Edexcel Maths M3

Topic Questions from Papers

Strings & Springs

3.	A particle $P$ of mass $m$ is attached to one end of a light elastic string, of natural length $a$ and modulus of elasticity 3.6 $mg$ . The other end of the string is fixed at a point $O$ on a rough horizontal table. The particle is projected along the surface of the table from $O$ with speed $\sqrt{(2ag)}$ . At its furthest point from $O$ , the particle is at the point $A$ , where $OA = \frac{4}{3}a$ .	
	(a) Find, in terms of $m$ , $g$ and $a$ , the elastic energy stored in the string when $P$ is at $A$ . (3)	
	(b) Using the work-energy principle, or otherwise, find the coefficient of friction between $P$ and the table. (6)	
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A light elastic string of natural length $0.4$ m has one end $A$ attached to The other end of the string is attached to a particle $P$ of mass 2 kg. We equilibrium vertically below $A$ , the length of the string is $0.56$ m.	
(a) Find the modulus of elasticity of the string.	(3)
A horizontal force is applied to $P$ so that it is held in equilibrium with the an angle $\theta$ with the downward vertical. The length of the string is now 0.7	
(b) Find the angle $\theta$ .	(3)



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4.	4. A particle $P$ of mass $m$ lies on a smooth plane inclined at an angle 30° to the The particle is attached to one end of a light elastic string, of natural le modulus of elasticity $2mg$ . The other end of the string is attached to a fixed the plane. The particle $P$ is in equilibrium at the point $A$ on the plane and the of the string is $\frac{1}{4}a$ . The particle $P$ is now projected from $A$ down a line of growth plane with speed $V$ . It comes to instantaneous rest after moving a disconnection of the plane with speed $V$ .	
	By using the principle of conservation of energy,	
	(a) find $V$ in terms of $a$ and $g$ ,	(6)
	(b) find, in terms of $a$ and $g$ , the speed of $P$ when the string first becomes slack.	(4)

**(5)** 

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- 1. A particle *P* of mass 2 kg is attached to one end of a light elastic string, of natural length 1 m and modulus of elasticity 98 N. The other end of the string is attached to a fixed point *A*. When *P* hangs freely below *A* in equilibrium, *P* is at the point *E*, 1.2 m below *A*. The particle is now pulled down to a point *B* which is 0.4 m vertically below *E* and released from rest.
  - (a) Prove that, while the string is taut, P moves with simple harmonic motion about E with period  $\frac{2\pi}{7}$ s.
  - (b) Find the greatest magnitude of the acceleration of P while the string is taut. (1)
  - (c) Find the speed of P when the string first becomes slack. (3)
  - (d) Find, to 3 significant figures, the time taken, from release, for *P* to return to *B* for the first time.

(7)

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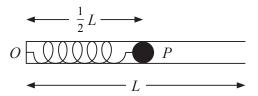


Figure 1

A light elastic spring, of natural length L and modulus of elasticity  $\lambda$ , has a particle P of mass m attached to one end. The other end of the spring is fixed to a point O on the closed end of a fixed smooth hollow tube of length L.

The tube is placed horizontally and P is held inside the tube with  $OP = \frac{1}{2}L$ , as shown

in Figure 1. The particle P is released and passes through the open end of the tube with speed  $\sqrt{(2gL)}$ .

(a) Show that  $\lambda = 8mg$ . (4)

The tube is now fixed vertically and P is held inside the tube with  $OP = \frac{1}{2}L$  and P above O. The particle P is released and passes through the open top of the tube with speed u.

(b)	Find <i>u</i> .	
		<b>(5)</b>



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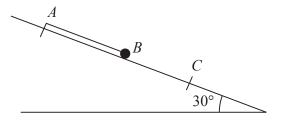


Figure 2

One end A of a light elastic string, of natural length a and modulus of elasticity 6mg, is fixed at a point on a smooth plane inclined at  $30^{\circ}$  to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and AB = a. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in Figure 2. Find

(a) the length AC,

**(5)** 

(b) the greatest speed attained by B as it moves from its initial position to C.

**(7)** 

Question 5 continued	



1.	A light elastic string has natural length 8 m and modulus of elasticity 80 N.	
	The ends of the string are attached to fixed points $P$ and $Q$ which are on the shorizontal level and 12 m apart. A particle is attached to the mid-point of the string hangs in equilibrium at a point 4.5 m below $PQ$ .	same g and
	(a) Calculate the weight of the particle.	(6)
		(6)
	(b) Calculate the elastic energy in the string when the particle is in this position.	(3)

7. A light elastic string has natural length a and modulus of elasticity  $\frac{3}{2}mg$ . A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically. When P has fallen a distance a + x, where x > 0, the speed of P is v.

(a) Show that  $v^2 = 2g(a+x) - \frac{3gx^2}{2a}$ .

(b) Find the greatest speed attained by P as it falls.

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After release, P next comes to instantaneous rest at a point D.

(c) Find the magnitude of the acceleration of P at D.

**(6)** 

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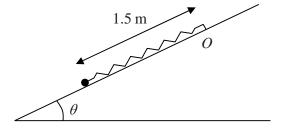


Figure 2

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity  $\lambda$  newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = \frac{3}{5}$ . The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O, as shown in Figure 2. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of $\lambda$ .	



