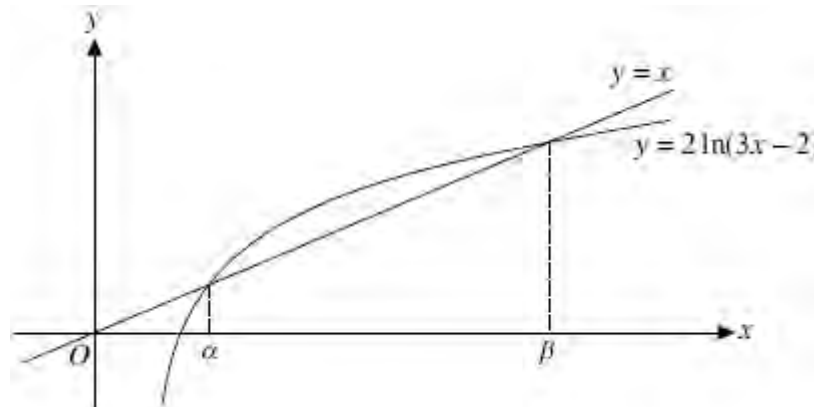


## FP2 Numerical Methods

### 1. [June 2010 qu. 7](#)



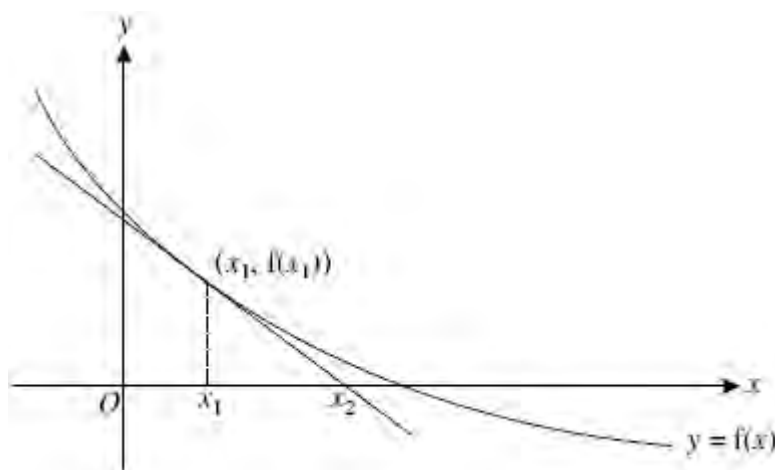
The line  $y = x$  and the curve  $y = 2 \ln(3x - 2)$  meet where  $x = \alpha$  and  $x = \beta$ , as shown in the diagram.

- (i) Use the iteration  $x_{n+1} = 2 \ln(3x_n - 2)$ , with initial value  $x_1 = 5.25$ , to find the value of  $\beta$  correct to 2 decimal places. Show all your working. [2]
- (ii) With the help of a 'staircase' diagram, explain why this iteration will not converge to  $\alpha$ , whatever value of  $x_1$  (other than  $\alpha$ ) is used. [3]
- (iii) Show that the equation  $x = 2 \ln(3x - 2)$  can be rewritten as  $x = \frac{1}{3}(e^{\frac{1}{2}x} + 2)$ . Use the Newton-Raphson method, with  $f(x) = \frac{1}{3}(e^{\frac{1}{2}x} + 2) - x$  and  $x_1 = 1.2$ , to find  $\alpha$  correct to 2 decimal places. Show all your working. [4]
- (iv) Given that  $x_1 = \ln 36$ , explain why the Newton-Raphson method would not converge to a root of  $f(x) = 0$ . [2]

### 2. [Jan 2010 qu.1](#)

It is given that  $f(x) = x^2 - \sin x$ .

