Sequences and Series - Edexcel Past Exam Questions MARK SCHEME

Question 1: Jan 05 Q6

Question Number	Scheme	Marks
	(a) $ar = 7.2, ar^3 = 5.832 \Rightarrow r^2 = \frac{5.832}{7.2} (= 0.81)$ $r = 0.9$	M1 A1 (2)
	(b) $a = \frac{7.2}{(a)}, = \underline{\underline{8}}$	M1, A1 (2)
	(c) $s_{50} = \frac{8(1-(0.9)^{50})}{1-0.9}$ $= \underline{\underline{79.588}}$ $(3dp)$	M1 A1 c.a.o (2)
	(d) $s_\infty = \frac{8}{1-0.9} (= 80)$ $s_\infty - s_{50} = 80 - (c) = 0.412$ (Awrt 3 dp)	M1 A1 $\sqrt{}$ (2) (8)
	(a) M1 for full method $\rightarrow r^2$ or r N.B. $ar^2 = 7.2, ar^4 = 5.832 \rightarrow r = 0.9$ scores M1A1 in part (a) but probably M0A0 in (b). (c) M1$\sqrt{}$ their "a", "r" in s_{50} formula (d) M1$\sqrt{}$ their "a", "r" in s_∞ A1$\sqrt{}$ for 80 – their (c) i.e.$\sqrt{}$ their (c) only	

Question 2: Jan 06 Q4

Question number	Scheme		Marks	
	(a) $\frac{a}{1-r}=480$		M1	
	$\frac{120}{1-r}=480 \Rightarrow 120=480(1-r)$		M1	
	$1-r=\frac{1}{4} \Rightarrow \quad \underline{r=\frac{3}{4}}$ *		A1cso	(3)
	(b) $u_5=120\times\left(\frac{3}{4}\right)^4$ [$=37.96875$] $u_6=120\times\left(\frac{3}{4}\right)^5$ [$=28.4765625$]	either	M1	
	Difference = $\underline{9.49}$	(allow ±)	A1	(2)
	(c) $S_7=\frac{120(1-(0.75)^7)}{1-0.75}$		M1	
	$=415.9277\ldots$	(AWRT) $\underline{416}$	A1	(2)
	(d) $\frac{120(1-(0.75)^n)}{1-0.75}>300$		M1	
	$1-(0.75)^n>\frac{300}{480}$	(or better)	A1	
	$n>\frac{\log(0.375)}{\log(0.75)}$	(=3.409…)	M1	
	$\underline{n=4}$		A1cso	(4)
				11
	(a) 1st M1 for use of S_∞ 2nd M1 substituting for a and moving $(1-r)$ to form linear equation in r.		<u>For Information</u> $u_1=120$ $u_2=90$	
	(b) M1 for some correct use of ar^{n-1}.[$120(\frac{3}{4})^5-120(\frac{3}{4})^6$ is M0]		$u_3=67.5$ $u_4=50.625$	
	(c) M1 for a correct expression (need use of a and r)		$S_2=210$	
	(d) 1st M1 for attempting $S_n>300$ [or $=300$] (need use of a and some use of r) 2nd M1 for valid attempt to solve $r^n=p(r, p<1)$, must give linear eqn in n. Any correct log form will do.		$S_3=277.5$ $S_4=328.125$ $S_5=366.09\ldots$	
Trial & Imp.	1st M1 for attempting at least 2 values of S_n, one $n<4$ and one $n\geq 4$. 2nd M1 for attempting S_3 and S_4. 1st A1 for both values correct to 2 s.f. or better. 2nd A1 for $n=4$.			

