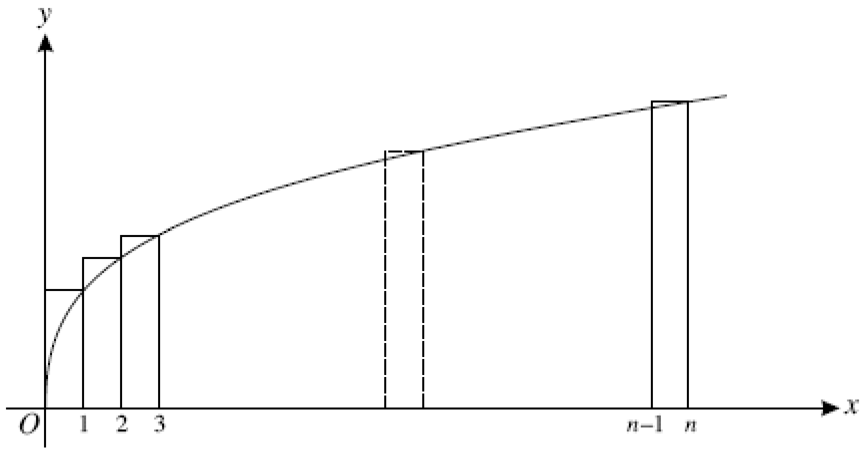
Areas using Rectangles

1. Jan 2010 qu. 7



The diagram shows the curve with equation $y = \sqrt[3]{x}$, together with a set of n rectangles of unit width.

(i) By considering the areas of these rectangles, explain why

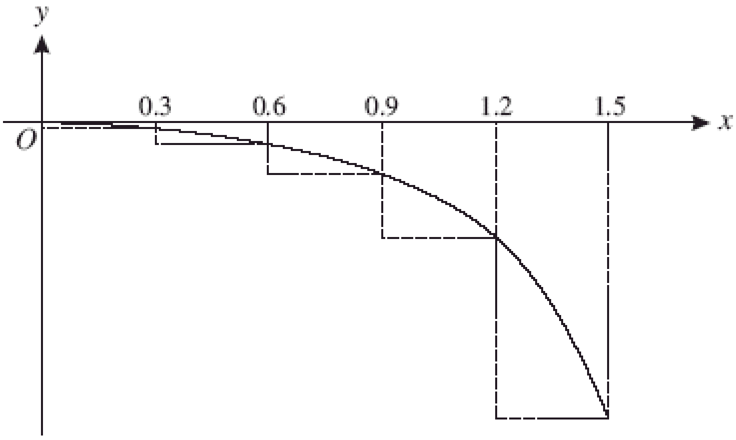
$$\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{3}+\ldots+\sqrt[3]{n} > \int_0^n \sqrt[3]{x}\,\mathrm{d}x.$$ [2]

(ii) By drawing another set of rectangles and considering their areas, show that

$$\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{3}+\ldots+\sqrt[3]{n} < \int_1^{n+1} \sqrt[3]{x}\,\mathrm{d}x.$$ [3]

(iii) Hence find an approximation to $\sum_{n=1}^{100} \sqrt[3]{n}$, giving your answer correct to 2 significant figures.[3]

2. June 2006 qu. 1



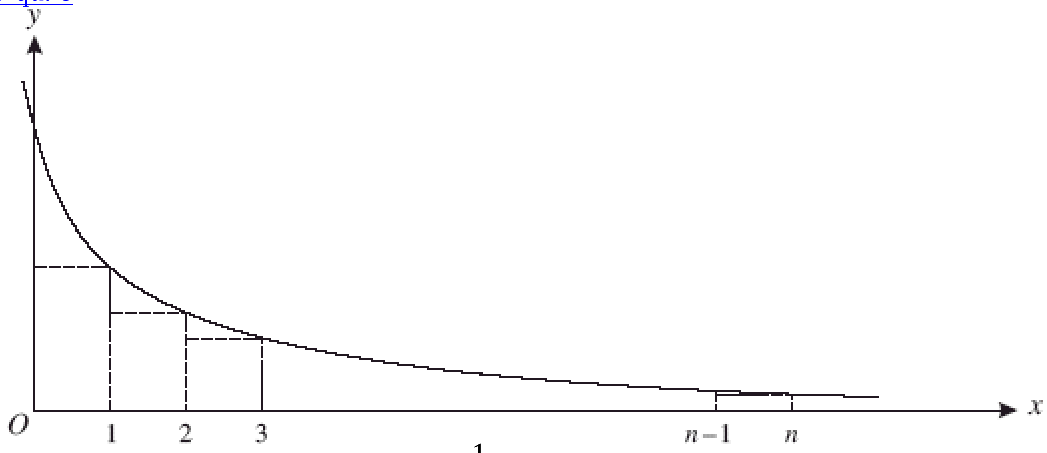
The diagram shows the curve with equation $y = \ln(\cos x)$, for $0 \le x \le 1.5$. The region bounded by the curve, the x-axis and the line $x = 1.5$ has area A. The region is divided into five strips, each of width 0.3.

