Evelyn Lamb:	00:00	Hello, and welcome to the Lathisms podcast. I'm your host, Evelyn Lamb. In each episode, we invite Hispanic or Latinxs mathematicians to share their journey in mathematics.
Evelyn Lamb:	00:21	Today, I'm very happy to be talking with Enrique Trevino. Hi, how are you today?
Enrique Trevino:	00:27	Very good. Thank you for having me here.
Evelyn Lamb:	00:30	Could you tell us a little bit about yourself?
Enrique Trevino:	00:32	Yes. My name is Enrique Trevino, and I'm from Sierra Juarez, Mexico. Well, I grew up on the border between Juarez and El Paso. I studied before undergrad in Mexico, and then undergrad in the US. I studied in the University of Texas El Paso, while still living in Juarez. So, I've crossed the border over 1000 times. I did my PhD at Dartmouth College. My area of specialization is number theory.
Enrique Trevino:	01:03	I then worked two years in Swarthmore. Then, I've been at Lake Forest College for the last six years.
Evelyn Lamb:	01:10	Nice. Maybe we can jump right into your research. You say you specialize in number theory. What kinds of questions in number theory really excite you?
Enrique Trevino:	01:20	I really like questions that are easy to understand. A lot of my papers concern what's called the least quadratic non residue.
Enrique Trevino:	01:29	When I was in high school, my first eye-opening experience was finding out that the sequence two, five, eight, 11, 14, and so on, where the difference between consecutive elements is three. That sequence does not contain any perfect squares. I was really inspired by this. When I saw the proof, it's very simple proof using congruence modulo three.
Evelyn Lamb:	01:57	Wow. I'm sitting here thinking, really? Is that really true? I didn't know this fact. I'm going to sit down later, and try to write down, and see if I can prove it wrong.
Enrique Trevino:	02:09	Yes. It's pretty simple. If X is ... Well, maybe you want to prove it yourself, so I won't spoil it. Or, should I?
Evelyn Lamb:	02:17	You can go ahead, yeah.

Evelyn Lamb:	02:19	Dear listeners, if you don't want the spoiler, just skip forward 15 seconds.
Enrique Trevino:	02:24	Okay. If X is zero mod three, then X squared is zero. If X is one mod three, then X squared is one. If X is two mod three, then X squared is four, which is one. You always get zero, one, you never get two mod three.
Evelyn Lamb:	02:38	Oh, okay. I guess I won't try to prove it wrong. I'm pretty convinced at this point.
Enrique Trevino:	02:46	Then, we say that two is not a quadratic resident mod three, because of this property. If you pick a prime P, then there're some remainders that can be squares, and some that cannot be squares. They're actually spread equally. If you avoid zero, then the rest, half of them, are quadratic residues, and half of them are not.
Enrique Trevino:	03:14	Interesting problem is, which one is the least quadratic non residue? That's something I've written a lot about.
Evelyn Lamb:	03:24	So, that means, given a prime P, you're looking for what the smallest number is, for that prime, that isn't a quadratic residue mod P?
Enrique Trevino:	03:34	Correct.
Enrique Trevino:	03:35	For example, if you take the prime seven, then the smallest quadratic non residue this three. One is obviously a quadratic residue, because one squared is one. And, two modulo seven is a quadratic residue, because three squared is nine, which is two mod seven. So, the first one that is not is three mod seven.
Evelyn Lamb:	03:59	I suppose one natural question would be whether there's a pattern to which numbers are these least quadratic residues. Is that correct?
Enrique Trevino:	04:10	That is a nice question. If you just fix ... Always, the least quadratic non reside is a prime. That's because this thing is multiplicative. If you multiply two residues, you get a residue. If you multiply two non residues, you get a residue. It's like a parity thing, it forces the least one to be prime.
Enrique Trevino:	04:40	Then, suppose you say, well I want the least quadratic non residue to be five. What do the primes have to satisfy? Then, you can use something called quadratic reciprocity, which was Gauss' favorite theorem, and figure out what remainders would

work. What properties does a prime have to have so that it's least quadratic number residue is five? You can answer that precisely.