Question 3: June 06 Q9

Question number	Scheme	Marks	
	(a) $ar = 4$, $\frac{a}{1-r} = 25$ (These can be seen elsewhere)	B1, B1	
	$a = 25(1-r)$ $25r(1-r) = 4$ M: Eliminate a	M1	
	$25r^2 - 25r + 4 = 0$ (*)	A1cso	(4)
	(b) $(5r-1)(5r-4) = 0$ $r = \dots$, $\frac{1}{5}$ or $\frac{4}{5}$	M1, A1	(2)
	(c) $r = \dots \Rightarrow a = \dots$, 20 or 5	M1, A1	(2)
	(d) $S_n = \frac{a(1-r^n)}{1-r}$, but $\frac{a}{1-r} = 25$, so $S_n = 25(1-r^n)$ (*)	B1	(1)
	(e) $25(1-0.8^n) > 24$ and proceed to $n = \dots$(or $>$, or $<$) with no unsound algebra.	M1	
	$\left(n > \frac{\log 0.04}{\log 0.8} \quad (=14.425\dots)\right)$ $n = 15$	A1	(2)
			11
	(a) The M mark is not dependent, but both expressions must contain both a and r. (b) <u>Special case:</u> One correct r value given, with no method (or perhaps trial and error): B1 B0. (c) M1: Substitute one r value back to find a value of a. (d) Sufficient here to verify with just one pair of values of a and r. (e) Accept "=" rather than inequalities throughout, and also allow the <u>wrong</u> inequality to be used at any stage. M1 requires use of <u>their</u> larger value of r. A correct answer with no working scores both marks. For "trial and error" methods, to score M1, a value of n between 12 and 18 (inclusive) must be tried.		

Question 4: Jan 07 Q10

Question Number	Scheme	Marks
(a)	$\{S_n =\}\ a + ar + \ldots + ar^{n-1}$	B1
	$\{rS_n =\}\ ar + ar^2 + \ldots + ar^n$	M1
	$(1-r)S_n = a(1-r^n)$	dM1
	$S_n = \dfrac{a(1-r^n)}{1-r}$ (✱)	A1cso (4)
(b)	$a = 200,\ r = 2,\ n = 10,\quad S_{10} = \dfrac{200(1-2^{10})}{1-2}$	M1, A1
	$= 204{,}600$	A1 (3)
(c)	$a = \dfrac{5}{6},\ r = \dfrac{1}{3}$	B1
	$S_\infty = \dfrac{a}{1-r},\qquad S_\infty = \dfrac{\frac{5}{6}}{1-\frac{1}{3}}$	M1
	$= \dfrac{5}{4}$ o.e.	A1 (3)
(d)	$-1 < r < 1$ (or $\lvert r \rvert < 1$)	B1 (1) **(11)**

<u>Notes</u>

10(a)	
S_n not required. The following must be seen: at least one + sign, a, ar^{n-1} and one other intermediate term. No extra terms (usually ar^n).	B1
Multiply by r; rS_n not required. At least 2 of their terms on RHS correctly multiplied by r.	M1
Subtract both sides: LHS must be $\pm(1-r)S_n$, RHS must be in the form $\pm a(1 - r^{pn+q})$. Only award this mark if the line for $S_n = \ldots$ or the line for $rS_n = \ldots$ contains a term of the form ar^{cn+d} Method mark, so may contain a slip but not awarded if last term of their S_n = last term of their rS_n.	dM1
Completion c.s.o. N.B. Answer given in question	A1 cso

10(a)	
S_n not required. The following must be seen: at least one + sign, a, ar^{n-1} and one other intermediate term. No extra terms (usually ar^n).	B1
On RHS, multiply by $\dfrac{1-r}{1-r}$ Or Multiply LHS and RHS by $(1-r)$	M1

Question 5: Jan 08 Q2

Question Number	Scheme	Marks
(a)	Complete method, using terms of form ar^k, to find r [e.g. **Dividing** $ar^6 = 80$ by $ar^3 = 10$ to find r; $r^6 - r^3 = 8$ is M0] $r = 2$	M1 A1 (2)
(b)	Complete method for finding a [e.g. Substituting value for r into equation of form $ar^k = 10$ or 80 and finding a value for a.] ($8a = 10$) $a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25)	M1 A1 (2)
(c)	Substituting their values of a and r into **correct** formula for sum. $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this)	M1 A1 (2) **[6]**
Notes:	(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly) (b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^3} \leftarrow \frac{10}{r_c^2} \leftarrow \frac{10}{r_c} \leftarrow 10$ In (a) and (b) correct answer, with no working, allow both marks. (c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula **not** quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.	

