FP2 Integration

1. June 2010 qu. 3

Use the substitution $t = \tan\frac{1}{2}x$ to show that $\int_0^{\frac{1}{3}\pi} \frac{1}{1-\sin x}\,dx = 1 + \sqrt{3}$. [6]

2. June 2010 qu. 5

It is given that, for $n \geq 0$, $I_n = \int_0^{\frac{1}{2}} (1-2x)^n \, e^x \, dx$.

(i) Prove that, for $n \geq 1$, $I_n = 2nI_{n-1} - 1$. [4]

(ii) Find the exact value of I_3. [4]

3. Jan 2010 qu.6

(i) Express $\frac{4}{(1-x)(1+x)(1+x^2)}$ in partial fractions. [5]

(ii) Show that $\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1-x^4}\,dx = \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{1}{3}\pi$. [4]

4. June 2009 qu. 5

It is given that $I = \int_0^{\frac{1}{2}\pi} \frac{\cos\theta}{1+\cos\theta}\,d\theta$.

(i) By using the substitution $t = \tan\frac{1}{2}\theta$, show that $I = \int_0^1 \left(\frac{2}{1+t^2} - 1\right) dt$. [5]

(ii) Hence find I in terms of π. [2]

5. June 2009 qu. 6

Given that $\int_0^1 \frac{1}{\sqrt{16+9x^2}}\,dx + \int_0^2 \frac{1}{\sqrt{9+4x^2}}\,dx = \ln a$, find the exact value of a. [6]

6. June 2009 qu. 9

(i) It is given that, for non-negative integers n, $I_n = \int_0^{\frac{1}{2}\pi} \sin^n \theta \, d\theta$.

Show that, for $n \geq 2$, $nI_n = (n-1)I_{n-2}$. [4]

(ii) The equation of a curve, in polar coordinates, is $r = \sin^3\theta$, for $0 \leq \theta \leq \pi$.

(a) Find the equations of the tangents at the pole and sketch the curve. [4]

(b) Find the exact area of the region enclosed by the curve. [6]

7. Jan 2009 qu.4

(i) By means of a suitable substitution, show that $\int \frac{x^2}{\sqrt{x^2-1}}\,dx$

can be transformed to $\int \cosh^2\theta\, d\theta$. [2]

(ii) Hence show that $\int \frac{x^2}{\sqrt{x^2-1}}dx = \frac{1}{2}x\sqrt{x^2-1} + \frac{1}{2}\cosh^{-1}x + c$. [4]

8. Jan 2009 qu.9

A curve has equation $y = \frac{4x-3a}{2(x^2+a^2)}$, where a is a positive constant.

(i) Explain why the curve has no asymptotes parallel to the y-axis. [2]

(ii) Find, in terms of a, the set of values of y for which there are no points on the curve. [5]

(iii) Find the exact value of $\int_a^{2a} \frac{4x-3a}{2(x^2+a^2)}\,dx$, showing that it is independent of a. [5]

9. June 2008 qu. 3

By using the substitution $t = \tan\frac{1}{2}x$, find the exact value of $\int_0^{\frac{1}{2}\pi} \frac{1}{2-\cos x}\,dx$,

giving the answer in terms of π. [6]

10. June 2008 qu. 5

It is given that, for $n \geq 0$, $I_n = \int_0^{\frac{1}{4}\pi} \tan^n x\,dx$.

(i) By considering $I_n + I_{n-2}$, or otherwise, show that, for $n \geq 2$, $(n-1)(I_n + I_{n-2}) = 1$. [4]

(ii) Find I_4 in terms of π. [4]

11. Jan 2008 qu.7

It is given that, for integers $n \geq 1$, $I_n = \int_0^1 \frac{1}{(1+x^2)^n}dx$

(i) Use integration by parts to show that $I_n = 2^{-n} + 2n\int_0^1 \frac{x^2}{(1+x^2)^{n+1}}dx$ [3]

(ii) Show that $2nI_{n+1} = 2^{-n} + (2n-1)I_n$. [3]

(iii) Find I_2 in terms of π. [3]

12. Jan 2008 qu.9

(i) Prove that $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$ [3]

(ii) Hence, or otherwise, find $\int \frac{1}{\sqrt{4x^2 - 1}}\,dx$. [2]

(iii) By means of a suitable substitution, find $\int \sqrt{4x^2 - 1}\,dx$. [6]

13. June 2007 qu. 5

It is given that, for non-negative integers n, $\quad I_n = \int_1^e (\ln x)^n\,dx$.

(i) Show that, for $n \geq 1$, $\quad I_n = e - nI_{n-1}$. [4]

(ii) Find I_3 in terms of e.

[4]

14. Jan 2007 qu.5

It is given that, for non-negative integers n, $\quad I_n = \int_0^{\frac{1}{2}\pi} x^n \cos x\,dx$

(i) Prove that, for $n \geq 2$, $\quad I_n = \left(\frac{1}{2}\pi\right)^n - n(n-1)I_{n-2}$. [5]

(ii) Find I_4 in terms of π. [4]

15. Jan 2007 qu.7

(i) Express $\frac{1-t^2}{t^2(1+t^2)}$ in partial fractions. [4]

(ii) Use the substitution $t = \tan\frac{1}{2}x$ to show that $\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{\cos x}{1-\cos x}\,dx = \sqrt{3} - 1 - \frac{1}{6}\pi$. [5]

16. June 2006 qu. 5

(i) Express $t^2 + t + 1$ in the form $(t + a)^2 + b$. [1]

(ii) By using the substitution $\tan\frac{1}{2}x = t$, show that $\int_0^{\frac{1}{2}\pi} \frac{1}{2+\sin x}\,dx = \frac{\sqrt{3}}{9}\pi$ [6]

17. June 2006 qu. 9

(i) Given that $y = \sinh^{-1}x$, prove that $y = \ln(x+\sqrt{x^2+1})$ [3]

(ii) It is given that, for non-negative integers n, $I_n = \int_0^a \sinh^n \theta \, d\theta$,

where $\alpha = \sinh^{-1} 1$. Show that $nI_n = \sqrt{2} - (n-1)I_{n-2}$, for $n \geq 2$. [6]

(iii) Evaluate I_4, giving your answer in terms of $\sqrt{2}$ and logarithms. [4]

18. Jan 2006 qu.6

(i) It is given that, for non-negative integers n, $I_n = \int_0^1 e^{-x} x^n \, dx$.

Prove that, for $n \geq 1$, $I_n = nI_{n-1} - e^{-1}$ [4]

(ii) Evaluate I_3, giving the answer in terms of e. [4]