Enrique Trevino:	05:09	What I usually study is how big can the least quadratic number residue be? That is harder, because it's a little more broad.
Evelyn Lamb:	05:22	Is it known that it could be arbitrarily large, or is that not the case?
Enrique Trevino:	05:29	It can be arbitrarily large, so the question is, how large with respect to P?
Evelyn Lamb:	05:34	Right, okay.
Enrique Trevino:	05:35	For example, there's a nice proof that it's less than, say, square root of P plus one. But, how small can you get it? If you think about it probabilistically, if you model by saying, okay, we'll just flip a coin and decide if a number is a residue or non residue, then this suggests, if you do some probability, that the balance should be Log times Log, Log. So, very small.
Enrique Trevino:	06:08	What we can actually prove is nowhere near that good. It's something like P to the one-sixth.
Evelyn Lamb:	06:14	Okay. Yeah, that's a lot of interesting questions for our listeners to think about.
Evelyn Lamb:	06:22	Backing up many years, how did you get interested in math?
Enrique Trevino:	06:28	I was always pretty good at math at school, I liked to be the fastest when we were in elementary school. Then, what really got me very interested was the Mathematical Olympiad. When I was a junior, or maybe a sophomore in high school, someone invited me to go to these special training sessions for math competitions. That's where I found out this property, that two, five, eight, never has a square. I really fell in love with math at that moment.
Enrique Trevino:	07:05	I always knew I was good in math, but I didn't realize that math had this incredibly creative side, and that it was something so exciting. I always was good at it, but I didn't know how interesting the field was, until I was exposed to it in the Math Olympiad.
Evelyn Lamb:	07:29	You competed, then, for Mexico?

Enrique Trevino:	07:31	Well, it depends what you count as competing.
Enrique Trevino:	07:35	The Mexican Math Olympiad, each state ... There's 32 states in Mexico, although when I was in high school there were 31 states, and the Federal District. Each state sent six people to the competition, and I was one of the six from Chihuahua. Then, at this competition, they select the best 16 in the country. I managed to pass that threshold, as well, in my senior year in high school. Then, out of those 16, they train us for months, and then they select the best six for the International Math Olympiad. I was ranked 10th, so I didn't make that final cut.
Evelyn Lamb:	08:21	Ah. The experience sounds like it was a pretty positive one for you, anyway?
Enrique Trevino:	08:24	Oh, it was extremely positive. Now, I'm extremely involved with it. Just last month, I was at the [Ibero 00:08:31] Math Olympiad, as the leader of the Mexican team. I've been training students for the competition since 2004.
Evelyn Lamb:	08:40	Do you feel like, once you saw this creative side of math, through being in the Olympiad, do you think that's what really ... Do you think at that point, you knew you wanted to be a mathematician?
Enrique Trevino:	08:53	When I was in high school, I loved programming. I started programming when I was in fifth grade, and I loved solving puzzles with respect to programming. I thought I wanted to be a programmer. The Math Olympiad made me consider math as an alternative possibility.
Enrique Trevino:	09:15	When I started undergrad, I was a math, and a computer science major. In my computer science classes, I realized the only thing I liked was programming, I didn't like any of the other stuff. Most of what I liked was already in math. I ended up just graduating with a math major and a CS minor.
Evelyn Lamb:	09:38	Did you go to grad school directly from that?
Enrique Trevino:	09:40	Yes, I went straight to grad school.
Evelyn Lamb:	09:44	Were you encouraged by your math professors, at UTEP, to pursue math as a career, and to pursue grad school?
Enrique Trevino:	09:53	Yes, I was ... I started, actually, not at UTEP. I started at another institution, and I missed home so I transferred back to live home.

Evelyn Lamb:	10:01	Okay.
Enrique Trevino:	10:03	One of the ... My dad was not in favor of me transferring, he thought I was changing to a weaker school. I had to convince myself that there were good reasons for the change, so I changed from a Liberal Arts college, to a bigger college. One of the reasons I thought was, I can take graduate classes, and I can be very involved with the Mexican Math Olympiad, if I live at home.
Enrique Trevino:	10:31	Immediately as a sophomore, I started taking grad classes, I took a lot of theory. The first fall I was at UTEP I was at a sophomore, and the professor thought I was very good, so he immediately encouraged me to apply to a scholarship. Then, I got a nice scholarship, so yeah I guess he was very encouraging. Then, this professor wrote letters of recommendation when I was applying to grad school.
Evelyn Lamb:	10:59	How was the transition from college to grad school, especially moving such a far distance away from your family?
Enrique Trevino:	11:07	Well, it was easy for me. I like traveling a lot, and I like new experiences. When I was an undergrad, I studied abroad in Budapest. I guess this sounds contradictory, in the sense that I moved back because I missed home in undergrad, and then it was easy in grad school. I guess, something changed in those three years.
Enrique Trevino:	11:33	I loved my experiences in grad school, and I loved experiencing snow for the first time. Well, sort of the first time, it does snow in Juarez. But, not the whole winter.
Evelyn Lamb:	11:48	Right.
Enrique Trevino:	11:48	It's one or two snows, yeah.
Enrique Trevino:	11:51	The change was not too difficult. I think, because of the Math Olympiad, I had been traveling a lot in Mexico to take students to competitions. I was very aware of math in different locations, and I loved the graduate school vibe. I loved being independent. Yeah, I really enjoyed that transition.
Evelyn Lamb:	12:17	I'd imagine, it sounds like you had a really wonderful time there, but there are always challenges in any career, whether at grad school, or when you're applying for jobs or anything. How have you dealt with challenges like that, in your career?

