A Biologically Inspired Search Algorithm for Optimal Network Design

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Abstract

Optimal road network design is a crucial component of the city planning process. The mixed network design problem (MNDP) aims to modify existing links and add new candidate links to optimize network performance, and solutions to it form an important part of road network planning. However, current solutions to the MNDP are computationally intensive and can impede the road planning process. We introduce a novel algorithm, BioNet, to solve the MNDP as well as related road network design problems. Inspired by the slime mold *Physarum*, BioNet makes iterative changes to the links of a network in order to optimize performance. We show that BioNet matches the state-of-the-art on multiple classes of road network design problems, including the MNDP. In particular, its computational time for the MNDP is much lower than that of existing solution methods to the MNDP and delivers a speedup of over 400 times. Due to BioNet's computational performance on the MNDP, it appears to be a promising approach for the rapid development of large network designs. Further directions include considering a wider variety of optimization objectives, such as fault tolerance, and generalizing BioNet to other classes of network design problems outside of road network design.

1 Introduction

A city's transportation system, particularly its road network, is a core component of a city's physical, economic, and social organization and can shape its development. The network design problem (NDP) is concerned with determining the optimal ways to improve an existing road network, given a fixed financial budget. Road networks are represented as graphs with nodes and links. Nodes represent high-density places along the network and are connected by directed links, which are each associated with capacities. Traffic on the network is expressed in terms of link flows, which are numbers associated with each link that represent the amount of traffic on the link. Together with capacities, link flows are a factor in determining the travel time on each link.

There are three main classifications of the NDP: the discrete network design problem (DNDP), the continuous network design problem (CNDP), and the mixed network design problem (MNDP). The DNDP aims to find the optimal set of links to add from a given set of candidate links, and the CNDP aims to find the optimal changes in link capacity to make to existing links. The MNDP mixes the two and considers both the discrete decision variables of the DNDP and the continuous decision variables of the CNDP. Table 1 shows the types of NDPs depending on the types of candidate improvements each problem considers. In general, the MNDP aims to determine the optimal capacity changes and link additions to make, given a network of nodes and links, demand between each origin-destination pair, a set of capacity improvement costs, a set of new candidate links, and budget constraints. Challenges of the problem include its non-linearity and non-convexity [5]. In this project, we propose BioNet, a novel solution method to the MNDP inspired by the slime mold *Physarum*.

2 Literature Review

The MNDP is typically formulated as a mathematical programming with equilibrium constraints problem and expressed as a mixed, nonlinear bi-level optimization problem [20]. The lower level problem is the traffic assignment or user equilibrium (UE) problem, in which link flows are predicted, and the upper level problem consists of expanding link capacities and adding new links. The lower-level problem

Tab. 1: Types of road network design problems by types of candidate projects considered.

Variation	Capacity expansion	New link addition
Discrete (DNDP)	No	Yes
Continuous (CNDP)	Yes	No
Mixed (MNDP)	Yes	Yes

can be solved using a deterministic user equilibrium model or a stochastic user equilibrium (SUE) model. The SUE model accounts for the fact that perceived travel times may not equal actual travel times [4]. However, calculations using SUE are more difficult, so most studies prefer to use the deterministic user equilibrium model instead [5]. For the overall MNDP, two common formulations appear in the literature: either the objective is a weighted sum of passenger travel times and network construction cost [13] [22], or the objective is the sum of passenger travel times, and construction cost is a constraint [13] [14].

The non-linear and non-convex nature of the problem, which arises from its discrete, multi-objective nature, has prevented the development of an exact solution algorithm that runs in reasonable time. Thus, research has focused almost exclusively on finding suitable heuristic and metaheuristic solutions. Examples of previous heuristics are the link-based mixed-integer linear program (LMILP) method [14]. This method finds linear approximations to the non-linear objective function and solves the simplified linear programming problem using an exact method. Metaheuristics applied to the MNDP include genetic algorithms, simulated annealing, tabu search, and scatter search [2] [9]. Out of these metaheuristics, genetic algorithms and tabu search achieve the highest objective functions on the tested networks.

However, many limitations exist on current solution methods to the MNDP. Finding solutions is still computationally intensive, with the LMILP method taking 35 minutes to run on a medium-sized network [14]. Improving computational speed would expedite the overall road network planning process. Furthermore, the heuristics applied depend heavily on the approximations derived from the mathematical formulation of the MNDP and cannot be readily transferred to related NDPs, such as those for bus and railway network design.

In other fields, biologically inspired algorithms and methods, particularly neural networks and new metaheuristics, have led to significant computational advances. Initially inspired by the brain, neural networks have recently been applied to solve difficult tasks in a wide variety of domains, such as image classification, speech recognition, text generation, and medical diagnosis. Similarly, inspired by the process of evolution, genetic algorithms [10] have been used for a wide variety of optimization problems, including the MNDP itself. Other metaheuristics, such as particle swarm optimization–inspired by the social behavior of swarms [11]–and ant colony optimization–inspired by the behavior of ant colonies–have also been applied to a number of combinatorial optimization problems.

