Edexcel Maths S2

Topic Questions from Papers

Hypothesis Testing

Leave blank

7. A drugs company claims that 75% of patients suffering from depression recover when treated with a new drug.

A random sample of 10 patients with depression is taken from a doctor's records.

(a) Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new drug.

(2)

Given that the claim is correct,

(b) find the probability that the treatment will be successful for exactly 6 patients.

(2)

The doctor believes that the claim is incorrect and the percentage who will recover is lower. From her records she took a random sample of 20 patients who had been treated with the new drug. She found that 13 had recovered.

(c) Stating your hypotheses clearly, test, at the 5% level of significance, the doctor's belief.

(6)

(d) From a sample of size 20, find the greatest number of patients who need to recover for the test in part (c) to be significant at the 1% level.

(4)

Leave blank

Question 7 continued

Leave blank

4. Breakdowns occur on a particular machine at random at a mean rate of 1.25 per week.

(a) Find the probability that fewer than 3 breakdowns occurred in a randomly chosen week.

(4)

Over a 4 week period the machine was monitored. During this time there were 11 breakdowns.

(b) Test, at the 5% level of significance, whether or not there is evidence that the rate of breakdowns has changed over this period. State your hypotheses clearly.

(7)

Leave blank

Question 4 continued

Leave blank

7. It is known from past records that 1 in 5 bowls produced in a pottery have minor defects. To monitor production a random sample of 25 bowls was taken and the number of such bowls with defects was recorded.

(a) Using a 5% level of significance, find critical regions for a two-tailed test of the hypothesis that 1 in 5 bowls have defects. The probability of rejecting, in either tail, should be as close to 2.5% as possible.

(6)

(b) State the actual significance level of the above test.

(1)

At a later date, a random sample of 20 bowls was taken and 2 of them were found to have defects.

(c) Test, at the 10% level of significance, whether or not there is evidence that the proportion of bowls with defects has decreased. State your hypotheses clearly.

(7)

Leave blank

Question 7 continued

Leave blank

6. Past records from a large supermarket show that 20% of people who buy chocolate bars buy the family size bar. On one particular day a random sample of 30 people was taken from those that had bought chocolate bars and 2 of them were found to have bought a family size bar.

(a) Test at the 5% significance level, whether or not the proportion p, of people who bought a family size bar of chocolate that day had decreased. State your hypotheses clearly.

(6)

The manager of the supermarket thinks that the probability of a person buying a gigantic chocolate bar is only 0.02. To test whether this hypothesis is true the manager decides to take a random sample of 200 people who bought chocolate bars.

(b) Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.02. The probability of each tail should be as close to 2.5% as possible.

(6)

(c) Write down the significance level of this test.

(1)

Leave blank

Question 6 continued

Leave blank

2. Bacteria are randomly distributed in a river at a rate of 5 per litre of water. A new factory opens and a scientist claims it is polluting the river with bacteria. He takes a sample of 0.5 litres of water from the river near the factory and finds that it contains 7 bacteria. Stating your hypotheses clearly test, at the 5% level of significance, the claim of the scientist.

(7)

Leave blank

6. Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week, with the new driver, the taxi is late 3 times.

You may assume that the number of times a taxi is late in a week has a Binomial distribution.

Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly.

(7)

Leave blank

5. Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.

Test Dhriti's claim at the 5% level of significance. State your hypotheses clearly.

(7)

Leave blank

7. (a) Explain what you understand by

(i) a hypothesis test,

(ii) a critical region.

(3)

During term time, incoming calls to a school are thought to occur at a rate of 0.45 per minute. To test this, the number of calls during a random 20 minute interval, is recorded.

(b) Find the critical region for a two-tailed test of the hypothesis that the number of incoming calls occurs at a rate of 0.45 per 1 minute interval. The probability in each tail should be as close to 2.5% as possible.

(5)

(c) Write down the actual significance level of the above test.

(1)

In the school holidays, 1 call occurs in a 10 minute interval.

(d) Test, at the 5% level of significance, whether or not there is evidence that the rate of incoming calls is less during the school holidays than in term time.

(5)

Leave blank

Question 7 continued

Leave blank

3. A test statistic has a Poisson distribution with parameter λ.

Given that

$$H_0 : \lambda = 9,\ H_1 : \lambda \neq 9$$

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.

(3)

(b) State the probability of incorrectly rejecting H_0 using this critical region.

(2)

Leave blank

5. Sue throws a fair coin 15 times and records the number of times it shows a head.

(a) State the distribution to model the number of times the coin shows a head.

(2)

Find the probability that Sue records

(b) exactly 8 heads,

(2)

(c) at least 4 heads.

(2)

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

(d) Test Sue's belief at the 1% level of significance. State your hypotheses clearly.

(6)

Leave blank

Question 5 continued

Leave blank

6. A call centre agent handles telephone calls at a rate of 18 per hour.

(a) Give two reasons to support the use of a Poisson distribution as a suitable model for the number of calls per hour handled by the agent.

