Introduction to atomic physics

Atomic clock

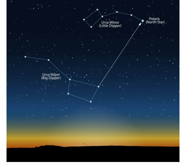
Previous lectures

- Energy levels → $\frac{E}{h^2}$ → fine structure, → hyperfine structure
- magnetic field / electric / e-m field
- NMR proton in magnetic field $B(t)$

Today - applications

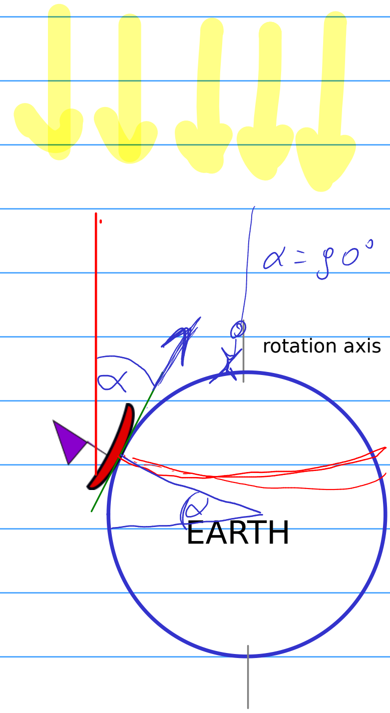
Atomic clock
Ramsey Interferometry

Quiz



Timekeeping - Motivation

Latitude

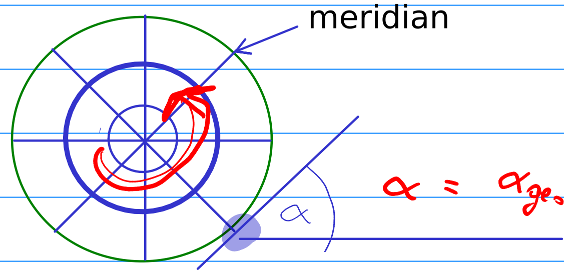


Longitude

How to measure on the see?

BIG PROBLEM

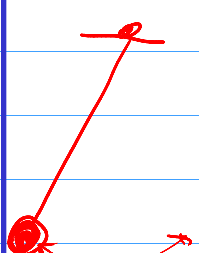
Idea of measruring longitude



$$\alpha = \alpha_{geo} + \frac{2\pi}{24h} \cdot t$$

Earth from top

John Harris



Map of Ptolemy, II century

The second is defined as: ω

"the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom" (at a temperature of 0 K)"

$|e\rangle$, $|g\rangle$ $\quad \frac{E_e - E_g}{\hbar} = \omega$ $\quad n = 0$

How to use this definition!?

Rabi pulses

Reminder: Lecture 9, Movies 5A-5C

B_0

$\hat{\vec{\mu}} \quad B_1(t) = B_1[\cos\omega t, \sin\omega t, 0]$

$\hat{\vec{\mu}} = \gamma \begin{bmatrix} \hat{\sigma}_x \\ \hat{\sigma}_y \\ \hat{\sigma}_z \end{bmatrix}$ $\quad \hat{H} = -\hat{\vec{\mu}} \cdot B(t)$

$= -\gamma \hat{\sigma}_x B_1 \cos\omega t$

$\boxed{-\gamma \hat{\sigma}_y B_1 \sin\omega t - \gamma \hat{\sigma}_z B_0}$

$T_R = \frac{\pi}{\Omega}$ $\quad t = \frac{T_R}{2}$ $\quad \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$

$|\psi(t)\rangle = \cos(\Omega t)|\uparrow\rangle + \cdot \sin(\Omega t)|\downarrow\rangle$

Reminder: Lecture 11, Movies 6A-6C

$H_A = -\hat{\vec{d}} \cdot \vec{\mathcal{E}} + \underline{E_e |e\rangle\langle e| + E_g |g\rangle\langle g|}$

$\frac{E_e + E_g}{2}(|e\rangle\langle e| + |g\rangle\langle g|)$

$+ \frac{E_e - E_g}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$ $\quad \hat{\sigma}_z$

