IB Calculus Internal Assessment:

How Air Pressure Affects a Soccer Ball

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Introduction:

I have played soccer for almost the entirety of my life starting at the age of four years old. The sport has had a massive impact on my life and continues to challenge me and provide me with new opportunities to meet new people everyday. Soccer has made such a great impact on my life that I wanted to dive into how a soccer ball actually moves and how the pressure inside of the soccer ball affects its movement.

There are many aspects of the game that can affect how it is played such as the weather, who is playing, and even the type of soccer ball used. Over the years I was always told that soccer cannot be played with a flat ball--but what is considered flat? It is required to have the ball checked by the referees at the beginning of each game to avoid playing with deflated balls. My teammates have also sometimes proclaimed that they prefer "hard" soccer balls, and this is also my personal opinion. Many times my team would completely stop playing if we felt the ball was too soft and needed to be replaced. This made me think about the air pressure in soccer balls and how air pressure affects the game's pace and playability. Obviously you cannot use a ball that isn't even round due to so much deflation, but I have always wondered if slight differences in the air pressure in a soccer ball affect how fast and how far the ball moves during play.

This investigation will include throwing the same exact soccer ball at a wet surface. My independent variable will be the pressure inside the ball. Each different pressure amount will be used multiple times in order to formulate an average for more accurate data. My goal for this investigation is to find the optimal pressure for a soccer ball based on the fact that if a ball of a certain pressure travels with a greater velocity, it is better for a fast paced level of gameplay. A faster ball allows for players to move up and down the field more quickly, potentially catching opponents off guard. Quick movement of the ball could end up being a huge advantage for a team; however, at younger ages, a slower ball may be preferred to compensate for the lack of skill, but that is not my focus. In order to find the ideal pressure for a soccer ball, I plan to measure the distance traveled from the place the ball hits a wet surface to where it comes in contact with the ground again. The amount of time taken to travel from the wet surface to where the ball rejoins the ground will also be measured so that a displacement versus time graph can be constructed. The distance versus time graph will then allow me to create a velocity versus time graph using derivative rules, which will provide me with the fastest ball based on air pressure inside it. A faster ball allows for a faster game which is greatly preferred at higher levels of play.

After some research, I discovered that the recommended pressure of a soccer ball for professional games by FIFA, Fédération Internationale de Football Association, is 8.5 to 15.6 PSI or pounds per inch ("Frequently Asked Questions"). The range is to compensate for all levels of play, but for the investigation I focused on the higher PSI end. My air pressure test includes values above and below the recommended range to get a full idea of whether this is an accurate range, but I had more values on the high end because, generally speaking, a ball with higher pressure will travel farther and faster. The next thing I discovered was that printed on the ball I was using was a recommended manufacturer air pressure of 8.6 to 19.6 PSI. I assume that the manufacturer recommended air pressure is greater than what FIFA recommends because that is most likely the maximum air pressure the soccer ball can handle before losing its shape and getting deformed. This also provided me with my maximum air pressure for my soccer ball.

Data Collection:

In order to collect my data, I had to carry out an experiment that would help me produce a displacement versus time graph. My setup consisted of laying a flat board on a grass field in order to have a flat surface to bounce the soccer balls off of. Next to the board I had a pipe angled at about 30 degrees to guide my throwing. This allowed me to be as consistent as possible and minimize as much human error as I could. I then poured water on top of the board in order to mimic wet grass, and also reduce the amount of friction acting on the ball. I decided to make the surface wet because when a soccer ball bounces off of a wet surface it tends to skip. The distance traveled and the ball's velocity increase after this skip amplifying the miniscule differences between distances of the differently pressured soccer balls that were hard to observe. I felt that having the ball skip would provide me with more concise and accurate data because the movement of the soccer ball will be exaggerated making it easier to record. Once the board was all setup, I used a tripod to prop up my phone so that I could record all of my trials. This reduced the amount of error in my measurements and also aided me when trying to construct my distance versus time graphs. In addition to the recording, I placed other soccer balls on the field to mark every 10 feet. This made my graphs much more accurate because I was able to see the specific distance of the ball at a specific time when looking back through the recordings. I was not able to get as accurate data for the specific times and distances of the ball while it was in flight because the measurements on the tape measure were not completely visible in the recording.

In order to get the exact final distance of the ball, I had one of my friends keep an eye on where the ball first came in contact with the ground after I threw it. A ballpark estimate of this was not too detrimental to my data because the ball had a big surface area, so an exact point was not needed. After I threw the ball, I used a tape measure to measure the distance from the edge of the board to where the ball first made contact with the ground. I then repeated this ten times for each different pressure of my soccer ball. Multiple trials allowed me to get an average distance and time, which minimized the effect of extraneous data on my results. Looking back, I would have liked to have had access to a machine that would have been able to fully control my angle of release and the amount of force that acted on the ball. An image of my data collection set up and my raw data tables with all trials during the experiment are provided below:

Raw Data Tables:

Distance (ft)											
Pressure (PSI)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10	Average

8	19.8	20.7	20.0	23.6	24.6	21.3	18.0	24.5	25.5	22	22
10	24.6	21.9	22.4	22.7	24.6	25.7	21.7	22.8	20.9	20.7	22.8
12	23.2	21.9	27.8	26.5	25.0	27.8	28.9	27.5	24.0	24.7	25.7
14	25.0	25.5	30.1	29.0	30.7	27.5	29.7	31.2	28.8	28.6	28.6
16	26.5	29.2	28.1	27.8	32.2	26.4	25.7	33.9	35.0	29.8	29.5
18	30.0	31.7	33.6	30.8	34.2	30.3	31.0	29.5	29.9	34.4	31.5