(2)

(b) Find the probability that in any randomly selected 15 minute interval the agent handles

(i) exactly 5 calls,

(ii) more than 8 calls.

(5)

The agent received some training to increase the number of calls handled per hour. During a randomly selected 30 minute interval after the training the agent handles 14 calls.

(c) Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that the rate at which the agent handles calls has increased. State your hypotheses clearly.

(6)

Leave blank

Question 6 continued

Leave blank

3. A single observation x is to be taken from a Binomial distribution B(20, p).

This observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test.
The probability of rejecting either tail should be as close as possible to 2.5%.

(3)

(b) State the actual significance level of this test.

(2)

The actual value of x obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.

(2)

Leave blank

6. A web server is visited on weekdays, at a rate of 7 visits per minute. In a random one minute on a Saturday the web server is visited 10 times.

(a) (i) Test, at the 10% level of significance, whether or not there is evidence that the rate of visits is greater on a Saturday than on weekdays. State your hypotheses clearly.

(ii) State the minimum number of visits required to obtain a significant result.

(7)

(b) State an assumption that has been made about the visits to the server.

(1)

In a random two minute period on a Saturday the web server is visited 20 times.

(c) Using a suitable approximation, test at the 10% level of significance, whether or not the rate of visits is greater on a Saturday.

(6)

Leave blank

Question 6 continued

Leave blank

2. An effect of a certain disease is that a small number of the red blood cells are deformed. Emily has this disease and the deformed blood cells occur randomly at a rate of 2.5 per ml of her blood. Following a course of treatment, a random sample of 2 ml of Emily's blood is found to contain only 1 deformed red blood cell.

Stating your hypotheses clearly and using a 5% level of significance, test whether or not there has been a decrease in the number of deformed red blood cells in Emily's blood.

(6)

Leave blank

4. Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager questions whether or not there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(a) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's question. You should state the probability of rejection in each tail which should be less than 0.05.

(5)

(b) Write down the actual significance level of a test based on your critical region from part (a).

(1)

The manager found that 11 customers from the sample of 20 had bought baked beans in single tins.

(c) Comment on this finding in the light of your critical region found in part (a).

(2)

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

6. (a) Define the critical region of a test statistic.

(2)

A discrete random variable X has a Binomial distribution B(30, p). A single observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005

(5)

(c) Write down the actual significance level of the test.

(1)

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region.

(2)

Leave blank

Question 6 continued

(Total 10 marks)

Q6

Leave blank

5. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company's website at their first attempt.

(a) Explain why the Poisson distribution may be a suitable model in this case.

(1)

Find the probability that, in a randomly chosen **2 hour** period,

(b) (i) all users connect at their first attempt,

(ii) at least 4 users fail to connect at their first attempt.

(5)

The company suffered from a virus infecting its computer system. During this infection it was found that the number of users failing to connect at their first attempt, over a 12 hour period, was 60.

(c) Using a suitable approximation, test whether or not the mean number of users per hour who failed to connect at their first attempt had increased. Use a 5% level of significance and state your hypotheses clearly.

(9)

Leave blank

Question 5 continued

Leave blank

6. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample.

(2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025

(3)

(c) Find the actual significance level of this test.

(2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company's claim in the light of this value. Justify your answer.

(2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.

(6)

Leave blank

Question 6 continued

Leave blank

2. A student takes a multiple choice test. The test is made up of 10 questions each with 5 possible answers. The student gets 4 questions correct. Her teacher claims she was guessing the answers. Using a one tailed test, at the 5% level of significance, test whether or not there is evidence to reject the teacher's claim.
State your hypotheses clearly.

(6)

Leave blank

4. Richard regularly travels to work on a ferry. Over a long period of time, Richard has found that the ferry is late on average 2 times every week. The company buys a new ferry to improve the service. In the 4-week period after the new ferry is launched, Richard finds the ferry is late 3 times and claims the service has improved. Assuming that the number of times the ferry is late has a Poisson distribution, test Richard's claim at the 5% level of significance. State your hypotheses clearly.

(6)

Leave blank

2. A traffic officer monitors the rate at which vehicles pass a fixed point on a motorway. When the rate exceeds 36 vehicles per minute he must switch on some speed restrictions to improve traffic flow.

(a) Suggest a suitable model to describe the number of vehicles passing the fixed point in a 15 s interval.

(1)

The traffic officer records 12 vehicles passing the fixed point in a 15 s interval.

(b) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not the traffic officer has sufficient evidence to switch on the speed restrictions.

(6)

(c) Using a 5% level of significance, determine the smallest number of vehicles the traffic officer must observe in a 10 s interval in order to have sufficient evidence to switch on the speed restrictions.

(3)

Leave blank

Question 2 continued

Leave blank

6. A shopkeeper knows, from past records, that 15% of customers buy an item from the display next to the till. After a refurbishment of the shop, he takes a random sample of 30 customers and finds that only 1 customer has bought an item from the display next to the till.

(a) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not there has been a change in the proportion of customers buying an item from the display next to the till.