$H_A = -\vec{d}\,\vec{\mathcal{E}} + \frac{\hbar\omega_0}{2}\hat{\sigma}_z$

$+ \text{const}$

$\omega_0 \{ \; |e\rangle, \; |g\rangle$

$\mathcal{E}(t) = \mathcal{E}_0[\cos\omega t, \sin\omega t, 0]$

$T_R = \frac{2\pi}{\Omega_R}$ $\quad 2\pi$ pulse

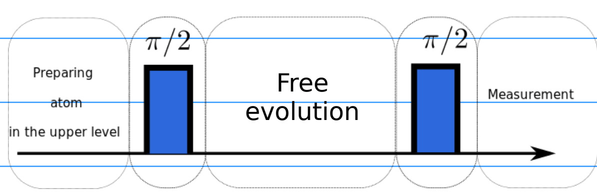
$\frac{1}{2} T_R = \frac{\pi}{\Omega_R}$ $\quad \pi$ pulse

$\frac{1}{4} T_R = \frac{\pi}{2\Omega_R}$ $\quad \frac{\pi}{2}$ pulse $\quad |e\rangle$, $|g\rangle$ $\quad \frac{|e\rangle + |g\rangle}{\sqrt{2}}$

$|e\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|g\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad \frac{\pi}{2}$ pulse $\quad \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ $\quad e^{-iHt}$

Ramsey Interferometer

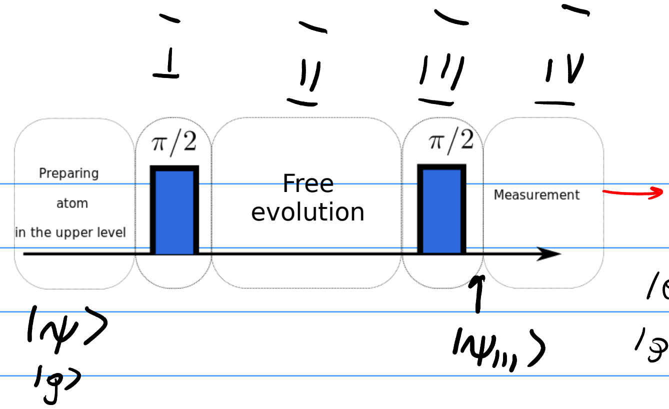


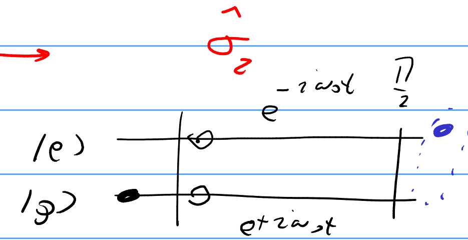
$|e\rangle$

$|g\rangle$

$|\psi\rangle$

$|g\rangle$





$$\bar{\text{I}} \quad U_{\frac{\pi}{2}}|g\rangle = \frac{|g\rangle + |e\rangle}{\sqrt{2}}$$

$$\bar{\text{II}} \quad |\psi_{II}\rangle = e^{-iHt/\hbar}\, U_{\frac{\pi}{2}}|g\rangle = e^{-i\omega_0 t/2\, \hat{\sigma}_z}\frac{|g\rangle + |e\rangle}{\sqrt{2}} = \frac{e^{+i\omega_0 t/2}|g\rangle + e^{-i\omega_0 t/2}|e\rangle}{\sqrt{2}}$$

$$H = \frac{\hbar\omega_0}{2}\hat{\sigma}_z \qquad \hat{\sigma}_z|g\rangle = -|g\rangle \qquad \hat{\sigma}_z|e\rangle = |e\rangle$$

$$\bar{\text{III}} \quad |\psi_{III}\rangle = U_{\frac{\pi}{2}}|\psi_{II}\rangle = \left(e^{+i\omega_0 t/2}\frac{(|g\rangle + |e\rangle)}{\sqrt{2}} + e^{-i\omega_0 t/2}\left(\frac{|e\rangle - |g\rangle}{\sqrt{2}}\right)\right)\frac{1}{\sqrt{2}}$$

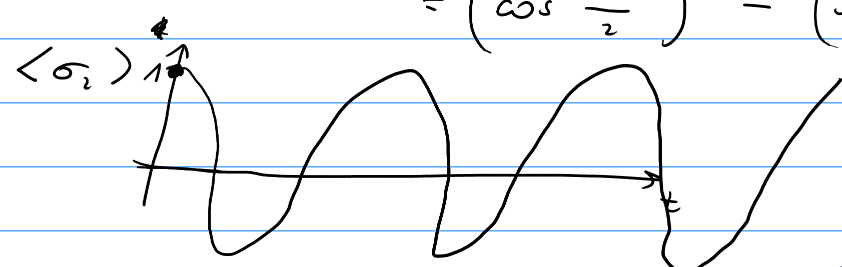
$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad = \cos\left(\frac{\omega_0 t}{2}\right)|e\rangle + i\sin\left(\frac{\omega_0 t}{2}\right)|g\rangle$$