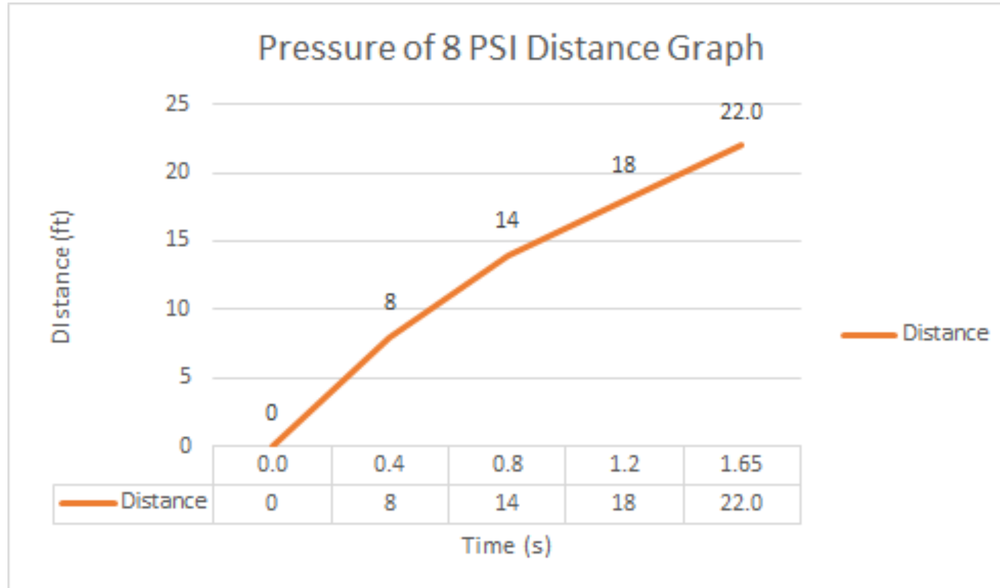
Time (s)											
Pressure (PSI)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10	Average
8	1.65	1.67	1.64	1.70	1.68	1.63	1.58	1.64	1.70	1.65	1.65
10	1.71	1.69	1.70	1.73	1.72	1.74	1.70	1.69	1.68	1.70	1.71
12	1.74	1.73	1.76	1.75	1.7	1.74	1.77	1.76	1.70	1.71	1.74
14	1.71	1.70	1.75	1.74	1.76	1.74	1.76	1.77	1.75	1.74	1.74
16	1.71	1.75	1.72	1.77	1.79	1.70	1.74	1.79	1.80	1.74	1.75
18	1.76	1.78	1.80	1.75	1.75	1.78	1.77	1.76	1.78	1.81	1.77

Data Analysis:

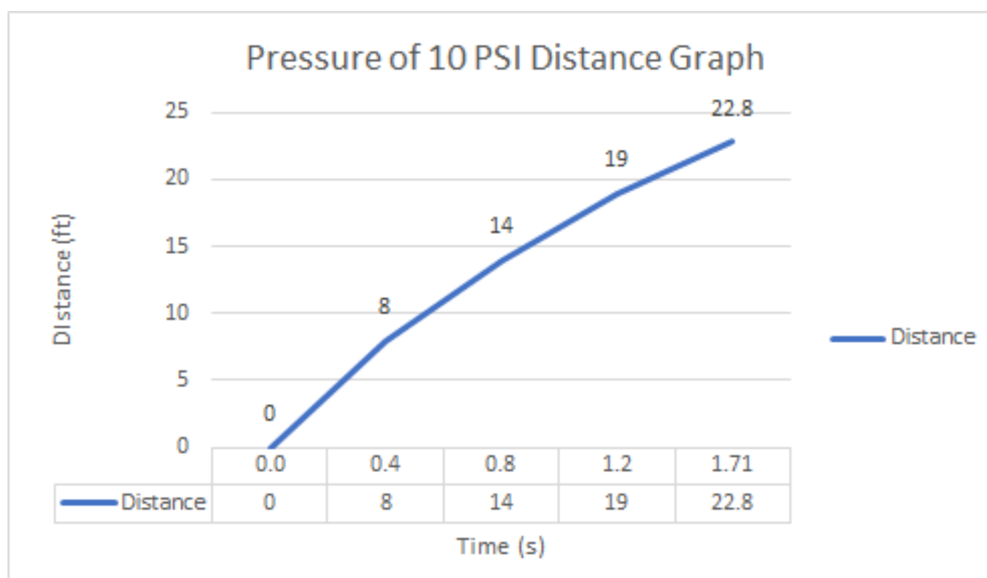
My experiment provided me with distance versus time graphs for each pressure that I used in my soccer ball; however, my aim was to figure out what pressure provides the greatest velocity, which allows for a faster paced game. After I collected all of my data, I needed to calculate an average time and distance for each pressure. For both measurements, time and distance, I saw a direct relationship between the amount of pressure in the ball and these two variables. As I increased the pressure of my soccer ball, it traveled a farther distance which increased its air time.

In order to construct an accurate distance versus time graph for each amount of pressure, I took the average distance and average time for all the trials for balls of the same pressure. I then used a recording of one trial that was of a ball with a similar flight path to the average. For example, a ball with a pressure of 8 PSI traveled, on average, 22 feet in 1.65 seconds. Using my recordings of this balls' flight, I was able to visually see, by stopping the video at certain times and lining up the ball in the air with a measurement on the tape measure, that after 0.4 seconds the ball had traveled about 8 feet, after 0.8 seconds its traveled 14 feet, and so on until it hit the ground at 22 feet. The distance versus time graphs, and the table of data used to form the graphs, for all of the different pressures used are shown on the next page.

