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REVIEW OF 'QUANTUM MAPPINGS AND DESIGNS'

Gregorz Rajchel-Mieldzioć has submitted a dissertation with the above title, and I have been asked to provide a review.

The thesis is full of interesting content. A main theme is how ideas from combinatorics, notably combinatorial designs, can be generalized to the quantum domain. A main result is the construction of an *absolutely maximally entangled* or AME state build from four quhexes (i.e. six dimensional quantum states). AME states are maximally entangled in the sense that all possible bipartitions of the state result in maximally entangled bipartite states. There are several reasons why a considerable amount of effort have gone into constructing such states, notably their usefulness in connection with quantum error correcting codes. In many cases they have been constructed using combinatorial ideas such as orthogonal Latin squares, but for the case considered in this thesis one runs into a classical problem first announced by Euler: the relevant orthogonal Latin squares do not exist. This is not the end of the story however, because quantum generalisations of the notion of a Latin square are available. A large portion of the thesis is taken up by an account of the search for the desired AME(4,6) state. Along the way, many interesting observations were made, and towards the end of chapter 4 the solution is presented. It turns out to be remarkably simple and elegant.

Some further explorations of quantum combinatorial designs are presented in chapter 5, and the reader finds an interesting list of open problems in the concluding chapter. I think it is fair to say that this thesis has opened up new avenues to explore.

Among the ingredients of chapter 4 is the *entangling power* of unitary gates. Chapter 3, which is short, contains one of the few presently available analytical results concerning entangling power in the tripartite case.

Chapter 2 is somewhat different in flavour, and as it happens concerns a subject on which I have spent a considerable amount of time myself, hence it was also of special interest to me. *Bistochastic* maps are maps that take one probability distribution to another and obey some special restrictions that make them useful for instance when one wants to describe the time evolution of a probability distribution, perhaps in a Markov chain. Given a unitary matrix one can construct a bistochastic matrix, and bistochastic matrices that arise in this way are called *unistochastic*. This notion is again a quantum twist of a classical one, and it has a surprisingly diverse range of application. But now we encounter two hard problems: to decide whether a given bistochastic matrix arises in this way, and to characterize the unistochastic subset of the set of bistochastic matrices. They had so far been resolved only for 2×2 and 3×3 matrices. In the thesis the first of these two problems is solved for the 4×4 case (based on a suggestion by the late Uffe Haagerup). The thesis also contributes some substantial progress on the second problem, again for the 4×4 case.

I find the results of the thesis impressive. I believe it is appropriate to add that the thesis is not only very well written, but it is also well and thoughtfully illustrated. The author has made the thesis a pleasure to read.

In view of the above I conclude that the presented dissertation meets the formal requirements for a PhD thesis and I recommend admission of the Candidate to the subsequent stages of the procedure, including the public defence. Moreover, I recommend the dissertation as being worthy of distinction.

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