FP1 Matrices Questions

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(ii) Calculate the matrix product $\mathbf{A}^2$. *(2 marks)*

(b) The matrix $\mathbf{B}$ is defined by

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(i) Calculate $\mathbf{B}^2 - \mathbf{A}^2$. *(3 marks)*

(ii) Calculate $(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A})$. *(3 marks)*

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5 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix:

(i) $\mathbf{M}^2$; *(3 marks)*

(ii) $\mathbf{M}^4$. *(1 mark)*

(c) Find the matrix $\mathbf{M}^{2006}$. *(3 marks)*

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2 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{BA}$. *(3 marks)*

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1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. *(2 marks)*

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

FP1 Matrices Answers

(ii)	$\mathbf{A}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	M1A1	2	M1A0 for three correct entries
(b)(i)	$\mathbf{B}^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$	M1A1		M1A0 for three correct entries
	$\mathbf{B}^2 - \mathbf{A}^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$	A1√	3	ft errors, dependent on both M marks
(ii)	$(\mathbf{B}+\mathbf{A})(\mathbf{B}-\mathbf{A}) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$	B1		
	$\ldots = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$	M1 A1√	3	ft one error; M1A0 for three correct (ft) entries

5(a)(i)	$\mathbf{M}^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	M1 A2,1	3	M1 if 2 entries correct M1A1 if 3 entries correct
(ii)	$\mathbf{M}^4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	B1√	1	ft error in $\mathbf{M}^2$ provided no surds in $\mathbf{M}^2$
(c)	Awareness of $\mathbf{M}^8 = \mathbf{I}$ $\mathbf{M}^{2006} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	M1 m1 A1√	3	OE; NMS 2/3 complete valid method ft error in $\mathbf{M}^2$ as above

2(a)(i)	$\mathbf{A}+\mathbf{B} = \begin{bmatrix} \sqrt{3} & 0 \\ 1 & 0 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct; Condone $\frac{2\sqrt{3}}{2}$ for $\sqrt{3}$
(ii)	$\mathbf{BA} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	B3,2,1	3	Deduct one for each error; **SC** B2,1 for **AB**

1(a)	$\mathbf{M} = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}$	B2,1	2	B1 if subtracted the wrong way round
(c)	$\mathbf{M}^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ $\ldots = 9\mathbf{I}$	B1F B1F	 2	Or by geometrical reasoning; ft as before ft as before