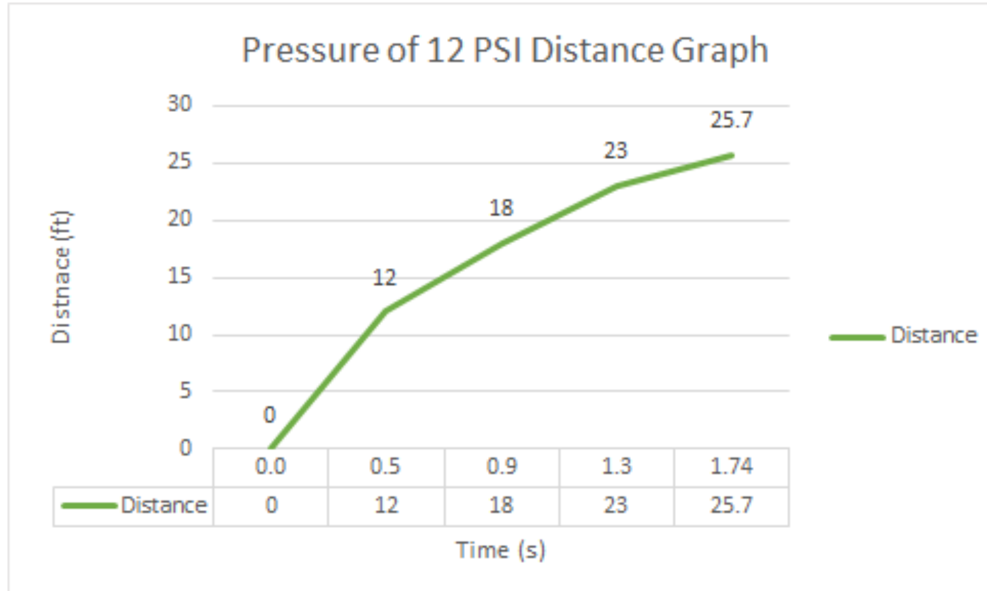
Pressure of 8 PSI	
Time (s)	Distance (ft)
0.0	0
0.4	8
0.8	14
1.2	18
1.65	22.0



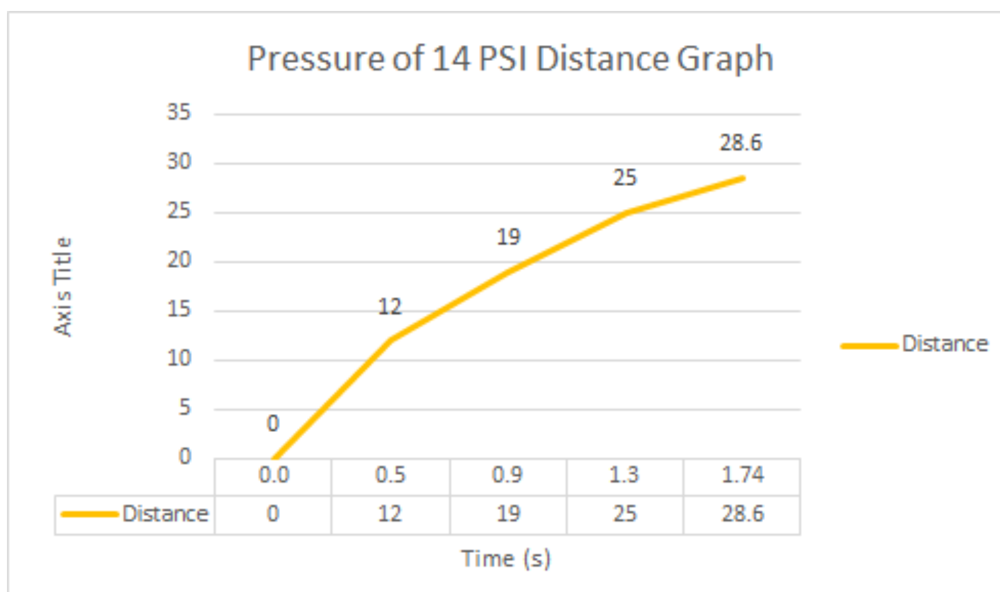
Pressure of 10 PSI	
Time (s)	Distance (ft)
0.0	0
0.4	8
0.8	14
1.2	19
1.71	22.8



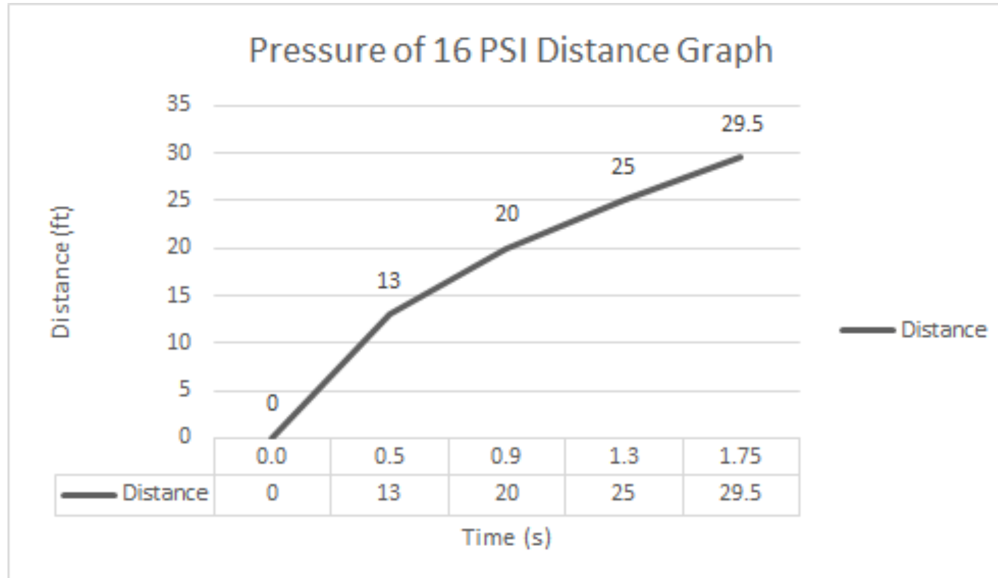
Pressure of 12 PSI	
Time (s)	Distance (ft)
0.0	0
0.5	12
0.9	18
1.3	23
1.74	25.7