Question 6: June 08 Q6

Question Number	Scheme	Marks
(a)	$T_{20} = 5 \times \left(\frac{4}{5}\right)^{19} = 0.072$	M1 A1 (2)
(b)	$S_\infty = \frac{5}{1 - 0.8} = 25$	M1 A1 (2)
(c)	$\frac{5(1 - 0.8^k)}{1 - 0.8} > 24.95$	M1
	$1 - 0.8^k > 0.998$ or equivalent	A1
	$k \log 0.8 < \log 0.002$ or $k > \log_{0.8} 0.002$	M1
	$k > \frac{\log 0.002}{\log 0.8}$	A1 cso (4)
(d)	$k = 28$	B1
		(9 marks)

Question 7: Jan 09 Q9

Question Number	Scheme	Marks
(a)	Initial step: Two of: $a = k+4,\ ar = k,\ ar^2 = 2k-15$ Or one of: $r = \frac{k}{k+4},\quad r = \frac{2k-15}{k},\quad r^2 = \frac{2k-15}{k+4}$, Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$	M1
	$k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$	M1, A1
	Proceed to $k^2 - 7k - 60 = 0$ (*)	A1 (4)
(b)	$(k-12)(k+5) = 0 \qquad k = 12$ (*)	M1 A1 (2)
(c)	Common ratio: $\frac{k}{k+4}$ or $\frac{2k\text{-}15}{k} = \frac{12}{16}\left(= \frac{3}{4} \text{ or } 0.75\right)$	M1 A1 (2)
(d)	$\frac{a}{1-r} = \frac{16}{\left(\frac{1}{4}\right)} = 64$	M1 A1 (2) [10]
(a)	**M1**: The 'initial step', scoring the first M mark, may be implied by next line of proof **M1**: Eliminates a and r to give valid equation in k only. Can be awarded for equation involving fractions. **A1** : need some correct expansion and working and answer equivalent to required quadratic but with uncollected terms. Equations involving fractions do not get this mark. (No fractions, no brackets – could be a cubic equation) **A1**: as answer is printed this mark is for cso (Needs = 0) **All four marks must be scored in part (a)**	
(b)	**M1**: Attempt to solve quadratic **A1**: This is for correct factorisation or solution and $k = 12$. Ignore the extra solution ($k = -5$ or even $k = 5$), if seen. Substitute and verify is **M1 A0** **Marks must be scored in part (b)**	
(c)	**M1**: Complete method to find r Could have answer in terms of k **A1**: 0.75 or any correct equivalent **Both Marks must be scored in (c)**	
(d)	**M1**: Tries to use $\frac{a}{1-r}$, (even with r>1). Could have an answer still in terms of k. **A1**: This answer is 64 cao.	

Question 8: June 09 Q5

Question Number	Scheme	Marks
(a)	$324r^3 = 96$ or $r^3 = \frac{96}{324}$ or $r^3 = \frac{8}{27}$	M1
	$r = \frac{2}{3}$ (*)	A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324$ or $a\left(\frac{2}{3}\right)^5 = 96$ $a = \dots,$ 729	M1, A1 (2)
(c)	$S_{15} = \frac{729\left(1-\left[\frac{2}{3}\right]^{15}\right)}{1-\frac{2}{3}}, = 2182.00\dots$ (AWRT 2180)	M1A1ft, (3)
(d)	$S_\infty = \frac{729}{1-\frac{2}{3}}, = 2187$	M1, A1 (2)
		[9]
(a)	M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction. A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp and the final answer 2/3 is seen. Alternative: (verification) M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times). A1 Obtaining 96 (cso). (A conclusion is not required). $324 \times \left(\frac{2}{3}\right)^3 = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.	
(b)	M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their r) twice from 324 (or 5 times from 96). Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)… but no other exceptions are allowed.	
(c)	M1 for use of sum to 15 terms formula with values of a and r. If the wrong power is used, e.g. 14, the M mark is scored only if the correct sum formula is stated. 1st A1ft for a correct expression or correct ft their a with $r = \frac{2}{3}$. 2nd A1 for awrt 2180, even following 'minor inaccuracies'. Condone missing brackets round the $\frac{2}{3}$ for the marks in part (c). Alternative: M1 for adding 15 terms and 1st A1ft for adding the 15 terms that ft from their a and $r = \frac{2}{3}$.	
(d)	M1 for use of correct sum to infinity formula with their a. For this mark, if a value of r different from the given value is being used, M1 can still be allowed providing $\lvert r \rvert < 1$.	