(i) By considering the set of rectangles indicated in the diagram, find an upper bound for A. Give the answer correct to 3 decimal places. [2]

(ii) By considering another set of five suitable rectangles, find a lower bound for A. Give the answer correct to 3 decimal places. [2]

(iii) How could you reduce the difference between the upper and lower bounds for A? [1]

3. Jan 2009 qu. 8



The diagram shows the curve with equation $y = \frac{1}{x+1}$. A set of n rectangles of unit width is drawn, starting at $x = 0$ and ending at $x = n$, where n is an integer.

(i) By considering the areas of these rectangles, explain why $\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n+1} < \ln(n+1)$. [5]

(ii) By considering the areas of another set of rectangles, show that

$$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n} > \ln(n+1).$$ [2]

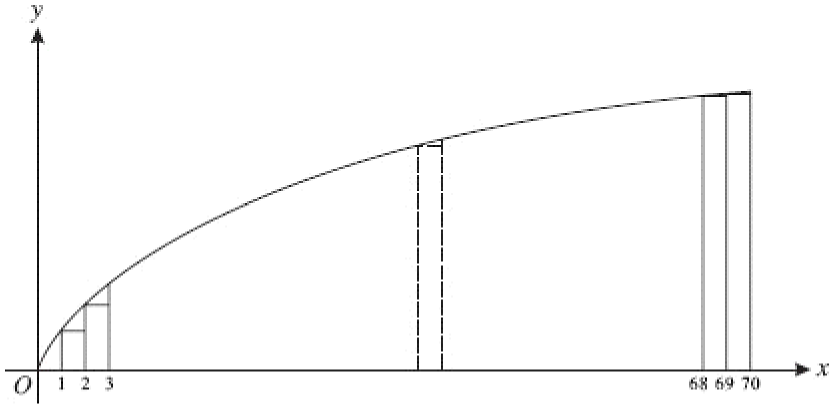
(iii) Hence show that $\ln(n+1) + \frac{1}{n+1} < \sum_{r=1}^{n+1}\frac{1}{r} < \ln(n+1) + 1$. [2]

(iv) State, with a reason, whether $\sum_{r=1}^{\infty}\frac{1}{r}$ is convergent. [2]

4. June 2008 qu. 9

(i) Prove that $\int_0^N \ln(1+x)\,dx = (N+1)\ln(N+1) - N$, where N is a positive constant. [4]

(ii)



The diagram shows the curve $y = \ln(1 + x)$, for $0 \le x \le 70$, together with a set of rectangles of unit width.

(a) By considering the areas of these rectangles, explain why

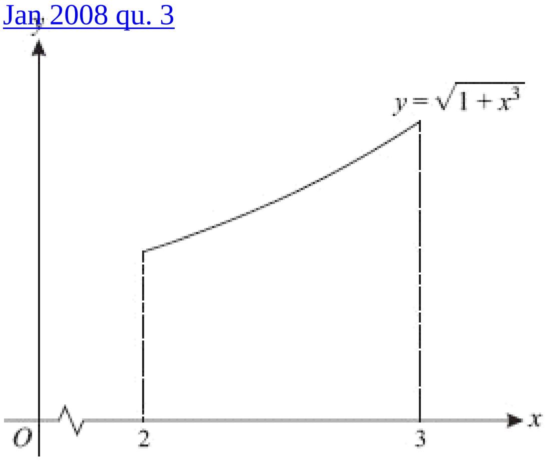
$$\ln 2 + \ln 3 + \ln 4 + \ldots + \ln 70 < \int_0^{70} \ln(1+x)\,dx.$$ [2]

(b) By considering the areas of another set of rectangles, show that

$$\ln 2 + \ln 3 + \ln 4 + \ldots + \ln 70 > \int_0^{69} \ln(1+x)\,dx. \qquad [3]$$

(c) Hence find bounds between which ln(70!) lies. Give the answers correct to 1 decimal place. [3]