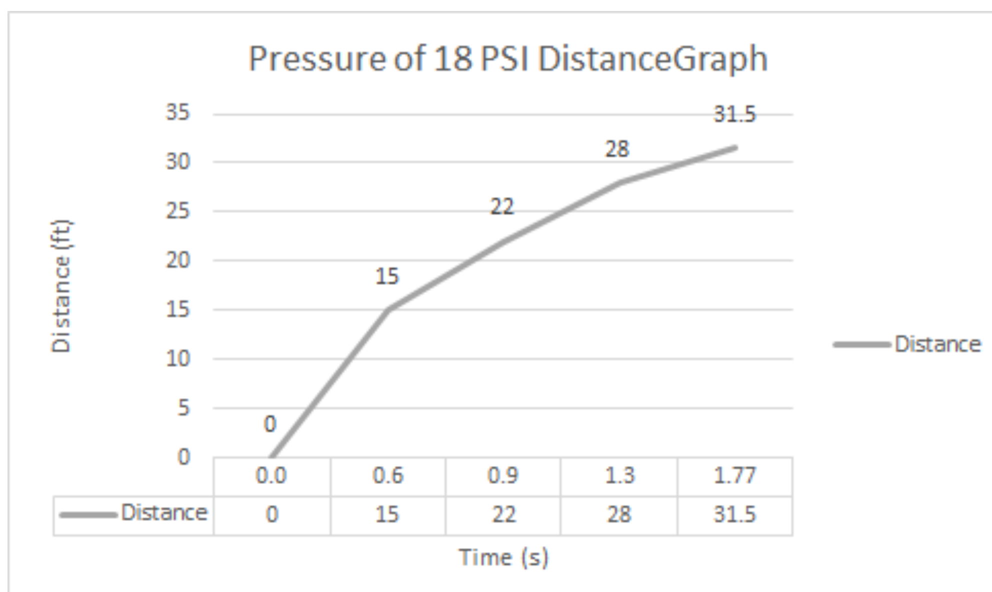
Pressure of 14 PSI	
Time (s)	Distance (ft)
0.0	0
0.5	12
0.9	19
1.3	25
1.74	28.6



Pressure of 16 PSI	
Time (s)	Distance (ft)
0.0	0
0.5	13
0.9	20
1.3	25
1.75	29.5



Pressure of 18 PSI	
Time (s)	Distance (ft)
0.0	0
0.6	15
0.9	22
1.3	28
1.77	31.5



Based on the graphs, I thought that the distance traveled could be modeled by a square root function. However after modeling my graphs based on a square root function, I realized this was not the most accurate so I decided to look at a general power function. The next step was to figure out the equation for this power function graph. I did this by using the general form of a power function, $f(x) = a(x)^b + c$ where a,b, and c are all constants. I first figured out that since my y-intercept of every graph was at the origin, that the value of c will be zero for each equation because c represents a vertical translation up or down from the origin. My next step was to input the data into the analysis software and linearize the data by taking the common logarithm of both the air time of the soccer ball and the distance traveled. Then, I graphed the transformed data to verify that it indeed was linear, and subsequently ran a linear regression on the transformed data. The steps to linearize the data were:

$$y = a(x)^b$$

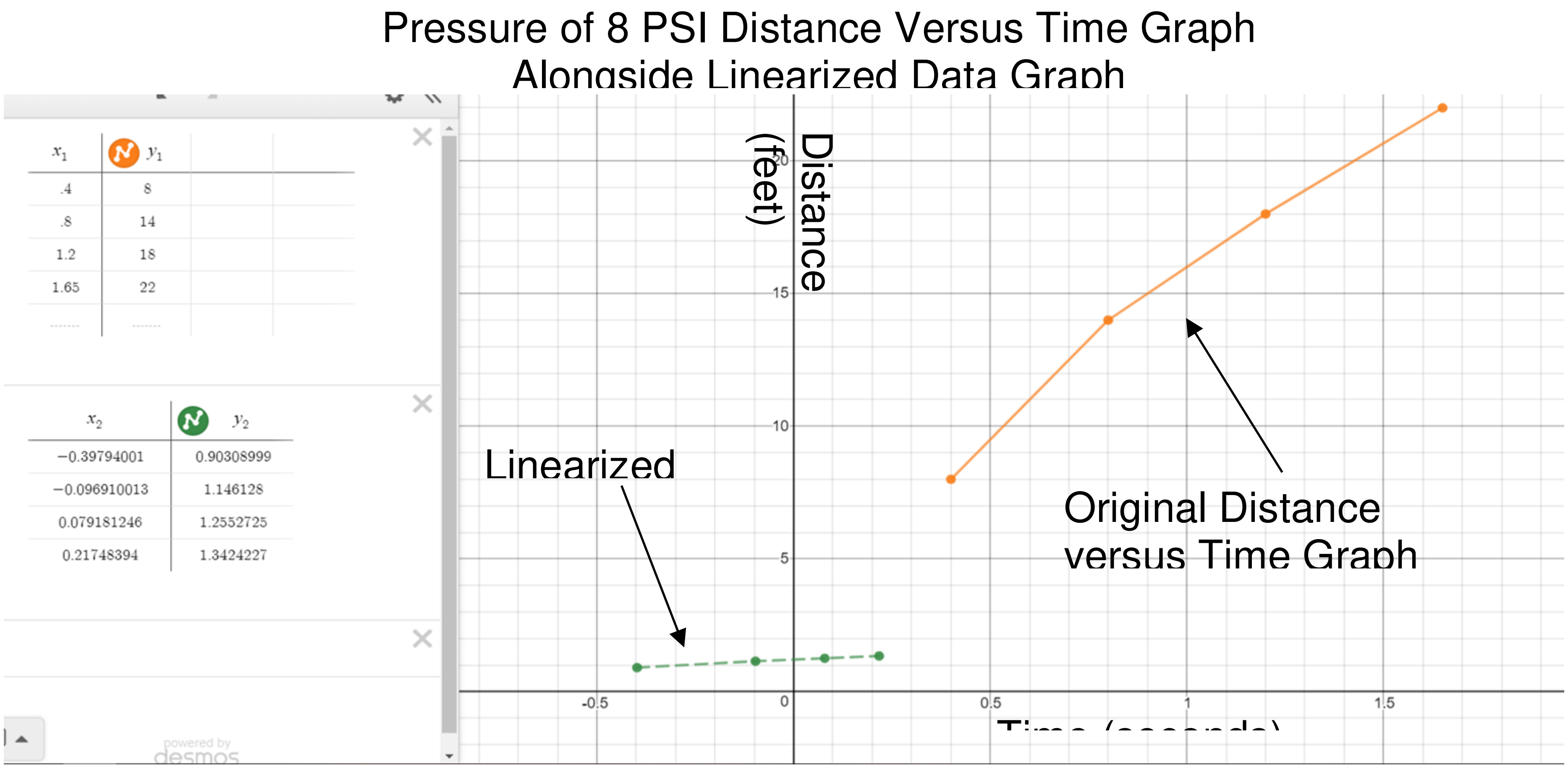
$$log(y) = log(a(x)^b)$$

$$log(y) = log(a) + log(x)^b$$

$$log(y) = log(a) + b * log(x)$$

The process to linearize the data took a good amount of time, not because it was difficult to complete, but because it took some time to input all of my data into the analysis software for all different pressures of the soccer ball.

Shown below is a table with my values for airtime, distance traveled, and the common logarithm of both of these values for a ball at a pressure of 8 PSI. I then graphed the original data in orange and the transformed data in green to verify that it was linear.



<u>Linearized Data:</u>

log (x)	*log (y)*
-0.39794001	0.90308999
-0.096910013	1.146128
0.079181246	1.2552725
0.21748394	1.3424227

The next process was to do a linear regression on the transformed data, so that it was in the form $log\ (\hat{y})\ =\ a\ +\ b * \ log(x)$where $log(\hat{y})$ is the common logarithm of the predicted distance traveled by a soccer ball after *log(x)* amount of time. The common logarithm had to be taken of both time and distance to linearize the data. I used an analysis software to accomplish the regression.

y =	*a* + *bx*
a =	1.19713481
b =	0.7146159087
r =	0.9975486252

The *r* value represents the strength, form, and direction of the transformed data compared to a linear function. Since a value of 1 for *r* represents a perfectly linear function in the positive direction, a value of 0.9975 means that the transformed data strongly fits a positive linear function, further verifying that the transformed data is linear.

My final step in finding a function to model the distance versus time data was to back transform the linearized function in order to obtain a power model. This was done by reversing the common logarithms with exponents. Calculations for a pressure of 8 PSI are shown for an example below.

$$log\ (\hat{y}) = a + b * log(x)$$

$$10^{log\ (\hat{y})} = 10^{a} + 10^{log(x)^{b}}$$

$$\hat{y} = 10^{1.19713481} * x^{0.7146159087}$$

$$\hat{y} = 15.74471523(x)^{0.7146159087}$$

This was the correct function that accurately modeled my data. I completed this same exact process for all of my other pressures of the soccer ball. I noticed that as the soccer ball's pressure increased, the value of *a* also increased; however, the value of *b* was a little more inconsistent but generally stayed in the 0.62 to 0.72 value range. This is because each soccer ball decreased distance travelled per second at about the same rate. All of my models for each amount of pressure inside the soccer ball are shown below.

Amount of Pressure (PSI)	Power Function Model
8	$\hat{y}$=15.74471523$x^{0.714615909}$
10	$\hat{y}$=15.79301475$x^{0.7195098998}$
12	$\hat{y}$=18.85060062$x^{0.624772064}$
14	$\hat{y}$=20.0304933$x^{0.710477368}$
16	$\hat{y}$=20.84540086x$^{0.656702556}$
18	$\hat{y}$=22.37304757x$^{0.690082485}$

After I found all of my equations for each of my graphs based off of a power function, I needed to find the equations of the velocity graphs for each pressure value. I did this by finding the derivative, $f\ '(x)$, or the instantaneous rate of change of the distance graph. I used the simple power rule for derivatives which states that for a function $f(x) = ax^n$ the derivative is $f\ '(x) = (a * n)(x)^{(n-1)}$ where *a* and *n* are constants. I used the power rule to formulate my derivative or velocity equation of that specific pressure. My calculations for the 8 PSI pressure ball are shown below for an example.

