1. $f(\theta) = 4\cos^2\theta - 3\sin^2\theta$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta\, f(\theta)\, d\theta$

(7)
(Total 10 marks)

2. The curve C has the equation

$$\cos 2x + \cos 3y = 1, \qquad -\frac{\pi}{4} \le x \le \frac{\pi}{4},\ 0 \le y \le \frac{\pi}{6}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.

(b) Find the value of y at P.

(3)

(c) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)
(Total 9 marks)

3. On separate diagrams, sketch the curves with equations

(a) $y = \arcsin x, \quad -1 \le x \le 1,$

(b) $y = \sec x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$, stating the coordinates of the end points of your curves in each case.

(4)

Use the trapezium rule with five equally spaced ordinates to estimate the area of the region bounded by the curve with equation $y = \sec x$, the x-axis and the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, giving your answer to two decimal places.

(4)
(Total 8 marks)

1. (a) $\mathrm{f}(\theta) = 4\cos^2\theta - 3\sin^2\theta$

$= 4\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$ M1 M1

$= \frac{1}{2} + \frac{7}{2}\cos 2\theta \quad *$ cso A1 3

(b) $\int \theta\cos 2\theta \,\mathrm{d}\theta = \frac{1}{2}\theta\sin 2\theta - \frac{1}{2}\int \sin 2\theta \,\mathrm{d}\theta$ M1 A1

$= \frac{1}{2}\theta\sin 2\theta + \frac{1}{4}\cos 2\theta$ A1

$\int \theta\,\mathrm{f}(\theta)\,\mathrm{d}\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta\sin 2\theta + \frac{7}{8}\cos 2\theta$ M1 A1

$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16} + 0 - \frac{7}{8}\right] - \left[0 + 0 + \frac{7}{8}\right]$ M1

$= \frac{\pi^2}{16} - \frac{7}{4}$ A1 7

[10]

2. (a) $-2\sin 2x - 3\sin 3y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ M1 A1

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}$, $\frac{-2\sin 2x}{3\sin 3y}$ A1 3

(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$ M1

$\cos 3y = \frac{1}{2}$ A1

$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349 A1 3

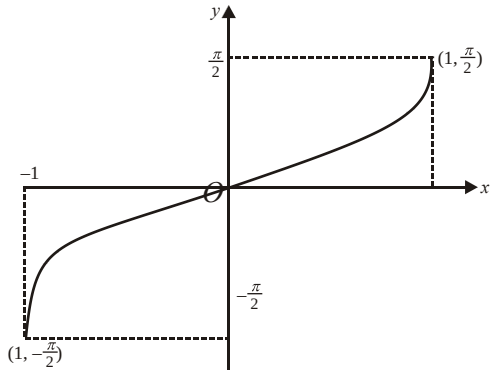
(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$ M1

$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$ M1

Leading to $6x + 9y - 2\pi = 0$ A1 3

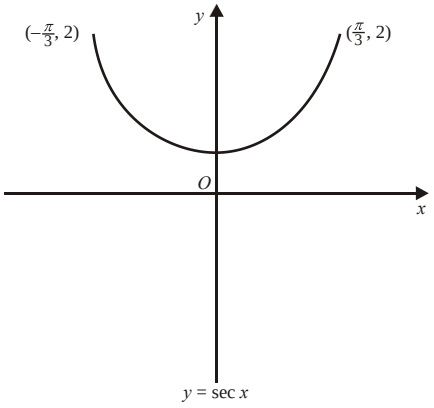
[9]

3. (a)



$y = \arcsin x$

(a) Shape correct
passing through O: G1;
end-points: G1 2



(b)

Shape correct,
symmetry in Oy: G1
end-points: G1 2

(c)

x	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
sec x	2	1.155	1	1.155	2

Area estimate = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec x \, dx = \frac{\pi}{6}\left[\frac{2+2}{2} + 1.155 + 1 + 1.155\right]$ M1 A1 A1

= 2.78 (2 d.p.) A1 4

[8]

1. Candidates tended either to get part (a) fully correct or make no progress at all. Of those who were successful, most replaced the $\cos^2\theta$ and $\sin^2\theta$ directly with the appropriate double angle formula. However many good answers were seen which worked successfully via $7\cos^2\theta - 3$ or $4 - 7\sin^2\theta$.

Part (b) proved demanding and there were candidates who did not understand the notation θf(θ). Some just integrated f(θ) and others thought that θf(θ) meant that the argument 2θ in $\cos 2\theta$ should be replaced by θ and integrated $\frac{1}{2}\theta + \frac{7}{2}\cos\theta$. A few candidates started by writing $\int\theta\text{f}(\theta)\text{d}\theta = \theta\int\text{f}(\theta)\text{d}\theta$, treating θ as a constant. Another error seen several times was $\int\theta\,\text{f}(\theta)\text{d}\theta = \int\left(\frac{1}{2}\theta + \frac{7}{2}\cos 2\theta^2\right)\text{d}\theta$.

Many candidates correctly identified that integration by parts was necessary and most of these were able to demonstrate a complete method of solving the problem. However there were many errors of detail, the correct manipulation of the negative signs that occur in both integrating by parts and in integrating trigonometric functions proving particularly difficult. Only about 15% of candidates completed the question correctly.

2. As has been noted in earlier reports, the quality of work in the topic of implicit differentiation has improved in recent years and many candidates successfully differentiated the equation and rearranged it to find $\frac{\text{d}y}{\text{d}x}$. Some, however, forgot to differentiate the constant. A not infrequent, error was candidates writing $\frac{\text{d}y}{\text{d}x} = -2\sin 2x - 3\sin 3y\ \frac{\text{d}y}{\text{d}x}$ and then incorporating the superfluous $\frac{\text{d}y}{\text{d}x}$ on the left hand side of the equation into their answer. Errors like $\frac{\text{d}y}{\text{d}x}\ (\cos 3y) = -\frac{1}{3}\sin 3y$.

were also seen. Part (b) was very well done. A few candidates gave the answer 20° , not recognising that the question required radians. Nearly all knew how to tackle part (c) although a few, as in Q2, spoilt otherwise completely correct solutions by not giving the answer in the form specified by the question.

3. No Report available for this question.