- (i) The iteration  $x_{n+1} = \sqrt{\sin x_n}$ , with  $x_1 = 0.875$ , is to be used to find a real root,  $\alpha$ , of the equation  $f(x) = 0$ . Find  $x_2$ ,  $x_3$  and  $x_4$ , giving the answers correct to 6 decimal places. [2]
- (ii) The error  $e_n$  is defined by  $e_n = \alpha - x_n$ . Given that  $\alpha = 0.876\,726$ , correct to 6 decimal places, find  $e_3$  and  $e_4$ . Given that  $g(x) = \sqrt{\sin x}$ , use  $e_3$  and  $e_4$  to estimate  $g'(\alpha)$ . [3]

3. [Jan 2010 qu.3](#)

A curve with no stationary points has equation  $y = f(x)$ . The equation  $f(x) = 0$  has one real root  $\alpha$ , and the Newton-Raphson method is to be used to find  $\alpha$ .

The tangent to the curve at the point  $(x_1, f(x_1))$  meets the  $x$ -axis where  $x = x_2$  (see diagram).

- (i) Show that  $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ . [3]
- (ii) Describe briefly, with the help of a sketch, how the Newton-Raphson method, using an initial approximation  $x = x_1$ , gives a sequence of approximations approaching  $\alpha$ . [2]
- (iii) Use the Newton-Raphson method, with a first approximation of 1, to find a second approximation of the root of  $x^2 - 2 \sinh x + 2 = 0$ . [2]

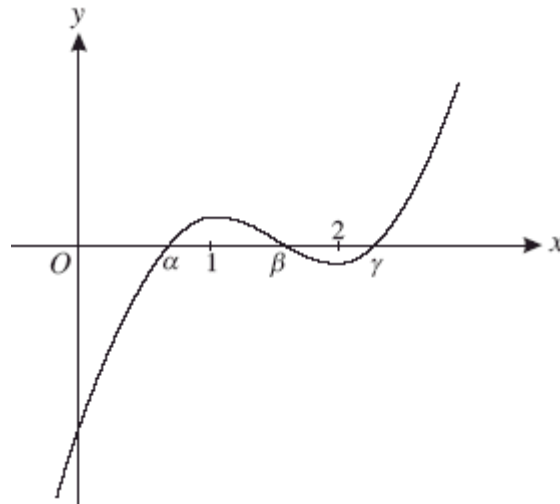
4. [June 2009 qu. 7](#)

- (i) Sketch the graph of  $y = \coth x$ , and give the equations of any asymptotes. [3]
- (ii) It is given that  $f(x) = x \tanh x - 2$ . Use the Newton-Raphson method, with a first approximation  $x_1 = 2$ , to find the next three approximations  $x_2$ ,  $x_3$  and  $x_4$  to a root of  $f(x) = 0$ . Give the answers correct to 4 decimal places. [4]
- (iii) If  $f(x) = 0$ , show that  $\coth x = \frac{1}{2}x$ . Hence write down the roots of  $f(x) = 0$ , correct to 4 decimal places. [3]

5. [Jan 2009 qu.2](#)

It is given that  $\alpha$  is the only real root of the equation  $x^5 + 2x - 28 = 0$  and that  $1.8 < \alpha < 2$ .

- (i) The iteration  $x_{n+1} = \sqrt[5]{28 - 2x_n}$ , with  $x_1 = 1.9$ , is to be used to find  $\alpha$ . Find the values of  $x_2$ ,  $x_3$  and  $x_4$ , giving the answers correct to 7 decimal places. [3]
- (ii) The error  $e_n$  is defined by  $e_n = \alpha - x_n$ . Given that  $\alpha = 1.891\,574\,9$ , correct to 7 decimal places, evaluate  $\frac{e_3}{e_2}$  and  $\frac{e_4}{e_3}$ . Comment on these values in relation to the gradient of the curve with equation  $y = \sqrt[5]{28 - 2x}$  at  $x = \alpha$ . [3]

6. [Jan 2009 qu.5](#)

The diagram shows the curve with equation  $y = f(x)$ , where  $f(x) = 2x^3 - 9x^2 + 12x - 4.36$ .