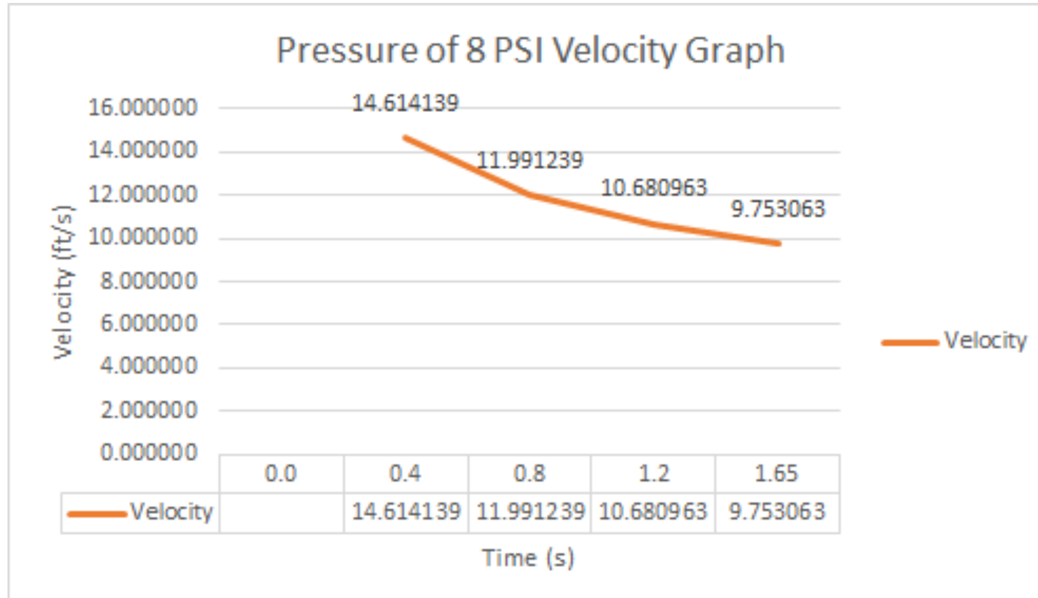
$$\hat{y} = 15.74471523(x)^{0.7146159087}$$

$$f\ '(x) = (15.74471523 * 0.7146159087)(x)^{(0.7146159081\ -\ 1)}$$

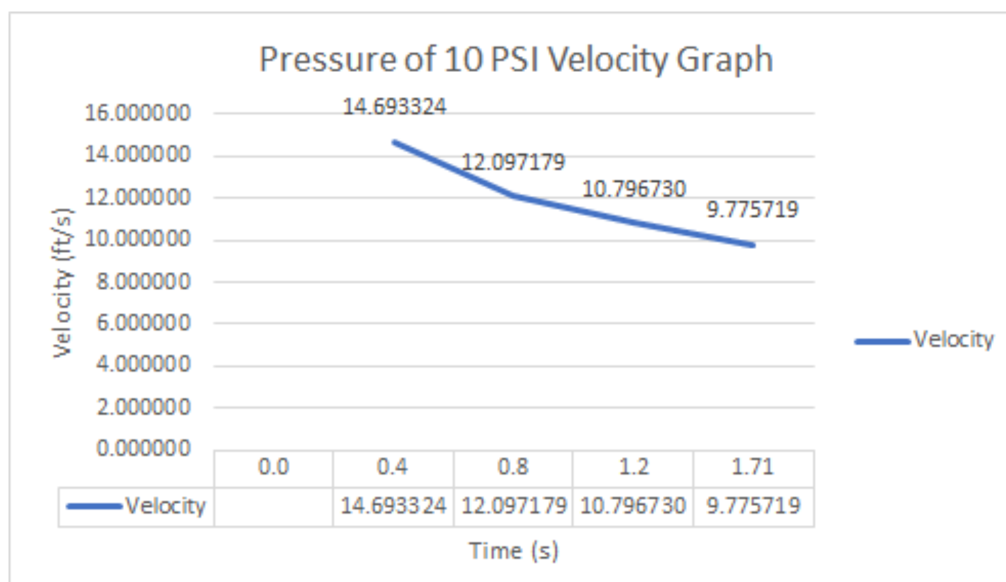
$$f\ '(x) = 11.25142399(x)^{-0.285384091}$$

Once I completed my calculations and found all of the equations for each of my velocity graphs, I was able to substitute the same time values used for the distance graphs into various derivative functions. A velocity at time 0 is not provided because, based on my experiment, the initial velocity would not be equal to 0. This is because I applied force to the soccer ball before a distance of 0 feet, but my function does not support this initial velocity reading. This provided me with data points showing the specific velocity of the ball at a specific time during its flight. All of the velocity versus time graphs are shown below.