| Enrique Trevino: | 12:34 | Yes, I had ... For example, I had a serious challenge in my second year in grad school, when I interpreted the rules a little differently than the professors there. I thought the classes were optional, so I took a summer job doing research with undergrads, at Trinity University. The University wanted me to take an algebraic topology class. They took away my stipend for a summer, and I was very upset about it. But, I decided to ... I was a little stubborn, so then I decided to prove them wrong, that this was a bad decision. I still worked in Trinity, and then I learned algebraic topology on my own. Then, my goal was to be the first to finish the quals, and I succeed in my goal by two days. I just wanted to prove them wrong, that they shouldn't have taken the stipend, that I was correct in doing research over the summer, and that I could learn on my own. |

| Enrique Trevino: | 13:45 | I think, usually, in these cases, I try to prove people wrong, if they think I'm not good enough for something. |

| Evelyn Lamb: | 13:55 | Yeah. Something I've enjoyed talking about with many of the people on this podcast is the importance of mentoring in their careers. Do you have people, earlier in your career, that you really consider good mentors? |

| Enrique Trevino: | 14:13 | Yes, certainly. I have many. |

| Enrique Trevino: | 14:16 | In the Math Olympiad, there's David Castillo, he's the person that taught me this, two, five, eight thing I mentioned. He was very influential in me being in the Math Olympiad. Essentially, I went into math because of his lessons. He's still a very good friend of mine, and we work together as coaches for over a decade. |

| Enrique Trevino: | 14:41 | At UTEP, Professor Boyd Chikowski, was really encouraging, and he gave me that nice scholarship. Then, he would always, whenever he would see an opportunity, he would tell me about it. |

| Enrique Trevino: | 14:56 | In grad school, my advisor was excellent. I think he's an extremely kind person, and very patient with me. It was a great experience, and he's been very supportive, found great problems for me, and pointed me in the right direction, multiple times. |

| Enrique Trevino: | 15:19 | Yeah, I'm very grateful to them. |

| Evelyn Lamb: | 15:21 | What are some things you try to think about when you are mentoring other people? |

Enrique Trevino:	15:27	Well, many times, I guess I think of, what would I have liked to have heard? Usually, my go-to thing is to listen to what the student really wants, and try to see how I can help them achieve that. I try to be very kind.
Enrique Trevino:	15:54	For example, Budapest was a great experience for me. I try to tell my students about that opportunity. I also like to take them to conferences, whenever I can, which is usually the Illinois Math Sectional, from the MAA. I meet weekly with students for Putnam problems, and we talk about other things like grad school, et cetera.
Enrique Trevino:	16:25	Yes, usually it's just listen to what kind of things they want. If I can help them, then help them. Sometimes I don't know about ... Because I went straight from undergrad to grad school, I don't really know much about the world outside of academia, so I have to point them in other directions if that's the help they need.
Evelyn Lamb:	16:45	Right. Yeah, I know a lot of academics can have trouble advising students who are going to non-academic paths.
Enrique Trevino:	16:53	Yeah. Luckily, I have colleagues that have been outside academia more than me. It's easy to just point them in their direction.
Evelyn Lamb:	17:04	Do you have any advice that you would give to math students, who are thinking about how to decide whether to pursue a career in math?
Enrique Trevino:	17:16	I mean, I think loving the subject is very useful. Graduate school is not that easy, if you're doing something you don't love. Perseverance is important, patience is very important, but I would think that loving the subject is key.
Evelyn Lamb:	17:43	Well, thanks a lot for taking the time to talk with me this morning.
Enrique Trevino:	17:47	You're welcome.
Evelyn Lamb:	17:50	Thank you for listening to the Lathisms podcast. It's produced by me, Evelyn Lamb, and made possible by a Tensor-SUMMA Grant from the Mathematical Association of America. Our music is [Volare 00:18:01] by [La Floresta 00:18:02].
Evelyn Lamb:	18:03	Lathisms is an initiative to celebrate the accomplishments of Hispanic and Latinx mathematicians. It was founded in 2016 by

Alexander Diaz Lopez, Pamela Harris, Alicia Prieto-Langarica, and Gabriel Sousa. You can find more information about the project at Lathisms.org. That's L-A-T-H-I-S-M-S. O-R-G.

Evelyn Lamb: 18:25   Join us next time to hear from another inspiring mathematician.