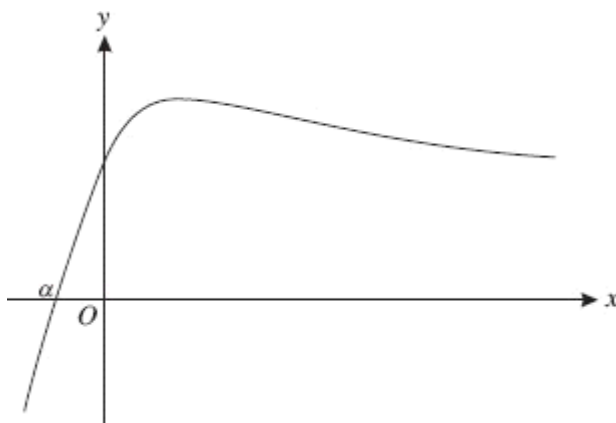
The curve has turning points at  $x = 1$  and  $x = 2$  and crosses the  $x$ -axis at  $x = \alpha$ ,  $x = \beta$  and  $x = \gamma$ , where  $0 < \alpha < \beta < \gamma$ .

- (i) The Newton-Raphson method is to be used to find the roots of the equation  $f(x) = 0$ , with  $x_1 = k$ .
  - (a) To which root, if any, would successive approximations converge in each of the cases  $k < 0$  and  $k = 1$ ? [2]
  - (b) What happens if  $1 < k < 2$ ? [2]
- (ii) Sketch the curve with equation  $y^2 = f(x)$ . State the coordinates of the points where the curve crosses the  $x$ -axis and the coordinates of any turning points. [4]

7. [June 2008 qu. 6](#)

It is given that  $f(x) = 1 - \frac{7}{x^2}$ .

- (i) Use the Newton-Raphson method, with a first approximation  $x_1 = 2.5$ , to find the next approximations  $x_2$  and  $x_3$  to a root of  $f(x) = 0$ . Give the answers correct to 6 decimal places.
- (ii) The root of  $f(x) = 0$  for which  $x_1$ ,  $x_2$  and  $x_3$  are approximations is denoted by  $\alpha$ . Write down the exact value of  $\alpha$ . [1]
- (iii) The error  $e_n$  is defined by  $e_n = \alpha - x_n$ . Find  $e_1$ ,  $e_2$  and  $e_3$ , giving your answers correct to 5 decimal places. Verify that  $e_3 \approx \frac{e_2^3}{e_1^2}$ . [3]

8. [Jan 2008 qu.5](#)

The diagram shows the curve with equation  $y = xe^{-x} + 1$ . The curve crosses the  $x$ -axis at  $x = \alpha$ .

- (i) Use differentiation to show that the  $x$ -coordinate of the stationary point is 1. [2]

$\alpha$  is to be found using the Newton-Raphson method, with  $f(x) = xe^{-x} + 1$ .

- (ii) Explain why this method will not converge to  $\alpha$  if an initial approximation  $x_1$  is chosen such that  $x_1 > 1$ . [2]
- (iii) Use this method, with a first approximation  $x_1 = 0$ , to find the next three approximations  $x_2$ ,  $x_3$  and  $x_4$ . Find  $\alpha$ , correct to 3 decimal places. [5]

9. [June 2007 qu. 8](#)

The iteration  $x_{n+1} = \frac{1}{(x_n + 2)^2}$ , with  $x_1 = 0.3$ , is to be used to find the real root,  $\alpha$ , of the equation  $x(x + 2)^2 = 1$ .

- (i) Find the value of  $\alpha$ , correct to 4 decimal places. You should show the result of each step of the iteration process. [4]
- (ii) Given that  $f(x) = \frac{1}{(x + 2)^2}$ , show that  $f'(\alpha) \neq 0$ . [2]
- (iii) The difference,  $\delta_r$ , between successive approximations is given by  $\delta_r = x_{r+1} - x_r$ . Find  $\delta_3$ . [1]
- (iv) Given that  $\delta_{r+1} \approx f'(\alpha)\delta_r$ , find an estimate for  $\delta_{10}$ . [3]

10. [Jan 2007 qu.2](#)

It is given that  $f(x) = x^2 - \tan^{-1}x$ .