Question 9: Jan 11 Q3

Question Number	Scheme	Marks
(a)	$ar = 750$ and $ar^4 = -6$ (could be implied from later working in either (a) or (b)).	B1
	$r^3 = \frac{-6}{750}$	M1
	$r = -\frac{1}{5}$ Correct answer from no working, except for special case below gains all three marks.	A1
		(3)
(b)	$a(-0.2) = 750$	M1
	$a\left\{= \frac{750}{-0.2}\right\} = -3750$	A1 ft
		(2)
(c)	Applies $\frac{a}{1-r}$ correctly using both their a and their $\lvert r \rvert < 1$. Eg. $\frac{-3750}{1--0.2}$	M1
	So, $S_\infty = -3125$	A1
		(2) [7]
	Notes	
(a)	B1: for both $ar = 750$ and $ar^4 = -6$ (may be implied from later working in either (a) or (b)). M1: **for eliminating *a*** by either dividing $ar^4 = -6$ by $ar = 750$ or dividing $ar = 750$ by $ar^4 = -6$, to achieve an equation in r^3 or $\frac{1}{r^3}$ Note that $r^4 - r = -\frac{6}{750}$ is M0. Note also that any of $r^3 = \frac{-6}{750}$ or $r^3 = \frac{750}{-6}\{=-125\}$ or $\frac{1}{r^3} = \frac{-6}{750}$ or $\frac{1}{r^3} = \frac{750}{-6}\{=-125\}$ are fine for the award of M1. SC: $ar^\alpha = 750$ and $ar^\beta = -6$ leading to $r^\delta = \frac{-6}{750}$ or $r^\delta = \frac{750}{-6}\{=-125\}$ or $\frac{1}{r^\delta} = \frac{-6}{750}$ or $\frac{1}{r^\delta} = \frac{750}{-6}\{=-125\}$ where $\delta = \beta - \alpha$ and $\delta \geq 2$ are fine for the award of M1. SC: $ar^2 = 750$ and $ar^5 = -6$ leading to $r = -\frac{1}{5}$ scores B0M1A1.	
(b)	M1 for inserting their r into either of their original correct equations of either $ar = 750$ or $\{a =\}\frac{750}{r}$ or $ar^4 = -6$ or $\{a =\}\frac{-6}{r^4}$ – in ***both a* and *r***. No slips allowed here for M1. A1 for either $a = -3750$ or a equal to the correct follow through result expressed either as an exact integer, or a fraction in the form $\frac{c}{d}$ where both c and d are integers, or correct to awrt 1 dp.	
(c)	M1 for applying $\frac{a}{1-r}$ correctly (only a slip in substituting r is allowed) using both their a and their $\lvert r \rvert < 1$. Eg. $\frac{-3750}{1--0.2}$. A1 for -3125 In parts (a) or (b) or (c), the correct answer with no working scores full marks.	

Question 10: June 11 Q6

Question Number	Scheme		Marks
(a)	$\{ar = 192 \text{ and } ar^2 = 144\}$		
	$r = \frac{144}{192}$	Attempt to eliminate a. (See notes.)	M1
	$r = \frac{3}{4}$ or 0.75	$\frac{3}{4}$ or 0.75	A1
			[2]
(b)	$a(0.75) = 192$		M1
	$a\left\{= \frac{192}{0.75}\right\} = 256$	256	A1
			[2]
(c)	$S_\infty = \frac{256}{1-0.75}$	Applies $\frac{a}{1-r}$ correctly using both their a and their $\lvert r \rvert < 1$.	M1
	So, $\{S_\infty =\}$ 1024	1024	A1 cao
			[2]
(d)	$\frac{256(1-(0.75)^n)}{1-0.75} > 1000$	Applies S_n with their a and r and "uses" 1000 at any point in their working. (Allow with = or <).	M1
	$(0.75)^n < 1 - \frac{1000(0.25)}{256}\ \left\{= \frac{6}{256}\right\}$	Attempt to isolate $+(r)^n$ from S_n formula. (Allow with = or >).	M1
	$n\log(0.75) < \log\left(\frac{6}{256}\right)$	Uses the power law of logarithms correctly. (Allow with = or >). (See notes.)	M1
	$n > \frac{\log\left(\frac{6}{256}\right)}{\log(0.75)} = 13.0471042... \Rightarrow n = 14$	See notes and $n = 14$	A1 cso
			[4] 10