Previously, the slime mold *Physarum polycephalum* has been used to find solutions to the railway NDP for medium-sized networks [17]. In the NDP considered by Tero et al., a new network is built without starting from an existing network, unlike in the MNDP. In these experimental models, food sources were placed at nodes in a dish containing the slime mold. Initially, the organism grew in all directions and branched out to cover the entire dish. Over the next few days, regions of the organism that did not receive nutrients disappeared, until all that remained were tunnels connecting different food sources, whose capacities varied with the amount of food at each tunnel's endpoints. This model of growth, where all links are initialized at first, and their corresponding capacities are changed, has not been applied to the road MNDP, and computational models inspired by *Physarum* could offer a new method.

3 Purpose

1. Implement a solution to the lower-level user equilibrium problem in the form of the Frank-Wolfe algorithm [6].

2. Implement the novel biologically inspired algorithm for the upper-level optimization problem.

3. Evaluate our methods against existing methods by comparing the metrics of including sum of travel times and network building cost.

4 Notation and Problem Formulation

Here, we mathematically formulate the mixed network design problem, the continuous network design problem, and the user equilibrium problem. In order for our method to solve the MNDP and CNDP, it must solve the user equilibrium problem to determine flows on each link.

4.1 Notation

Throughout this paper, we will use the following notation:

A_1	the set of unchanged links
A_2	the set of expanded links
A_3	the set of added candidate links
A	the set of links in the network, which is $A_1 \cup A_2 \cup A_3$
x_a	the traffic flow on link $a \in A$
$\mathbf{x}$	a vector whose elements are x_a
y_a	the incremental capacity on expanded link $a \in A_2$
$\mathbf{y}$	a vector whose elements are y_a
u_a	a 0-1 decision variable which is 1 if link $a \in A_3$ is added and 0 otherwise
$\mathbf{u}$	a vector whose elements are u_a
$g_a(y_a)$	the improvement cost function of link $a \in A_2$
d_a	the construction cost of adding link $a \in A_3$
$t_a(x_a)$	the travel time function of link $a \in A_1$
$t_a(x_a, y_a)$	the travel time function of link $a \in A_2$ with added capacity y_a
$t_a(x_a, u_a)$	the travel time function of link $a \in A_3$
K_{rs}	the set of paths from node r to node s
f_k^{rs}	the flow on path k from node r to node s
$\delta_{a,k}^{rs}$	a variable which equals 1 if link a is on path k from node r to node s and 0 otherwise
$q_r s$	the demand from node r to node s
R	the set of origin nodes, i.e. nodes from which there is demand
S	the set of destination nodes, i.e. nodes to which there is demand
Z	the objective function
θ	the weight of construction cost in Z
I	the construction budget

The travel time function t_a of a link is given by

$$t_a = A_a + B_a \left(\frac{x_a}{c_a} \right)^4,$$

where c_a is the total capacity of the link, and A_a and B_a are constants that vary from link to link.

4.2 Mixed network design problem

We solve two versions of the mixed network design problem. The first, which we call the unconstrained MNDP, aims to minimize an objective function Z, which is a weighted sum of total passenger travel time and network construction cost. It is formulated as

$$\begin{aligned}
\min_{\mathbf{y},\mathbf{u}} Z(\mathbf{y},\mathbf{u},\mathbf{x}) = &\sum_{a\in A_1} x_a t_a(x_a) + \sum_{a\in A_2} x_a t_a(x_a, y_a) + \sum_{a\in A_3} x_a t_a(x_a, u_a) \\
&+ \theta \left(\sum_{a\in A_2} g_a(y_a) + \sum_{a\in A_3} d_a u_a \right) \\
\text{s.t. } y_a \geq 0. \quad &\text{(all capacity changes are nonnegative)}
\end{aligned}$$

For the other version, which we call the constrained MNDP, the objective function Z is the sum of passenger travel times. The constrained MNDP aims to minimize Z while keeping the network construction cost below a predetermined budget I and is formulated as

$$\begin{aligned}
\min_{\mathbf{y},\mathbf{u}} Z(\mathbf{y},\mathbf{u},\mathbf{x}) &= \sum_{a\in A_1} x_a t_a(x_a) + \sum_{a\in A_2} x_a t_a(x_a, y_a) + \sum_{a\in A_3} x_a t_a(x_a, u_a) \\
\text{s.t. } &\sum_{a\in A_2} g_a(y_a) + \sum_{a\in A_3} d_a u_a \leq I \quad \text{(construction cost is under budget)} \\
&y_a \geq 0 \quad \text{(all capacity changes are nonnegative)}
\end{aligned}$$

