Abstract

Quantum information emerged in the 20^{th} century, fostering the rapid development of technologies that use the description of microscopic systems provided by quantum mechanics. The progress of this domain of science is a result of the efforts devoted to the study of phenomena not existing in the macroscopic domain. This includes the research of quantum protocols that use superposition, as well as quantum entanglement. Both of these properties of quantum systems make them compelling from the point of view of the creation of quantum devices that outperform the classical ones.

Employing these characteristics of the quantum domain, scientists and inventors have the ultimate goal of creating a quantum computer of a size that shall make it useful for computations that are not feasible on the classical computers that apply the Boolean logic. Nonetheless, to apply the physical principles that could lead experimentalists to succeed in their search for a groundbreaking quantum device, we need to understand the intricacies of the theoretical description. Therefore, the main goal of this thesis is to provide novel constructions useful for the comprehension of quantum mechanics from the perspective of mappings and designs.

The thesis is designed as follows. First, in the preliminary chapter, we introduce the necessary concepts from the field of quantum information, including quantum designs and mappings. In the subsequent part of the thesis, we investigate two instances of quantum mappings that were developed by the author and his collaborators.

The first one concerns the unistochasticity problem, which relates bistochastic and unitary matrices, both useful in the classical and the quantum domain, respectively. This fragment of the thesis dwells on the characterization of the unistochastic set, with a presentation of the algorithm that allows determining whether a given bistochastic matrix of size 4 is unistochastic. It contains also the proof that the simple bracelet condition is sufficient to decide the unistochasticity of a circulant matrix of size 4. Furthermore, we investigate the unistochasticity of certain sets inside the Birkhoff polytope of bistochastic matrices of an arbitrary dimension N and prove that the rays and counter-rays are unistochastic, provided there exists a robust Hadamard matrix of dimension N.

Moving on to the second instance of quantum mappings, we study the entangling power in the multipartite case. The main achievement shown in this chapter is an explicit analytical formula for the average entangling power of a tripartite orthogonal gate.

Using the notion of entangling power, we develop in the subsequent chapter novel ideas concerning the search for an absolutely maximally entangled (AME) state of four quhexes, the existence of which was first shown by the author and his collaborators. Several new methods in the search for AME states and other quantum designs are presented. In particular, we derived the Hessian for the entangling power and the average singular entropy of a unitary quantum gate. These can be used to evaluate the extremality of the solutions found. Furthermore, our research revealed a curious block-like structure of the newly discovered AME state of 4 subsystems 6 levels each, which has the potential to disclose more general facts about other quantum designs.

The final chapter concerning new research is devoted to an extension of the recently established notion of quantum Sudoku (SudoQ) designs. To characterize such objects we introduced the cardinality of a SudoQ as the number of different states forming the design. We characterized the cardinality also as a measure of "quantumness" of quantum Latin squares. Those of the highest cardinality yield families of quantum measurements of special properties. We characterize the problem in the general case of size N^2. A connection between SudoQ designs and mutually unbiased bases is demonstrated.