- (i) Show by calculation that the equation  $f(x) = 0$  has a root in the interval  $0.8 < x < 0.9$ . [2]
- (ii) Use the Newton-Raphson method, with a first approximation 0.8, to find the next approximation to this root. Give your answer correct to 3 decimal places. [4]

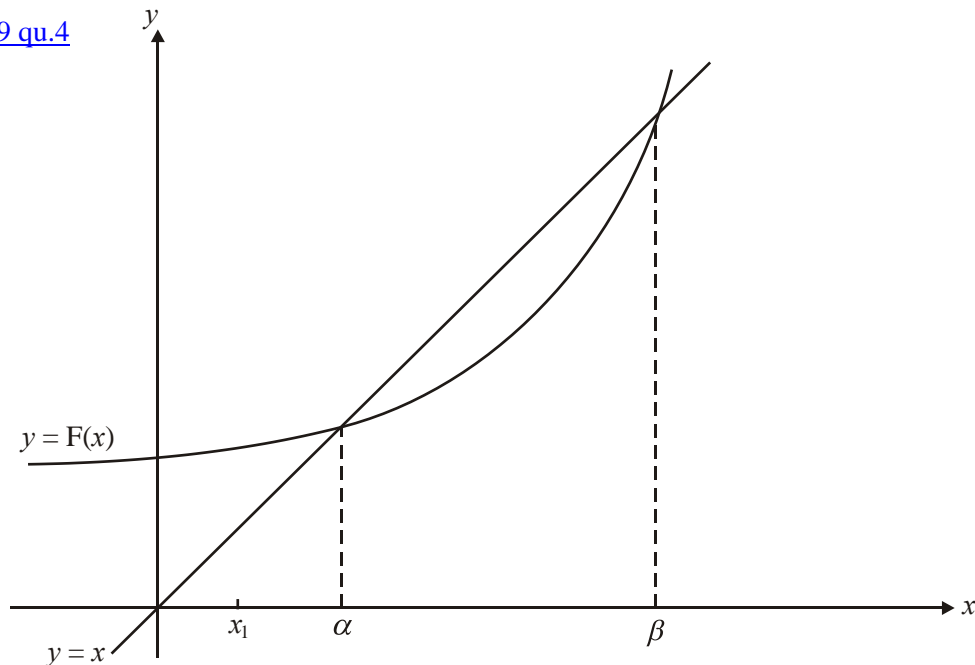
11. [June 2006 qu. 8](#)

The curve with equation  $y = \frac{\sinh x}{x^2}$ , for  $x > 0$ , has one turning point.

- (i) Show that the  $x$ -coordinate of the turning point satisfies the equation  $x - 2 \tanh x = 0$ . [3]
- (ii) Use the Newton-Raphson method, with a first approximation  $x_1 = 2$ , to find the next two approximations,  $x_2$  and  $x_3$ , to the positive root of  $x - 2 \tanh x = 0$ . [5]
- (iii) By considering the approximate errors in  $x_1$  and  $x_2$ , estimate the error in  $x_3$ . (You are not expected to evaluate  $x_4$ ) [3]

12. [Jan 2006 qu.2](#)

Use the Newton-Raphson method to find the root of the equation  $e^{-x} = x$  which is close to  $x = 0.5$ . Give the root correct to 3 decimal places. [5]

13. [Jan 2009 qu.4](#)

The sketch shows the curve with equation  $y = F(x)$  and the line  $y = x$ . The equation  $x = F(x)$  has roots  $x = \alpha$  and  $x = \beta$  as shown.

- (i) Show how an iteration of the form  $x_{n+1} = F(x_n)$ , with starting value  $x_1$  such that  $0 < x_1 < \alpha$  as shown, converges to the root  $x = \alpha$ .

.....  
 .....

[3]

- (ii) State what happens in the iteration in the following two cases.

- (a)  $x_1$  is chosen such that  $\alpha < x_1 < \beta$ .

.....

- (b)  $x_1$  is chosen such that  $x_1 > \beta$ .

.....

[3]