k+3)(k-1)[=0]$	M1
	Critical values are $k=1$ or -3	A1
	(choosing "outside" region)	M1
	$\underline{k>1 \text{ or } k<-3}$	A1 cao
		(4)
		7
	Notes	
(a)	1st M1 for attempt to find b^2-4ac with one of b or c correct 2nd M1 for a correct inequality symbol and an attempt to expand. A1cso no incorrect working seen	
(b)	1st M1 for an attempt to factorize or solve leading to $k=$ (2 values) 2nd M1 for a method that leads them to choose the "outside" region. Can follow through their critical values. 2nd A1 Allow "," instead of "or" $\geq$ loses the final A1 $1<k<-3$ scores M1A0 unless a correct version is seen before or after this one.	

Question 15: June 11 Q7

Question Number	Scheme	Marks
(a)	Discriminant: $b^2 - 4ac = (k+3)^2 - 4k$ or equivalent	M1 A1 (2)
(b)	$(k+3)^2 - 4k = k^2 + 2k + 9 = (k+1)^2 + 8$	M1 A1 (2)
(c)	For real roots, $b^2 - 4ac \geq 0$ or $b^2 - 4ac > 0$ or $(k+1)^2 + 8 > 0$ $(k+1)^2 \geq 0$ for all k, so $b^2 - 4ac > 0$, so roots are real for all k (or equiv.)	M1 A1 cso (2) **6**
	Notes (a) M1: attempt to find discriminant – substitution is required If formula $b^2 - 4ac$ is seen at least 2 of a, b and c must be correct If formula $b^2 - 4ac$ is **not** seen all 3 of a, b and c must be correct Use of $b^2 + 4ac$ is M0 A1: correct unsimplified (b) M1: Attempt at completion of square (see earlier notes) A1: both correct (no ft for this mark) (c) M1: States condition as on scheme **or** attempts to explain that their $(k+1)^2 + 8$ is greater than 0 A1: The final mark (A1cso) requires $(k+1)^2 \geq 0$ and conclusion. We will allow $(k+1)^2 > 0$ (or word positive) also allow $b^2 - 4ac \geq 0$ and conclusion.	