5. Jan 2008 qu. 3

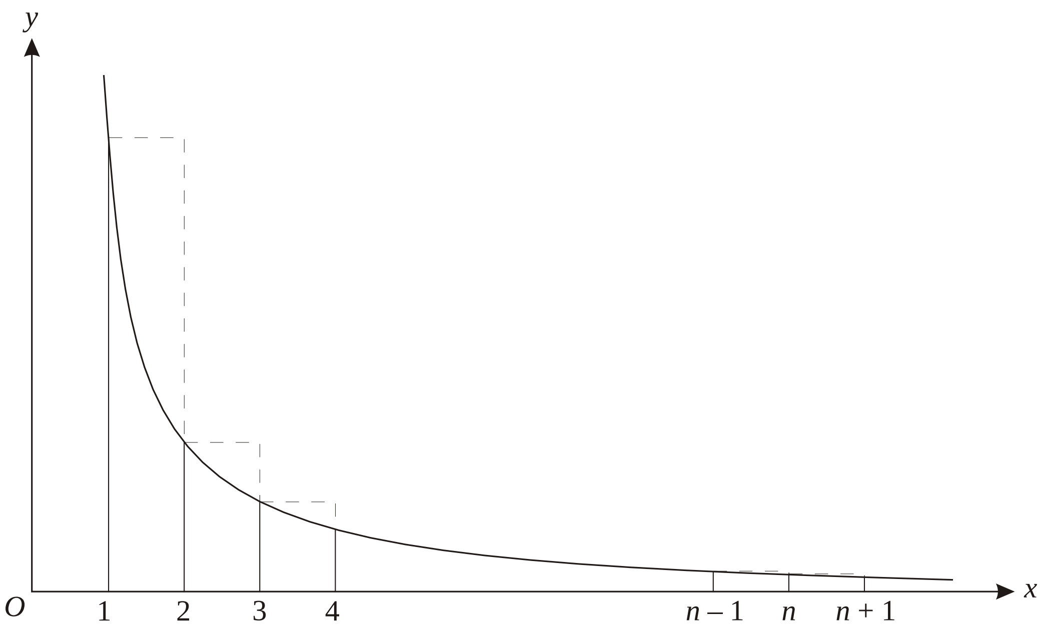


The diagram shows the curve with equation $y = \sqrt{1+x^3}$, for $2 \le x \le 3$. The region under the curve between these limits has area A.

(i) Explain why $3 < A < \sqrt{28}$. [2]

(ii) The region is divided into 5 strips, each of width 0.2. By using suitable rectangles, find improved lower and upper bounds between which A lies. Give your answers correct to 3 significant figures.

6. June 2007 qu. 6



The diagram shows the curve with equation $y = \dfrac{1}{x^2}$ for $x > 0$, together with a set of n rectangles of unit width, starting at $x = 1$.

(i) By considering the areas of these rectangles, explain why

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} > \int_1^{n+1} \frac{1}{x^2}\,dx. \qquad [2]$$

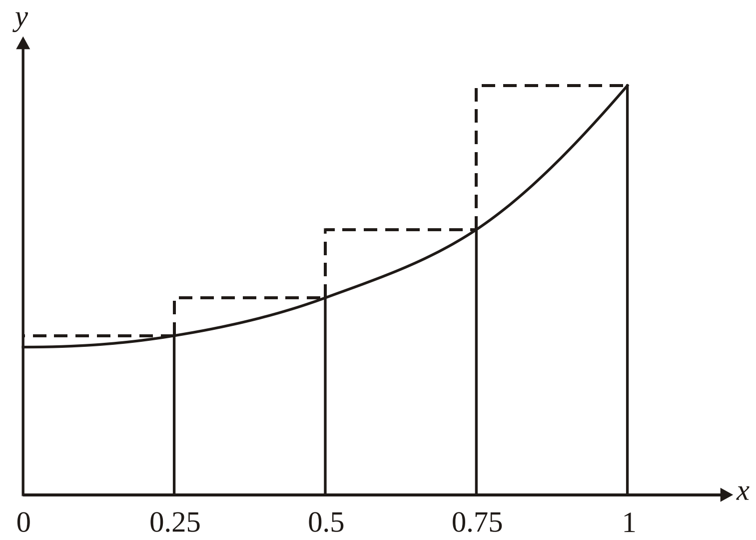
(ii) By considering the areas of another set of rectangles, explain why

$$\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots+\frac{1}{n^2}<\int_1^n \frac{1}{x^2}\,\mathrm{d}x\,. \qquad [3]$$

(iii) Hence show that $1-\frac{1}{n+1}<\sum_{r=1}^{n}\frac{1}{r^2}<2-\frac{1}{n}$. [4]

(iv) Hence give bounds between which $\sum_{r=1}^{\infty}\frac{1}{r^2}$ lies. [2]