4.3 Continuous network design problem

In the CNDP, there are no new candidate links to be added, and the only possible changes are capacity expansions of existing links. Thus, it can be thought of as a special case of the MNDP, so our method also solves the CNDP. The CNDP is usually stated in its unconstrained form, where the objective function Z is a weighted sum of total travel time and network construction cost. It is formulated as

$$\begin{aligned}
\min_{\mathbf{y},\mathbf{u}} Z(\mathbf{y},\mathbf{u},\mathbf{x}) &= \sum_{a\in A_1} x_a t_a(x_a) + \sum_{a\in A_2} x_a t_a(x_a, y_a) + \theta \sum_{a\in A_2} g_a(y_a) \\
\text{s.t. } y_a \geq 0. \quad &\text{(all capacity changes are nonnegative)}
\end{aligned}$$

4.4 User equilibrium problem

In the unconstrained MNDP, constrained MNDP, and CNDP, the link flows $\mathbf{x}$ are determined by solving the lower-level user equilibrium problem. We assume that route choice behavior follows Wardrop's user equilibrium model [19], in which travelers choose routes to minimize their individual travel times. The problem of determining the link flows given the network structure is formulated as

$$\begin{aligned}
\min_{\mathbf{x}} \; & \sum_{a \in A_1} \int_0^{x_a} t_a(w)\, dw + \sum_{a \in A_2} \int_0^{x_a} t_a(w, y_a)\, dw \\
& + \sum_{a \in A_3} \int_0^{x_a} t_a(w, u_a)\, dw \\
\text{s.t.} \; & \sum_{k \in K_{rs}} f_k^{rs} = q_{rs} \, \forall r \in R, s \in S \\
& f_k^{rs} \geq 0 \, \forall r \in R, s \in S, k \in K_{rs} \\
& x_a = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} f_k^{rs} \delta_{a,k}^{rs} \, \forall a \in A.
\end{aligned}$$

This problem is solved using the Frank-Wolfe algorithm [6]. In practice, although Frank-Wolfe may yield exact solutions, it takes too long to do so. As a result, the algorithm is usually stopped before an exact solution is reached.

5 BioNet, a Biologically Inspired Solution Method to the MNDP

We propose BioNet, a solution method to the MNDP inspired by the slime mold *Physarum*. Figure 1 shows *Physarum*'s growth over time. Initially, *Physarum polycephalum* grows evenly outward to fill its entire environment [18]. Pathways within the network that do not receive nutrients degenerate, and certain pathways that do transport nutrients are selectively strengthened and widened. Eventually, after enough time, the *Physarum* creates a stable network that only contains pathways connecting food sources. Since a pathway's thickness depends on the amount of nutrients it transports, the *Physarum* essentially attempts to optimize the network for the objective of effective nutrient transportation.

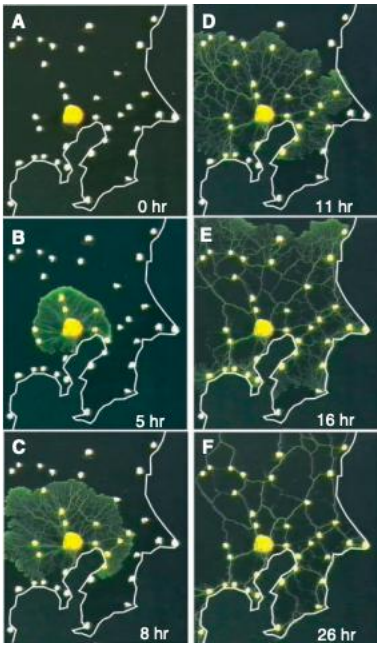

Fig. 1: Growth and network formation of *Physarum polycephalum*, reprinted from Tero et al., 2010. White dots represent food sources, and the organism grows outward from the center of the figure.

To simulate the growth of *Physarum*, which evenly explores its whole environment, the network is initialized with all links set to an arbitrarily high capacity. In practice, we set this arbitrarily high capacity to be 3 times the capacity of the link with the greatest capacity in the network. In each iteration, the method changes the capacities of each link that is targeted for improvement. These capacity changes are based off of the estimated benefit of improving that link, which we measure with an "effectiveness factor" μ_a calculated for each link in A_2 and A_3. The capacities of links with low μ_a values are reduced more than those of links with high μ_a values, and vice versa.

Since links in A_3 are controlled by a discrete decision variable, they can only either be added to the network with a fixed capacity or not added at all. For such a link $a \in A_3$, we keep track of a temporary capacity y_a, which does not necessarily equal the capacity that a would have if it were added. After the algorithm is complete, we add all links $a \in A_3$ such that y_a is above a certain threshold to the network with their respective candidate capacities.