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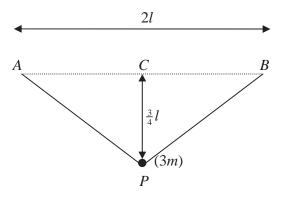


Figure 4

A small ball of mass 3m is attached to the ends of two light elastic strings AP and BP, each of natural length l and modulus of elasticity kmg. The ends A and B of the strings are attached to fixed points on the same horizontal level, with AB = 2l. The mid-point of AB is C. The ball hangs in equilibrium at a distance  $\frac{3}{4}l$  vertically below C as shown in Figure 4.

(a) Show that k = 10 (7)

The ball is now pulled vertically downwards until it is at a distance  $\frac{12}{5}l$  below C. The ball is released from rest.

(b) Find the speed of the ball as it reaches *C*.




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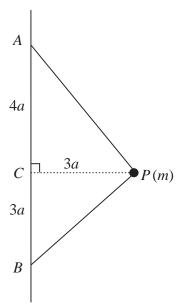


Figure 4

A light inextensible string has its ends attached to two fixed points A and B. The point A is vertically above B and AB = 7a. A particle P of mass m is fixed to the string and moves in a horizontal circle of radius 3a with angular speed  $\omega$ . The centre of the circle is C where C lies on AB and AC = 4a, as shown in Figure 4. Both parts of the string are taut.

(a) Show that the tension in AP is  $\frac{5}{7}m(3a\omega^2 + g)$ .

(b) Find the tension in BP.

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(c) Deduce that  $\omega \geqslant \frac{1}{2} \sqrt{\left(\frac{g}{a}\right)}$ . (2)

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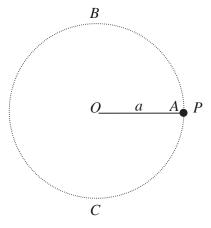


Figure 5

A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where OA = a and OA is horizontal. The point B is vertically above O and the point C is vertically below O, with OB = OC = a, as shown in Figure 5. The particle is projected vertically upwards with speed  $3\sqrt{(ag)}$ .

(a) Show that P will pass through B.

**(6)** 

(b) Find the speed of P as it reaches C.

**(2)** 

As P passes through C it receives an impulse. Immediately after this, the speed of P is  $\frac{5}{12}\sqrt{(11ag)}$  and the direction of motion of P is unchanged.

(c) Find the angle between the string and the downward vertical when P comes to instantaneous rest.

**(4)** 




Question 6 continued	Leave blank
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from rest at A and comes to instantaneous rest 1.1 m below	711.
Find the modulus of elasticity of the string.	(4)

4.	A light elastic string $AB$ has natural length 0.8 m and modulus of elasticity 19.6 N. The end $A$ is attached to a fixed point. A particle of mass 0.5 kg is attached to the end $B$ . The particle is moving with constant angular speed $\omega$ rad s <sup>-1</sup> in a horizontal circle whose cent is vertically below $A$ . The string is inclined at 60° to the vertical.	blank he he
	(a) Show that the extension of the string is 0.4 m.	5)
	(b) Find the value of $\omega$ .	5)
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6. A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is hanging in equilibrium at the point A, vertically below O, when it is set in motion with a horizontal speed  $\frac{1}{2}\sqrt{(11gl)}$ . When the string has turned through an angle  $\theta$  and the string is still taut, the tension in the string is T.

(a) Show that  $T = 3mg\left(\cos\theta + \frac{1}{4}\right)$ . (8)

At the instant when P reaches the point B, the string becomes slack. Find

(b) the speed of P at B,

**(3)** 

(c) the maximum height above B reached by P before it starts to fall.

**(4)** 




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A particle <i>B</i> of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.75 m and modulus of elasticity 24.5 N. The other end of the string is attached to a fixed point <i>A</i> . The particle is hanging in equilibrium at the point <i>E</i> , vertically below <i>A</i> .
(a) Show that $AE = 0.9 \text{ m}$ . (3)
The particle is held at $A$ and released from rest. The particle first comes to instantaneous rest at the point $C$ .
(b) Find the distance AC. (5)
(c) Show that while the string is taut, <i>B</i> is moving with simple harmonic motion with centre <i>E</i> .
(d) Calculate the maximum speed of <i>B</i> .
(2)



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7.	A particle $P$ of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity $\lambda$ newtons. The ends of the string are attached to two fixed points $A$ and $B$ , where $AB$ is horizontal and $AB = 0.48$ m. Initially $P$ is held at rest at the mid-point, $M$ , of the line $AB$ and the tension in the string is 240 N.				
	(a) Show that $\lambda = 400$				
	The particle is now held at rest at the point $C$ , where $C$ is 0.07 m vertically below $M$ . The particle is released from rest at $C$ .				
	(b) Find the magnitude of the initial acceleration of <i>P</i> . (6)				
	(c) Find the speed of $P$ as it passes through $M$ .				

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4.	A particle $P$ of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m. The other end of the string is attached to a fixed point $O$ on a rough horizontal	Dia
	plane. The coefficient of friction between P and the plane is $\frac{2}{5}$ . The particle is held at	
	rest at a point $B$ on the plane, where $OB = 1.5$ m. When $P$ is at $B$ , the tension in the string is 20 N. The particle is released from rest.	
	(a) Find the speed of $P$ when $OP = 1.2$ m. (7)	
	The particle comes to rest at the point $C$ .	
	(b) Find the distance <i>BC</i> . (2)	

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