(a)	M1: **for eliminating** a by eg. $192r = 144$ or by either dividing $ar^2 = 144$ by $ar = 192$ or dividing $ar = 192$ by $ar^2 = 144$, to achieve an equation in r or $\frac{1}{r}$ Note that $r^2 - r = \frac{144}{192}$ is M0. Note also that any of $r = \frac{144}{192}$ or $r = \frac{192}{144}\left\{= \frac{4}{3}\right\}$ or $\frac{1}{r} = \frac{192}{144}$ or $\frac{1}{r} = \frac{144}{192}$ are fine for the award of M1. **Note:** A candidate just writing $r = \frac{144}{192}$ with no reference to a can also get the method mark. **Note**: $ar^2 = 192$ and $ar^3 = 144$ leading to $r = \frac{3}{4}$ scores M1A1. This is because r is the ratio between any two consecutive terms. These candidates, however, will usually be penalised in part (b).
(b)	M1 for inserting their r into either of the correct equations of either $ar = 192$ or $\{a =\}\frac{192}{r}$ or $ar^2 = 144$ or $\{a =\}\frac{144}{r^2}$. No slips allowed here for M1. M1: can also be awarded for writing down $144 = a\left(\frac{192}{a}\right)^2$ A1 for $a = 256$ only. Note 256 from any working scores M1A1. **Note**: Some candidates incorrectly confuse notation to give $r = \frac{4}{3}$ or 1.33 in part (a) (getting M1A0). In part (b), they recover to write $a = 192 \times \frac{4}{3}$ for M1 and then 256 for A1.

Question Number	Scheme	Marks
(c)	M1: for applying $\frac{a}{1-r}$ correctly (no slips allowed!) using both their a **and** their r, where $\lvert r\rvert < 1$. A1: for 1024, cao. In parts (a) or (b) or (c), the correct answer with no working scores full marks.	
(d)	1[st] M1: For applying S_n with their a and either "the letter r" or their r and "uses" 1000. 2[nd] M1: For isolating $+(r)^n$ and not $(ar)^n$, (eg. $(192)^n$) as the subject of an equation or inequality. $+(r)^n$ must be derived from the S_n formula. 3[rd] M1: For applying the power law to $\lambda^k = \mu$ to give $k\log\lambda = \log\mu$ oe. where $\lambda, \mu > 0$. or 3[rd] M1: For solving $\lambda^k = \mu$ to give $k = \log_\lambda \mu$, where $\lambda, \mu > 0$. A1: cso If a candidate uses inequalities, a fully correct method with inequalities is required here. So, an <u>incorrect</u> inequality statement at any stage in a candidate's working for this part loses this mark. **Note:** Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. Or A1: cso Note a candidate can achieve full marks here if they do not use inequalities. So, if a candidate uses equations rather than inequalities in their working then they need to state in the final line of their working that $n = 13.04$ (truncated) or $n =$ awrt $13.05 \Rightarrow n = 14$ for A1. $n = 14$ from no working gets SC: M0M0M1A1. A method of $T_n > 1000 \Rightarrow 256(0.75)^{n-1} > 1000$ can score M0M0M1A0 for a correct application of the power law of logarithms. **<u>Trial & Improvement Method:</u>** For $a = 256$ and $r = 0.75$, apply the following scheme:	
	$S_{13} = \frac{256(1-(0.75)^{13})}{1-0.75} = 999.6725616...$ — Attempt to find either S_{13} or S_{14}.	M1
	EITHER (1) S_{13} = awrt 999.7 or truncated 999 OR (2) S_{14} = awrt 1005.8 or truncated 1005.	M1
	$S_{14} = \frac{256(1-(0.75)^{14})}{1-0.75} = 1005.754421...$ — Attempt to find both S_{13} and S_{14}.	M1
	So, $n = 14$. — BOTH (1) S_{13} = awrt 999.7 or truncated 999 AND (2) S_{14} = awrt 1005.8 or truncated 1005 AND $n = 14$.	A1