The process can be controlled by four parameters: m, ℓ, c_1, and c_2. The parameters m and ℓ control the size of the capacity changes made by the algorithm, and c_1 and c_2 control the threshold at which the algorithm accepts or rejects adding or improving links in the network. The procedure can be described as follows:

Step 0: *Initialization*. Set the capacity of all links in A_2 to an arbitrarily high capacity. Set $u_a = 1$ for all $a \in A_3$.

Step 1: *Objective checking*. If the objective function is below the target objective, or the construction cost is below budget, depending on the problem formulation, go to step 5.

Step 2: *Link evaluation*. Solve the user equilibrium problem, and update all link flows. For each link $a \in A_1 \cup A_2$, define the effectiveness factor

$$\mu_a \coloneqq \frac{x_a}{c_a},$$

where c_a is the capacity of link a. For a candidate link $a \in A_3$, define its effectiveness factor to be

$$\mu_a \coloneqq \frac{x_a}{c_a d_a}.$$

Step 3: *Update* μ_{max}. Find the maximum value of μ across all links. If this value is greater than μ_{max}, set μ_{max} to this value. If μ_{max} does not exist, set it to the maximum value of μ.

Step 4: *Update link capacities*. For all links $a \in A_2$, update their capacities by the rule

$$y_a \coloneqq y_a \gamma_a,$$

where

$$\gamma_a = \frac{(m-\ell)\mu + \ell\mu_{max}}{\mu_{max}}.$$

Step 5: *Clean links*. For all links $a \in A_2$ such that $y_a < c_1$, set $y_a = 0$. For all links $a \in A_3$ such that $y_a < c_2$, set $u_a = 0$. Go to step 1.

Step 6: *Re-initialize links in* A_3. For all links $a \in A_3$ such that $y_a > 0$, set $u_a = 1$.

Keeping track of a single μ_{max} value throughout the entire procedure prevents large capacity fluctuations as the algorithm proceeds.

6 Evaluations

In this section, we explain the tests that have been carried out on BioNet, the results of which are in the following section. We implemented the algorithm in Python 3.8.3 and used this implementation for all our tests. To compare BioNet to existing methods, we evaluate its performance on a small 16-link test network [7] and the medium-sized Sioux Falls network [12], which are commonly used for measuring NDP solution performance. Since there are significantly more solution methods for the CNDP than for the MNDP, we also compare BioNet's performance on the CNDP to other solution methods for the CNDP. Table 2 contains a list of methods to which we compare BioNet.

Tab. 2: Existing solution methods to the CNDP and MNDP.

Abbreviation	Algorithm	Problems solved	Source
H-J	Hooke-Jeeves	CNDP only	[1]
EDO	Equilibrium decomposed optimization	CNDP only	[16]
SA	Simulated annealing	CNDP only	[8]
SAB	Sensitivity analysis-based	CNDP only	[21]
GP	Gradient projection	CNDP only	[3]
CG	Conjugate gradient projection	CNDP only	[3]
QNEW	Quasi-Newton projection	CNDP only	[3]
PT	PARTAN gradient projection	CNDP only	[3]
LMILP	Link-based mixed-integer linear program	CNDP and MNDP	[14]
DDIA	Dimension-down iterative algorithm	CNDP and MNDP	[13]
-	BioNet	CNDP and MNDP	This paper

6.1 Notes on tests

To evaluate the total travel time of a network, the user equilibrium problem must be solved for that network. Since the Frank-Wolfe algorithm for UE problems takes an extremely long time to converge on a solution, different thresholds are used by different studies when calculating total travel time of a network. Therefore, to reduce any bias that may result from these differences, we calculate the objective function for all solutions using a common threshold of 100 Frank-Wolfe iterations. While more iterations may be desirable in real-world network design, we cap all testing at 100 Frank-Wolfe iterations to standardize comparisons between algorithms and to speed up tests.

When comparing results, we report the number of UE problems solved for our algorithm instead of computational time, since times can vary between different computing platforms. For reference, on a personal computer with a 1.6 GHz Intel Core i5 and 8 GB of RAM, one UE problem on the 16-link network takes around 20 ms to solve, and one UE problem on the Sioux Falls network takes around 375 ms to solve.

To choose values for the parameters m, ℓ, c_1, and c_2, we ran tests by solving the CNDP over a small 5-link network with a large number of combinations of different parameter values. Eventually, we selected a small number of feasible values for each parameter. When evaluating our method over the test networks, we ran tests for all combinations of the following parameter values:

$$\begin{aligned} m &= 0 \\ \ell &\in \{0.8, 1\} \\ c_1 &\in \{0, 0.1, 0.2\} \\ c_2 &\in \{0.05, 0.1\}. \end{aligned}$$

6.2 Continuous case

For the CNDP, we compare BioNet to the existing CNDP solution methods listed in Table 2. For each method, we evaluate the objective function Z as well as the total travel time and construction cost on the network.

