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Report on the PhD thesis
On the problem of quasi-local mass in the weak field regime of gravity
by Piotr Waluk

I find Piotr Waluk's thesis to be an excellent work. The thesis is based on 3 original research papers (references 42, 77 and 79) and concerns both the problem of defining the notion of quasi-local mass of gravitational field (chapter 3) and linear perturbations of the Kottler metric (chapter 4). Then, these two independent problems are nicely combined (chapter 5) in the discussion of quadratic approximation (based on Hamilton function for linearised gravity on the Kottler background) of quasi-local mass candidates from chapter 3, in particular the Hawking quasi-local mass. The tools for these studies are introduced in a concise chapter 1 and chapter 2 motivates a quest for the definition of quasi-local mass of gravitational field. Conclusions and perspectives for future research are given in chapter 6. The main text is accompanied with 7 appendices with clear presentation of more technical part of computations (as a reader I am very grateful for the appendix G with the index of all symbols used in the text).

I am deeply impressed by a broad perspective in presentation of the problems discussed in the thesis and massive computations that stand behind it - I do regret that the 12 weeks period was not enough for me to follow all these computations in detail. My area of expertise is closest to the content of chapter 4. Thus, I am really impressed by the elegance of the framework for studying linearised perturbations on spherically symmetric backgrounds and throughout discussion of the mono-dipole part and conserved charges of linearised gravitational field. I am less familiar with the problems studied in chapters 3 and 5 but I found these chapters very enlightening. I would strongly recommend to made the thesis publicly available by posting it on arXiv and I am convinced that the thesis deserve a distinction.

At the end I am presenting a short list of possible amendments and questions:

- The typos and misprint are inevitable - as a non-expert I found particularly annoying the misprint in the definition of ADM mass, eq. (2.5), where instead of g^{ab} contracting the bracket there should be η^{ab} - the inverse of the reference metric. In some places indices are miss-placed (e.g. (E.1)). It seems that to have the consistency with the notation of (4.93) one should include the gauged quantities on the RHS of the gauge action in (4.97) and (4.99)

- I would find it very useful for less-experienced reader to include some worked examples illustrating the definitions of quasi-local mass discussed in chapter 3, and the extrinsic curvature of 2-surfaces discussed in appendix A, on some concrete examples, e.g. Schwarzschild spacetime (see a worked example for the ADM mass definition in Sec.7.3.1 of arXiv:gr-qc/0703035). In particular, such examples are useful in case of misprints occurring in formal definitions, like one in (2.5).

- There are two common convention for extrinsic curvature: $K_{ab} = \pm(1/2)\mathcal{L}_n\, g_{ab}$. For a non-expert reader it would be useful to state explicitly that the minus sign convention is used in the thesis.

- Apart from Regge-Wheeler gauge, another convenient gauge choice is Detweiler (easy) gauge [Class. Quantum Grav. 34 (2017) 174001]. I wonder how the radiation polar part looks like in this gauge.

- In principle, the definition from Sec.4.2 $g_{\mu\nu} := \eta_{\mu\nu} + \epsilon h_{\mu\nu}$ could have been extended by adding higher order corrections to fulfill Einstein Equations to higher (then linear) order in ϵ. How can one be sure that such higher order corrections will not contribute to the quadratic (i.e. $\mathcal{O}\left(\epsilon^2\right)$) approximation for (5.7)?

- Being non-expert in Hamiltonian formulation of GR, I based my knowledge on Eric Poisson's introduction to the subject in *An advanced course in GR*, in particular lectures 13-18, and [E. Poisson : A Relativist's Toolkit, The Mathematics of Black-Hole Mechanics, Cambridge University Press, Cambridge (2004)] (see also Sec.7.3.1 of arXiv:gr-qc/0703035 and references therein), where the Einstein-Hilbert action is amended by adding a boundary term. The Candidate chooses a different path to gravitational Hamiltonian and seems reluctant to discuss the boundary term in the action for gravitational field. Being non-expert in the field I would like to ask the Candidate what is his attitude towards such boundary term and how it relates to the Hamiltonian formalism used in the thesis.

I conclude that the presented dissertation meets the formal requirements for a Ph.D. thesis and recommend admission of the Candidate to the subsequent stages of the procedure, including the public defense. I am also convinced that the presented dissertation is worthy of distinction.

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