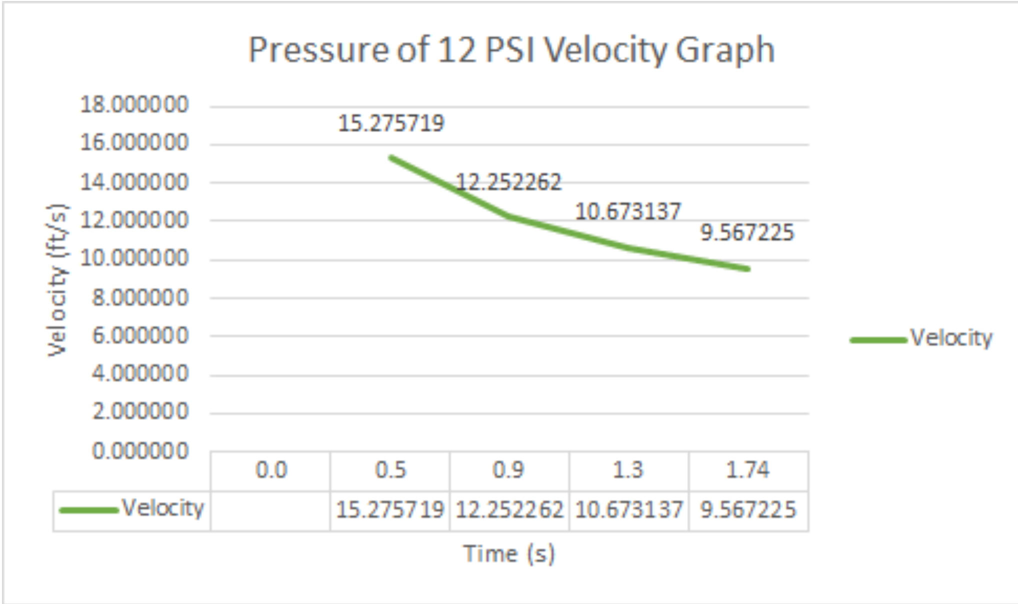
Pressure of 8 PSI	
Time (s)	Velocity (ft/s)
0.0	
0.4	14.614139
0.8	11.991239
1.2	10.680963
1.65	9.753063



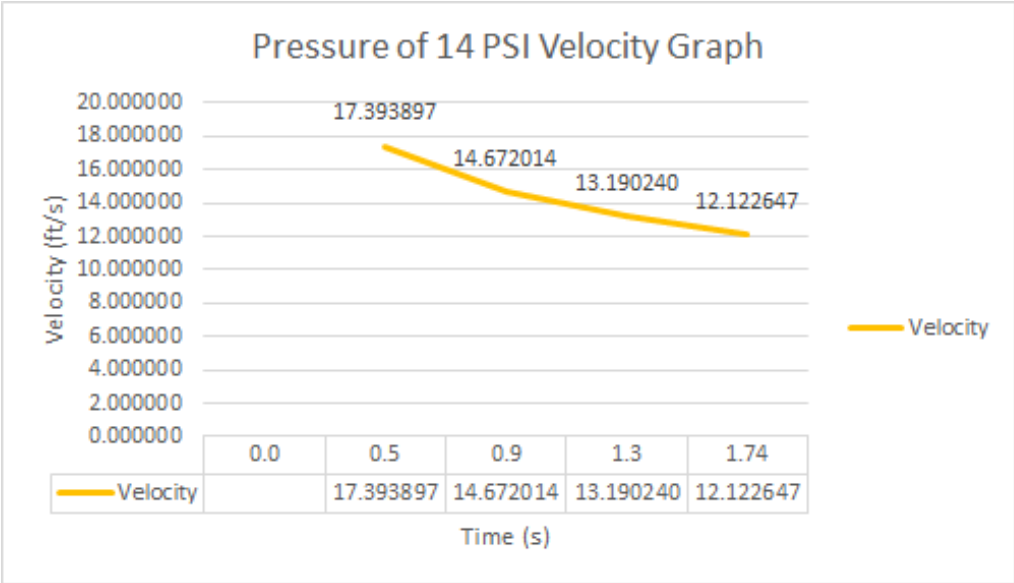
Pressure of 10 PSI	
Time (s)	Velocity (ft/s)
0.0	
0.4	14.693324
0.8	12.097179
1.2	10.796730
1.71	9.775719



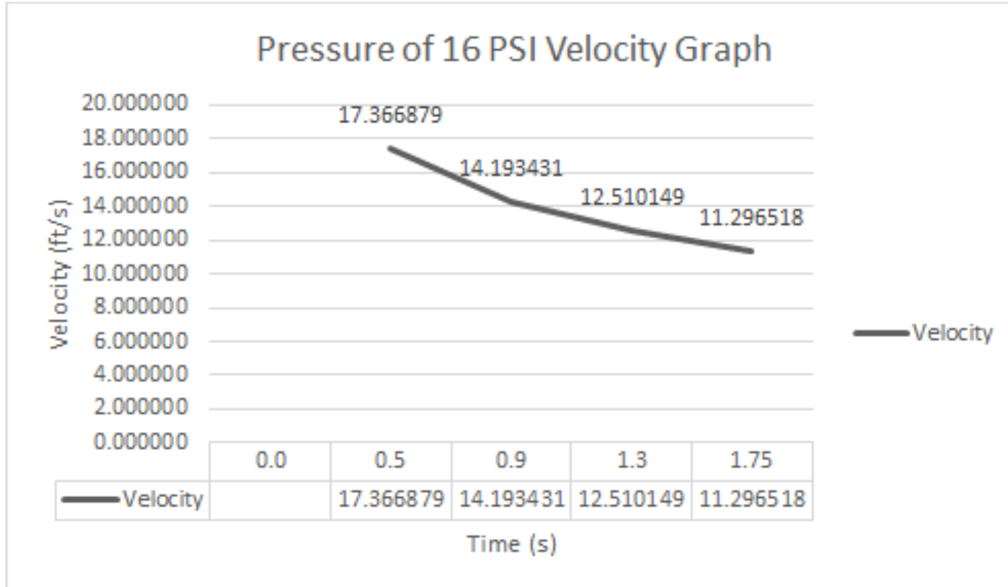
Pressure of 12 PSI	
Time (s)	Velocity (ft/s)
0.0	
0.5	15.275719
0.9	12.252262
1.3	10.673137
1.74	9.567225