The simple 16-link test network, which is shown in Figure 2, consists of 6 nodes. We use the input data given in Suwansirikul et al. (1987), which consists of the starting network, link improvement costs, and an O-D demand matrix.

There are only 2 origin-destination demand pairs, namely $(1, 6)$ and $(6, 1)$. We consider the case where these pairs have demand 5 and 10, respectively. The objective function to be minimized is

$$Z = \sum_{a \in A_1} x_a t_a(x_a) + \sum_{a \in A_2} x_a t_a(x_a, y_a) + \theta \sum_{a \in A_2} g_a(y_a),$$

where $\theta = 1$ and $g_a(y_a) = k_a y_a$ for a given link improvement cost k_a. The cost k_a varies from link to link and is defined for all 16 links in the network. We used parameter values of $m = 0$, $\ell = 0.8$, and $c_1 = 0.5$. Since no new links are being added, we did not need to provide a c_2 value.

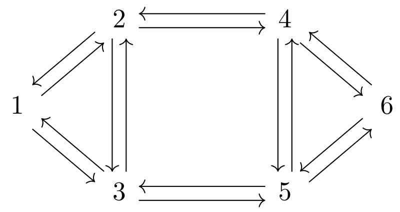

Fig. 2: The 16-link test network. All links are targeted for improvement.

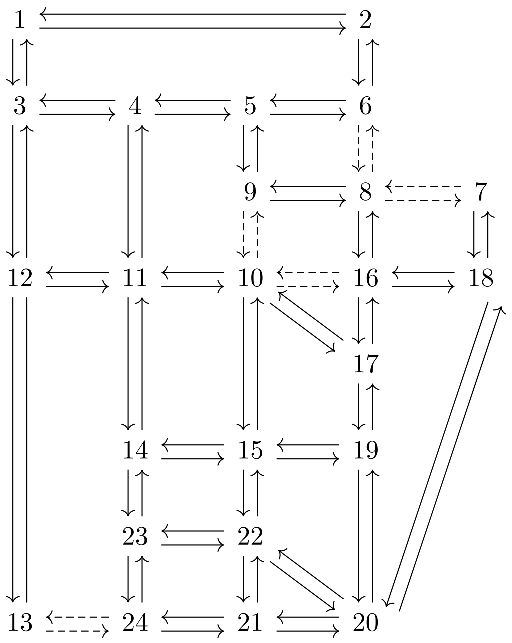

Fig. 3: The Sioux Falls test network. The dashed links are targeted for improvement.

The medium-sized Sioux Falls network, which is shown in Figure 3, consists of 24 nodes and 76 links. Again, we use the input data from Suwansirikul et al. (1987). For the CNDP on this network, the objective function to be minimized is the typical CNDP objective function

$$Z = \sum_{a \in A_1} x_a t_a(x_a) + \sum_{a \in A_2} x_a t_a(x_a, y_a) + \theta \sum_{a \in A_2} g_a(y_a),$$

where $\theta = 0.001$ and $g_a = k_a y_a^2$ for a given link improvement cost k_a. This cost k_a is only defined for the links that are targeted for improvement, which are the dashed links in Figure 3. We used parameter values of $m = 0$, $\ell = 1$, and $c_1 = 0.5$. Again, no new links are being added, so we did not need to provide a c_2 value.

6.3 Mixed case

Figure 4 shows the Sioux Falls network in the MNDP case, with 10 candidate links added. For the MNDP, we compare BioNet to three other solution methods for the MNDP. Due to the relative scarcity of network comparisons, we only evaluate BioNet's performance on the MNDP over the Sioux Falls network. We follow the approach of Luathep et al. (2011) and combine the link improvement costs of Suwansirikul

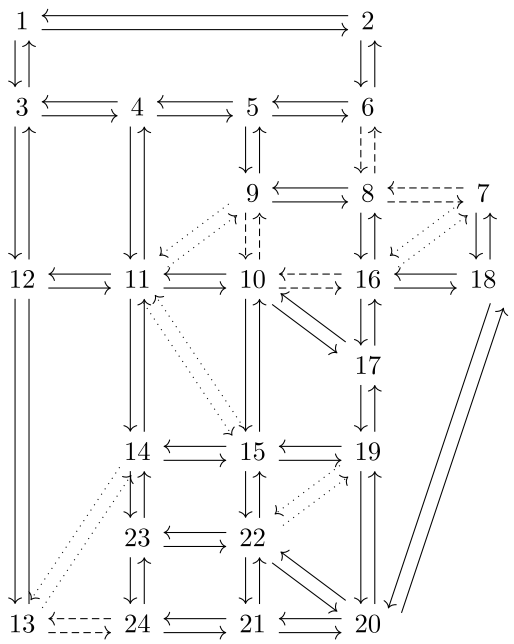

Fig. 4: The Sioux Falls test network for the MNDP case. Dashed links are targeted for capacity expansion, and dotted links are candidates for being added to the network.

et al. (1987) and the new candidate link projects of Poorzahedy and Turnquist (1982). We evaluate BioNet's performance on both the unconstrained and constrained MNDP.