7. Jan 2007 qu. 3

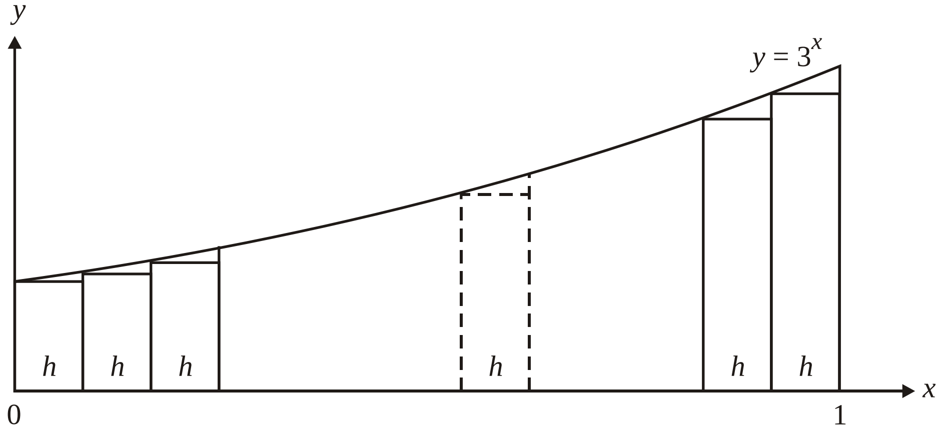


The diagram shows the curve with equation $y = \mathrm{e}^{x^2}$, for $0 \le x \le 1$. The region under the curve between these limits is divided into four strips of equal width. The area of this region under the curve is A.

(i) By considering the set of rectangles indicated in the diagram, show that an upper bound for A is 1.71. [3]

(ii) By considering an appropriate set of four rectangles, find a lower bound for A. [3]

8. June 2006 qu. 6



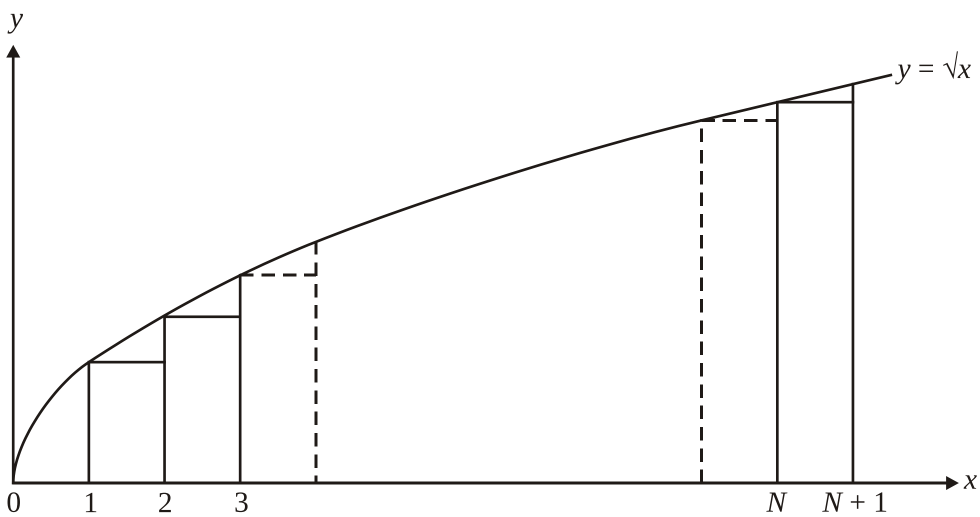
The diagram shows the curve with equation $y = 3^x$ for $0 \le x \le 1$. The area A under the curve between these limits is divided into n strips, each of width h where $nh = 1$.

(i) By using the set of rectangles indicated on the diagram, show that $A > \dfrac{2h}{3^h - 1}$. [3]

(ii) By considering another set of rectangles, show that $A < \dfrac{(2h)3^h}{3^h - 1}$ [3]

(iii) Given that $h = 0.001$, use these inequalities to find values between which A lies. [2]

9. Jan 2006 qu. 7



The diagram shows the curve with equation $y = \sqrt{x}$. A set of N rectangles of unit width is drawn, starting at $x = 1$ and ending at $x = N + 1$, where N is an integer (see diagram).

(i) By considering the areas of these rectangles, explain why

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{N} < \int_1^{N+1} \sqrt{x}\ dx$$ [3]

(ii) By considering the areas of another set of rectangles, explain why

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{N} > \int_0^{N} \sqrt{x}\ dx$$ [3]

(iii) Hence find, in terms of N, limits between which $\sum_{r=1}^{N} \sqrt{r}$ lies. [3]