1. Three warehouses *W*, *X* and *Y* supply televisions to three supermarkets *J*, *K* and *L*. The table gives the cost, in pounds, of transporting a television from each warehouse to each supermarket. The warehouses have stocks of 34, 57 and 25 televisions respectively, and the supermarkets require 20, 56 and 40 televisions respectively. The total cost of transporting the televisions is to be minimised.

	J	*K*	*L*
W	3	6	3
X	5	8	4
Y	2	5	7

Formulate this transportation problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

(Total 6 marks)

2. A two person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	–4	5	1
A plays 2	3	–1	–2
A plays 3	–3	0	2

Formulate the game as a linear programming problem for player A. Write the constraints as inequalities and define your variables.

(Total 7 marks)

3. Laura (L) and Sam (S) play a two-person zero-sum game which is represented by the following pay-off matrix for Laura.

	S plays 1	S plays 2	S plays 3
L plays 1	–2	8	–1
L plays 2	7	4	–3
L plays 3	1	–5	4

Formulate the game as a linear programming problem for Laura, writing the constraints as inequalities. Define your variables clearly.

(Total 7 marks)

4. Anna (A) and Roland (R) play a two-person zero-sum game which is represented by the following pay-off matrix for Anna.

	R plays 1	R plays 2	R plays 3
A plays 1	6	–2	–3
A plays 2	–3	1	2
A plays 3	5	4	–1

Formulate the game as a linear programming problem **for player R**. Write the constraints as inequalities. Define your variables clearly.

(Total 8 marks)

1. Let x_ij be number of units transported from i to j

Where $i \in \{W, X, Y\}$ and $j \in \{J, K, L]$

Warehouse Supermarket B1

Objective minimise "c" = $3x_{WJ} + 6x_{WK} + 3x_{WL} + 5x_{XJ} + 8x_{Xk} + 4x_{xl} + 2x_{YJ} + 5x_{Yk} + 7x_{Yl}$ B1

Subject to $x_{WJ} + x_{WK} + x_{WL} = 34$

$x_{XJ} + x_{XK} + x_{XL} = 57$ M1 A1

$x_{YJ} + x_{YK} + x_{YL} = 25$ A1

$x_{WJ} + x_{XJ} + x_{YJ} = 20$

$x_{WK} + x_{XK} + x_{YK} = 56$

$x_{WL} + x_{XL} + x_{YL} = 40$

$x_{ij} > 0 \ \forall i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ B1 6

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2. $\begin{bmatrix} -4 & 5 & 1 \\ 3 & -1 & -2 \\ -3 & 0 & 2 \end{bmatrix} \rightarrow$ add 5 to all entries $\begin{bmatrix} 1 & 10 & 6 \\ 8 & 4 & 3 \\ 2 & 5 & 7 \end{bmatrix}$ M1

Either	Or	
Define variables e.g. let p_1, p_2 and p_3 be the probability that A plays rows 1, 2 and 3 respectively.	Define variables e.g. let p_1, p_2 and p_3 be the probability that A plays rows 1, 2 and 3 respectively.	B1
	Let $x_i = \frac{p_i}{V}$	
Maximise P = V	Minimise P $= x_1 + x_2 + x_3$	B1
Subject to: $V - p_1 - 8p_2 - 2p_3 \le 0$	Subject to $x_1 + 8x_2 + 2x_3 \ge 1$	M1
$V - 10p_1 - 4p_2 - 5p_3 \le 0$	$10x_1 + 4x_2 + 5x_3 \ge 1$	A1
$V - 6p_1 - 3p_2 - 7p_3 \le 0$	$6x_1 + 3x_2 + 7x_3 \ge 1$	A1
$p_1 + p_2 + p_3 \le 1$	$x_1, x_2, x_3 \ge 0$	A1
$p_1, p_2, p_3 \ge 0$		

Note

1M1: Adding n (≥ 4) to all entries
1B1: Defining variables
1B1: Objective correct
2M1: At least 3 constraints, using columns, one of correct form
1A1ft: one correct constraint – excluding non-negativity constraint
2A1ft: two correct constraints – excluding non-negativity constraint
3A1: cao including non-negativity constraint

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3. E.g. Add 6 to make all elements positive $\begin{bmatrix} 4 & 14 & 5 \\ 13 & 10 & 3 \\ 7 & 1 & 10 \end{bmatrix}$ B1

Let Laura play 1, 2 and 3 with probabilities p_1, p_2 and p_3 respectively

Let V = value of game + 6 B1

e.g.