For the unconstrained MNDP, the objective function to be minimized is

$$Z(\mathbf{y},\mathbf{u},\mathbf{x}) = \sum_{a\in A_1} x_a t_a(x_a) + \sum_{a\in A_2} x_a t_a(x_a, y_a) + \sum_{a\in A_3} x_a t_a(x_a, u_a) + \theta\left(\sum_{a\in A_2} g_a(y_a) + \sum_{a\in A_3} d_a u_a\right),$$

where $\theta = 0.001$ and $g_a(y_a) = k_a y_a^2$. These values are the same as those used for the CNDP over the Sioux Falls network. We used parameter values of $m = 0$, $\ell = 0.8$, $c_1 = 0$, and $c_2 = 0.1$.

We considered the constrained MNDP with a budget of $I = 4000$, or 4 million dollars. The objective function to be minimized is the total travel time on the network, or

$$Z(\mathbf{y},\mathbf{u},\mathbf{x}) = \sum_{a\in A_1} x_a t_a(x_a) + \sum_{a\in A_2} x_a t_a(x_a, y_a) + \sum_{a\in A_3} x_a t_a(x_a, u_a).$$

We used parameter values of $m = 0$, $\ell = 0.8$, $c_1 = 0.2$, and $c_2 = 0.05$.

7 Results

In this section, we apply the proposed solution algorithm to the CNDP and MNDP and compare them to existing methods in the literature. We follow the tests described in the Evaluations section above. Capacity expansion values for each solution method and information on which links were added for each MNDP solution method can be found in the appendix.

7.1 Continuous case

7.1.1 16-link network

The results in Table 3 show that our method finds a solution with a rather low value for Z, outperforming the majority of the methods we compared ours to. Our value for Z is less than 1% greater than the Z value found by LMILP, the best-performing method on this network. The construction cost of our solution is also less than 4% greater than the least expensive method and is much lower than the costs achieved by all other solutions. However, we solve slightly more UE problems than other methods, so our method may run slower than others. However, our method runs in less than a second for the 16-link network, so any such differences would be minimal.

Tab. 3: Results for the CNDP case on the 16-link network.

Metric	H-J	EDO	SA	SAB	GP	CG	QNEW	PT	LMILP	BioNet
Total travel time (10^3 veh)	188.35	187.25	191.45	186.35	186.60	185.93	185.96	**185.88**	186.80	190.47
Construction cost (10^3 \$)	29.80	13.95	**9.89**	17.85	17.16	14.30	14.65	16.54	12.84	10.27
UE problems solved	54	10	18300	**6**	14	7	12	16	-	21
Z	218.15	201.20	201.34	204.21	203.76	200.23	200.62	202.42	**199.63**	200.74

Note: Luathep et al. (2011) did not provide the number of UE problems solved.
Lower travel time, lower construction cost, fewer UE problems solved, and a lower value of Z are better. The best value for each metric is bolded.

7.1.2 Sioux Falls network

Tab. 4: Results for the CNDP case on the Sioux Falls network.

Metric	H-J	EDO	SA	SAB	GP	CG	QNEW	PT	LMILP	DDIA	BioNet
Total travel time (10^3 veh)	76.547	80.255	75.791	**73.564**	78.134	76.226	76.753	77.876	76.246	76.127	77.397
Construction cost (10^3 \$)	5079	**3132**	5487	10796	5973	6626	6451	6296	5027	4910	5216
UE problems solved	58	12	3900	11	10	6	**4**	7	-	-	37
Z	81.626	83.387	81.278	84.360	84.108	82.852	83.204	84.172	81.273	**81.038**	82.612

Note: Luathep et al. (2011) and Liu and Chen (2016) did not provide the number of UE problems solved.
Lower travel time, lower construction cost, fewer UE problems solved, and a lower value of Z are better. The best value for each metric is bolded.

For the CNDP over the Sioux Falls network, Table 4 shows that our method matches performance with existing methods on the continuous problem. Our objective function is closest to that of the CG method. However, in comparison, the construction cost of the network generated by our method is 5.216 million dollars, which is much smaller than the 6.626 million dollar construction cost of the CG method. Although our method solves more equilibrium problems than some other methods and therefore may be slightly slower, our method takes 16 seconds to run on the Sioux Falls network, so differences would also be small.

7.2 Mixed case

For the unconstrained MNDP over the Sioux Falls network, we compare our method to DDIA, another MNDP solution method, and the ”solve all CNDPs” method proposed by Liu and Chen (2016). The ”solve

Tab. 5: Results for the unconstrained MNDP case on the Sioux Falls network.