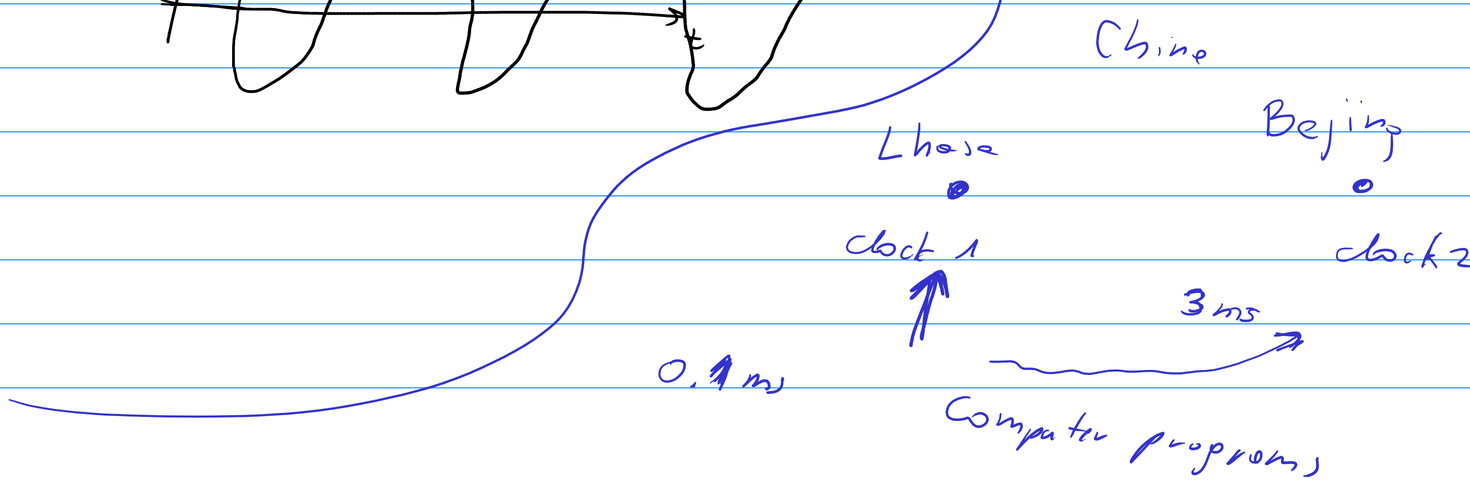
$\bar{\text{IV}}$ measurement

$$\hat{\sigma}_z|\psi_{II}\rangle$$

$$\langle\psi_{III}|\hat{\sigma}_z|\psi_{III}\rangle = \langle\psi_{III}|\left(+\cos\frac{\omega_0 t}{2}|e\rangle - i\sin\frac{\omega_0 t}{2}|g\rangle\right)$$

$$= \left(\cos\frac{\omega_0 t}{2}\right)^2 - \left(\sin\left(\frac{\omega_0 t}{2}\right)\right)^2 = \cos(\omega_0 t)$$

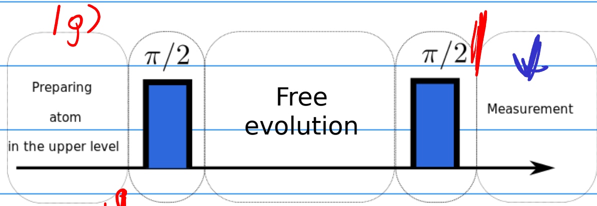




Clock and Ramsey Interferometer

10^4

$|g\rangle$

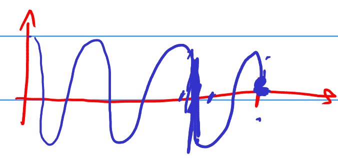


start

end

$\langle\sigma_z\rangle$

$\langle\sigma_z(\Delta t \cdot N_{pulses})\rangle$



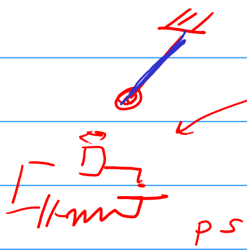
$T = \Delta t \cdot N_{pulses}$

N_{pulses}

Δt

Δt

optical comb



ps

Local oscillator
(regular pulses of a certain duration,
system of counting the number of pulses)

Application of the Ramsey interferometer

Atomic clock

Gravitometers

Magnetometers

GPS

stock market?