Maximise P = V

Subject to:

$V - 4p_1 - 13p_2 - 7p_3 \le 0$ M1

$V - 14p_1 - 10p_2 - p_3 \le 0$ A3 2ft 1ft 0

$V - 5p_1 - 3p_2 - 10p_3 \le 0$

$p_1 + p_2 + p_3 \le 1$

$p_1, p_2, p_3 \ge 0$

Note

1B1: Making all elements positive

2B1: Defining variables

3B1: Objective, cao word and function

1M1: At least one constraint in terms of their variables, must be going down columns.

Accept = here.

1A1ft: ft their table. One constraint in V correct.

2A1ft: ft their table. Two constraints in V correct.

3A1: CAO all correct.

Alt using x_i method

Now additionally need: let $x_i = \frac{Pi}{v}$ for 2B1

minimise (P) = $x_1 + x_2 + x_3 = \frac{1}{v}$

subject to:

$4x_1 + 13x_2 + 7x_3 \geq 1$

$14x_1 + 10x_2 + x_3 \geq 1$

$5x_1 + 3x_2 + 10x_3 \geq 1$

$x_i \geq 0$

[7]

4. Alt 1

Game from R's point of view.

	A1	A2	A3
R_1	–6	3	–5
R_2	2	–1	–4
R_3	3	–2	1

Add 7

	A1	A2	A3
R_1	1	10	2
R_2	9	6	3
R_3	10	5	8

B1, B1

Let R play 1 with probability P_1
2 with probability P_2
3 with probability P_3 B1
V = value of the game

Maximise P = V B1

Subject to $V - P_1 - 9P_2 - 10P_3 \leq 0$ M1A1ft
$V - 10P_1 - 6P_2 - 5P_3 \leq 0$ A1ft
$V - 2P_1 - 3P_2 - 8P_3 \leq 0$ A1ft
$P_1 + P_2 + P_3 \leq 1$ accept= A1 8
$V, P_1, P_2, P_3 \geq 0$

Alt 2

Add 4 to all entries B1

	R_1	R_2	R_3
A1	10	2	1
A2	1	5	6
A3	9	8	3

Let R play 1 with probability P_1
2 with probability P_2
3 with probability P_3 let V = value of game. B1

Let $x_1 = \frac{P_1}{V}, x_2 = \frac{P_2}{V}, x_3 = \frac{P_3}{V}$ B1

Maximise $P = x_1 + x_2 + x_3$ B1

Subject to $10x_1 + 2x_2 + x_3 \leq 1$ M1A1ft
$x_1 + 5x_2 + 6x_3 \leq 1$ A1ft
$9x_1 + 8x_2 + 3x_3 \leq 1$ A1
$x_1, x_2, x_3 \geq 0$ accept $P_i \geq 0$

[8]

1. No Report available for this question.

2. Candidates commonly either obtained very few marks for this question, or virtually full marks. Some did not attempt the question. Some attempted to solve the problem instead of formulating it as a linear programming problem. Many failed to add 5 to the elements and/or define their probabilities and/or state that they were maximising $P = v$. Many attempts involved 9 variables. Many used the rows of the table rather than the columns and some used slack variables and equalities rather than the inequalities requested. Some omitted the non-negativity constraints. Despite this a good number, nearly 20% were able to get full marks on this question.

3. This proved to be the most challenging question on the paper, with many attempts marking little progress, but very few blanks seen.. Many candidates did not answer the question as stated and did not give their answer from Laura's point of view and did not give their constraints as inequalities. Very few added 6 to all terms to make the entries positive. Most did attempt to define their variables, but some tried to structure the problem in terms of nine unknowns, confusing Game Theory with Transportation or Allocation. Many stated an objective but then either did not state if they were maximising or minimising or selected the wrong one. The majority of candidates used rows instead of columns to set up the constraints and those that sis correctly use columns often made mistakes with the inequality signs. Very many candidates unnecessarily made two attempts at the question, firstly in terms of V and p's and then in terms of x.

4. This question was poorly done by a large number of candidates. There are two common approaches to this question. The majority of candidates used the 'dividing by V' method (alt 2 on the mark scheme). Common errors included failing to add 4 to the matrix, not defining their probabilities and not realising that they should be maximizing 1/V. When setting up the inequalities, some candidates used columns instead of rows, some did not fully divide by V and some reversed the inequalities or incorrectly introduced slack variables. Some candidates also forgot the non-negativity constraints. Those candidates who chose the 'transposing' method (alt 1 on the mark scheme) generally produced a better attempt, although a significant number did not correctly perform all three operations on the matrix (transpose, change signs and add 7). Many weaker candidates produced highbred versions incorporating elements of both methods. A substantial minority introduced 9 unknowns, often treating it as an allocation problem.