Metric	DDIA	Solve all CNDPs	BioNet
Total travel time (10^3 veh)	56.708	56.709	**55.913**
Construction cost (10^3 \$)	**10208**	10249	12067
UE problems solved	-	-	3
Z	**66.916**	66.958	67.980

Note: Liu and Chen (2016) did not provide the number of UE problems solved.
Lower travel time, lower construction cost, fewer UE problems solved, and a lower value of Z are better. The best value for each metric is bolded.

all CNDPs" method enumerates all the possible combinations of adding new links and solves a CNDP for each one.

Table 5 shows the results for the unconstrained MNDP on the Sioux Falls network. Here, our method achieves a similar value on the objective function to DDIA. In addition, our method runs in fewer than 3 seconds. However, computational time is not reported for DDIA or the method of solving all CNDPs, so we cannot compare our time to theirs.

Tab. 6: Results for the constrained MNDP case on the Sioux Falls network.

Metric	LMILP	DDIA	BioNet
Construction cost (10^3 \$)	4000	3970	**3856**
UE problems solved	-	-	9
Z (total travel time, 10^3 veh)	**68.212**	68.651	68.87

Note: Luathep et al. (2011) and Liu and Chen (2016) did not provide the number of UE problems solved.
Lower travel time, lower construction cost, fewer UE problems solved, and a lower value of Z are better. The best value for each metric is bolded.

Table 6 shows the results for the constrained MNDP on the Sioux Falls network with a budget of $I = 4000$, or 4 million dollars. All three methods achieve very similar results. Our total travel time is less than 1% greater than that achieved by LMILP, the best-performing method, and our construction cost is also the lowest out of all the methods. Furthermore, our method achieves a greater than 400 times computational speedup over LMILP, with a runtime of only 5 seconds, compared to 35 minutes for LMILP. [14].

8 Conclusion and Discussion

The mixed network design problem aims to find improvements on an existing road network to optimize an objective function, usually a combination of total travel time and network construction cost. Because of the problem's non-linearity and non-convexity, heuristics and metaheuristics have been developed to solve the problem. However, these methods are computationally intensive and difficult to generalize to different objectives and types of networks. We propose BioNet, a method for solving the MNDP inspired by the slime mold *Physarum*. BioNet finds solutions by initializing all links and selectively keeping those that are effective, which we measure by calculating the ratio of link flow to link capacity. Our method can be applied to different classes of road network design problems, such as the CNDP and DNDP.

To evaluate the effectiveness of our method, we performed tests on a small 16-link network and the medium-sized Sioux Falls network. We compared our method's performance to existing methods on the CNDP, unconstrained MNDP, and constrained MNDP. For all classes of problems, BioNet matches the performance of current state-of-the-art methods, and its computational time either matches that of existing methods or is lower by far. On the CNDP, our method solves slightly more UE problems than some other methods, but the computational times are still small (< 20 seconds). On the unconstrained and constrained MNDPs, our method also performs similarly to existing methods on the objective function. However, it runs much faster than current solutions on the constrained MNDP.

For both the unconstrained and constrained MNDP, BioNet solves fewer UE problems than for the two CNDP tests. Thus, BioNet performs better with respect to computational time on the MNDP than for the CNDP. We believe that this difference may be due to BioNet's exploratory nature; it naturally considers the effects of adding or removing links, and limiting it to only consider capacity expansions hinders its performance. Furthermore, since BioNet does not solve significantly more problems for the Sioux Falls network than for the small 16-link network, the number of UE problems solved may depend strongly on network size. This may be due to the fact that BioNet changes the entire network at once, so improvements to different parts of the network can be made in parallel in the same number of iterations. As a result, computational time does not appear to vary strongly with network size, so our method may be able to offer fast performance on larger networks.

This paradigm of biologically inspired learning is the first of its kind to be applied to road network design, and it offers a number of benefits over existing methods. Based on the comparisons of computational speed that could be made, our method runs significantly faster than previously published state-of-the-art methods. The biologically inspired framework that our method uses is also more generalizable and can lead to new solution methods to other types of NDPs. Furthermore, our method allows for the consideration of alternative objective functions. While we consider the ratio of flow to capacity as well as improvement cost in μ, other metrics may be incorporated into the calculation of μ to optimize different objectives.

Future directions include considering a wider variety of objectives, evaluating BioNet's performance on larger networks, and incorporating more sophisticated traffic models. In particular, fault tolerance [17], environmental impact, and equity in travel time [5] have been identified as objectives that are of further interest in road network design. As for applying BioNet to larger networks, while we have not evaluated its performance on large networks, BioNet's performance on small and medium-sized test networks in this paper indicate that it may offer a promising approach for large networks. Finally, in order to better model traveler behavior, BioNet may be combined with more realistic models, such as elastic demand, which allows for changes in origin-destination demand over time, and stochastic user equilibrium, which corrects for the possibility that travelers' perceptions of travel time may not reflect actual travel time. Applying these models would make our framework even more applicable to real-life network design.