(6)

During the refurbishment a new sandwich display was installed. Before the refurbishment 20% of customers bought sandwiches. The shopkeeper claims that the proportion of customers buying sandwiches has now increased. He selects a random sample of 120 customers and finds that 31 of them have bought sandwiches.

(b) Using a suitable approximation and stating your hypotheses clearly, test the shopkeeper's claim. Use a 10% level of significance.

(8)

Leave blank

Question 6 continued

Leave blank

2. David claims that the weather forecasts produced by local radio are no better than those achieved by tossing a fair coin and predicting rain if a head is obtained or no rain if a tail is obtained. He records the weather for 30 randomly selected days. The local radio forecast is correct on 21 of these days.

Test David's claim at the 5% level of significance.

State your hypotheses clearly.

(7)

Leave blank

7. (a) Explain briefly what you understand by

(i) a critical region of a test statistic,

(ii) the level of significance of a hypothesis test.

(2)

(b) An estate agent has been selling houses at a rate of 8 per month. She believes that the rate of sales will decrease in the next month.

(i) Using a 5% level of significance, find the critical region for a one tailed test of the hypothesis that the rate of sales will decrease from 8 per month.

(ii) Write down the actual significance level of the test in part (b)(i).

(3)

The estate agent is surprised to find that she actually sold 13 houses in the next month. She now claims that this is evidence of an increase in the rate of sales per month.

(c) Test the estate agent's claim at the 5% level of significance. State your hypotheses clearly.

(5)

Leave blank

Question 7 continued

Leave blank

2. A test statistic has a distribution B(25, p).

Given that

$$H_0 : p = 0.5 \qquad H_1 : p \neq 0.5$$

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.

(3)

(b) State the probability of incorrectly rejecting H_0 using this critical region.

(2)

Leave blank

3. (a) Write down two conditions needed to approximate the binomial distribution by the Poisson distribution.

(2)

A machine which manufactures bolts is known to produce 3% defective bolts. The machine breaks down and a new machine is installed. A random sample of 200 bolts is taken from those produced by the new machine and 12 bolts were defective.

(b) Using a suitable approximation, test at the 5% level of significance whether or not the proportion of defective bolts is higher with the new machine than with the old machine. State your hypotheses clearly.

(7)

Leave blank

6. (a) Explain what you understand by a hypothesis.

(1)

(b) Explain what you understand by a critical region.

(2)

Mrs George claims that 45% of voters would vote for her.

In an opinion poll of 20 randomly selected voters it was found that 5 would vote for her.

(c) Test at the 5% level of significance whether or not the opinion poll provides evidence to support Mrs George's claim.

(4)

In a second opinion poll of n randomly selected people it was found that no one would vote for Mrs George.

(d) Using a 1% level of significance, find the smallest value of n for which the hypothesis $H_0 : p = 0.45$ will be rejected in favour of $H_1 : p < 0.45$

(3)

Leave blank

Question 6 continued

Leave blank

6. Frugal bakery claims that their packs of 10 muffins contain on average 80 raisins per pack. A Poisson distribution is used to describe the number of raisins per muffin.

A muffin is selected at random to test whether or not the mean number of raisins per muffin has changed.

(a) Find the critical region for a two-tailed test using a 10% level of significance. The probability of rejection in each tail should be less than 0.05

(4)

(b) Find the actual significance level of this test.

(2)

The bakery has a special promotion claiming that their muffins now contain even more raisins.

A random sample of 10 muffins is selected and is found to contain a total of 95 raisins.

(c) Use a suitable approximation to test the bakery's claim. You should state your hypotheses clearly and use a 5% level of significance.

(8)

Leave blank

Question 6 continued

Leave blank

3. An online shop sells a computer game at an average rate of 1 per day.

(a) Find the probability that the shop sells more than 10 games in a 7 day period.

(3)

Once every 7 days the shop has games delivered before it opens.

(b) Find the least number of games the shop should have in stock immediately after a delivery so that the probability of running out of the game before the next delivery is less than 0.05

(3)

In an attempt to increase sales of the computer game, the price is reduced for six months. A random sample of 28 days is taken from these six months. In the sample of 28 days, 36 computer games are sold.

(c) Using a suitable approximation and a 5% level of significance, test whether or not the average rate of sales per day has increased during these six months. State your hypotheses clearly.

(7)

Leave blank

Question 3 continued

Leave blank

6. In a manufacturing process 25% of articles are thought to be defective. Articles are produced in batches of 20

(a) A batch is selected at random. Using a 5% significance level, find the critical region for a two tailed test that the probability of an article chosen at random being defective is 0.25
You should state the probability in each tail which should be as close as possible to 0.025

(5)

The manufacturer changes the production process to try to reduce the number of defective articles. She then chooses a batch at random and discovers there are 3 defective articles.

(b) Test at the 5% level of significance whether or not there is evidence that the changes to the process have reduced the percentage of defective articles. State your hypotheses clearly.

(5)

Leave blank

Question 6 continued

Q6

(Total 10 marks)