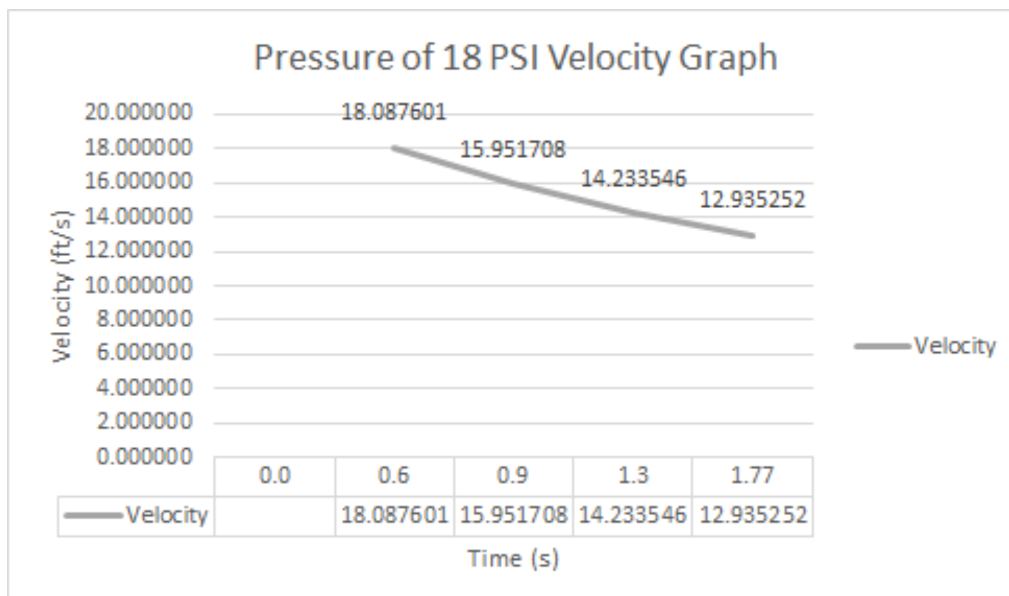
Pressure of 14 PSI	
Time (s)	Velocity (ft/s)
0.0	
0.5	17.393897
0.9	14.672014
1.3	13.190240
1.74	12.122647



Pressure of 16 PSI	
Time (s)	Velocity (ft/s)
0.0	
0.5	17.366879
0.9	14.193431
1.3	12.510149
1.75	11.296518



Pressure of 18 PSI	
Time (s)	Velocity (ft/s)
0.0	
0.6	18.087601
0.9	15.951708
1.3	14.233546
1.77	12.935252



Overall, my ending distances were consistent with the increase of pressure inside the soccer ball; the velocities at 0.4 seconds for each ball seemed to increase along with the pressure. I can assume that this is in line with my original hypothesis that the pressure will increase the soccer ball's velocity, but if I had applied more force to a ball the skip off of the wet surface would have created a greater initial velocity. Due to the absence of a machine that can control the force applied to the soccer ball, the initial velocities may be slightly skewed.

Conclusion:

In conclusion, based on my data, I could see a direct relationship between the increase of pressure inside a soccer ball and the increase of its distance and velocity. Although I can assume that as I infinitely increase the pressure it will infinitely increase its distance traveled, this is not possible because a soccer ball can only hold so much air before it is deformed or even pops. Looking back, there were two major changes I would have made to my data collection process. First, I would have liked to have a machine that could have controlled the release force and angle of the soccer ball, but without access to this type of machine I had to rely on doing multiple trials in order to eliminate as much human error as possible. Second, a system that could record specific distances of the soccer ball at specific times more accurately would have also aided my data collection process. I had to rely on a recording of the soccer ball's flight and stopping the video at certain times to observe the distance of the ball. This again was not totally accurate. As far as the mathematics were concerned, the process was extensive and should have been shortened. I needed to take more time to decide which function would be best to model my data off of instead of trying multiple different functions. Although my graphs seemed to follow a square root function, I later realized that a general power function would be more accurate because the power would be about 0.7 instead of 0.5. This slight variation in the power of my function drastically affected how accurate the model for my data was. Once I found the function that accurately modeled my distance versus time graphs, I was able to easily take the derivative of them to construct my velocity versus time graphs. One issue that I see with my velocity graphs is that they do not accurately show the initial velocity of the soccer ball at a time of zero seconds. This forced me to use the velocity at a time of 0.4 seconds as my “initial velocity.” I did not use a time of 0.1 seconds because I wanted to stay consistent with the times of the distance versus time graphs to show specific velocities at those distances.

All in all, I enjoyed performing my experiment, but the process of modeling my data with a function could have been simplified more if I took more time and fully understood the shape of my graphs. In addition the data collection process could have been more controlled if I had access to expensive machines that could have controlled the force applied to the ball and the angle of the ball coming in contact with the wet surface. Despite the long process to model my data, I was content with the experiment that I conducted and the results I got. This answers the question of whether of not slight differences in air pressure affect a soccer ball’s flight, and in fact they do. I saw a slight variation in initial velocities and the maximum distance traveled of each differently pressurized soccer ball. Now I have scientific reasoning to back up why my teammates and I prefer soccer balls that are more pumped up. They truly affect the pace at which the game is played at, and can greatly give an advantage to one team or the other throughout the duration of a soccer game.

Works Cited

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