9 Acknowledgements

I would like to thank my mentor at SSI, Dhaman Kaur, for providing valuable advice on both my project and this paper. I would also like to thank another mentor at SSI, Allison Tam, for her feedback and encouragement on my project and paper. Finally, I would like to thank Dhruvik Parikh, Anne Lee, and Franklyn Wang from SSI for their comments on this paper.

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A Solution details

Tab. 7: Capacity expansion values for the CNDP case on the 16-link network.

Variable	H-J	EDO	SA	SAB	GP	CG	QNEW	PT	LMILP	BioNet
Links to be expanded										
y_3	1.20	0.13								
y_6	3.00	6.26	3.1639	5.8352	5.8302	6.1989	6.0021	5.9502	5.24	4.531
y_{15}	3.00	0.13		0.9739	0.8700	0.0849	0.1846	0.5798	0.002	
y_{16}	2.80	6.26	6.7240	6.1762	6.1090	7.5888	7.5438	7.1064	7.585	5.743

Note: If no number is given in a cell, the solution method did not choose to expand the corresponding link.

Tab. 8: Capacity expansion values for the CNDP case on the Sioux Falls network.

Variable	H-J	EDO	SA	SAB	GP	CG	QNEW	PT	LMILP	DDIA	BioNet
Links to be expanded											
y_{16}	4.8	4.59	5.38	5.7392	5.4277	4.7691	5.3052	5.0237	5.362	3.9375	6.253
y_{17}	1.2	1.52	2.26	51.7182	5.3235	4.8605	5.0541	5.2158	2.057	1.6875	
y_{19}	4.8	5.45	5.50	4.9591	1.6825	3.0706	2.4415	1.8298	5.486	4.3125	6.254
y_{20}	0.8	2.33	2.01	4.9612	1.6761	2.6836	2.5442	1.5747	1.895	2.9375	
y_{25}	2.0	1.27	2.64	5.5066	2.8361	2.8397	3.9328	2.7947	2.556	2.5000	0.810
y_{26}	2.6	2.33	2.47	5.5199	2.7288	2.9754	4.0927	2.6639	2.618	2.1875	0.907
y_{29}	4.8	0.41	4.54	5.8024	5.7501	5.6823	4.3454	6.1879	3.741	2.9375	4.685
y_{39}	4.4	4.59	4.45	5.5902	4.9992	4.2726	5.2427	4.9624	4.551	4.0000	3.797
y_{48}	4.8	2.71	4.21	5.8439	4.4308	4.4026	4.7686	4.0674	3.741	4.2500	4.831
y_{74}	4.4	2.71	4.67	5.8662	4.3081	5.5183	4.0239	3.9199	4.489	3.875	3.760

Note: If no number is given in a cell, the solution method did not choose to expand the corresponding link.

Tab. 9: Capacity expansion values and added links for the unconstrained MNDP case on the Sioux Falls network.

Origin node	Destination node	DDIA	Solve all CNDPs	BioNet
Links to be expanded				
6	8	3.250	3.500	2.870
7	8	1.250	1.500	2.407
8	6	3.750	4.000	2.953
8	7	1.250	1.750	1.557
9	10	0.750	1.250	1.694
10	9	0.938	0.750	1.625
10	16	4.250	4.000	4.643
13	24	1.000	1.500	1.782
16	10	4.500	4.000	4.348
24	13	1.000	2.000	1.795
New candidate links				
7	16	−	−	+
9	11	+	+	+
11	9	+	+	+
11	15	+	+	+
13	14	+	+	+
14	13	+	+	+
15	11	+	+	+
16	7	−	−	+
19	22	+	+	+
22	19	+	+	+

Note: For candidate links, + indicates that the link was added, and − indicates that the link was not added.

Tab. 10: Capacity expansion values and added links for the constrained MNDP case on the Sioux Falls network.

Origin node	Destination node	LMILP	DDIA	BioNet
Links to be expanded				
6	8	3.173	4.625	2.300
7	8	1.084	1.875	0.338
8	6	2.919	3.000	2.325
8	7	1.078	1.500	0
9	10	1.316	2.250	0.400
10	9	1.564	2.250	0.342
10	16	3.232	3.000	2.054
13	24	2.867	2.000	0.731
16	10	3.232	2.000	2.042
24	13	2.638	2.000	0.722
New candidate links				
7	16	−	−	−
9	11	−	−	−
11	9	−	−	−
11	15	+	+	+
13	14	−	−	−
14	13	−	−	+
15	11	+	+	+
16	7	−	−	−
19	22	−	−	−
22	19	−	−	−

Note: For candidate links, + indicates that the link was added, and − indicates that the link was not added.