The Omega

Guidelines and insights

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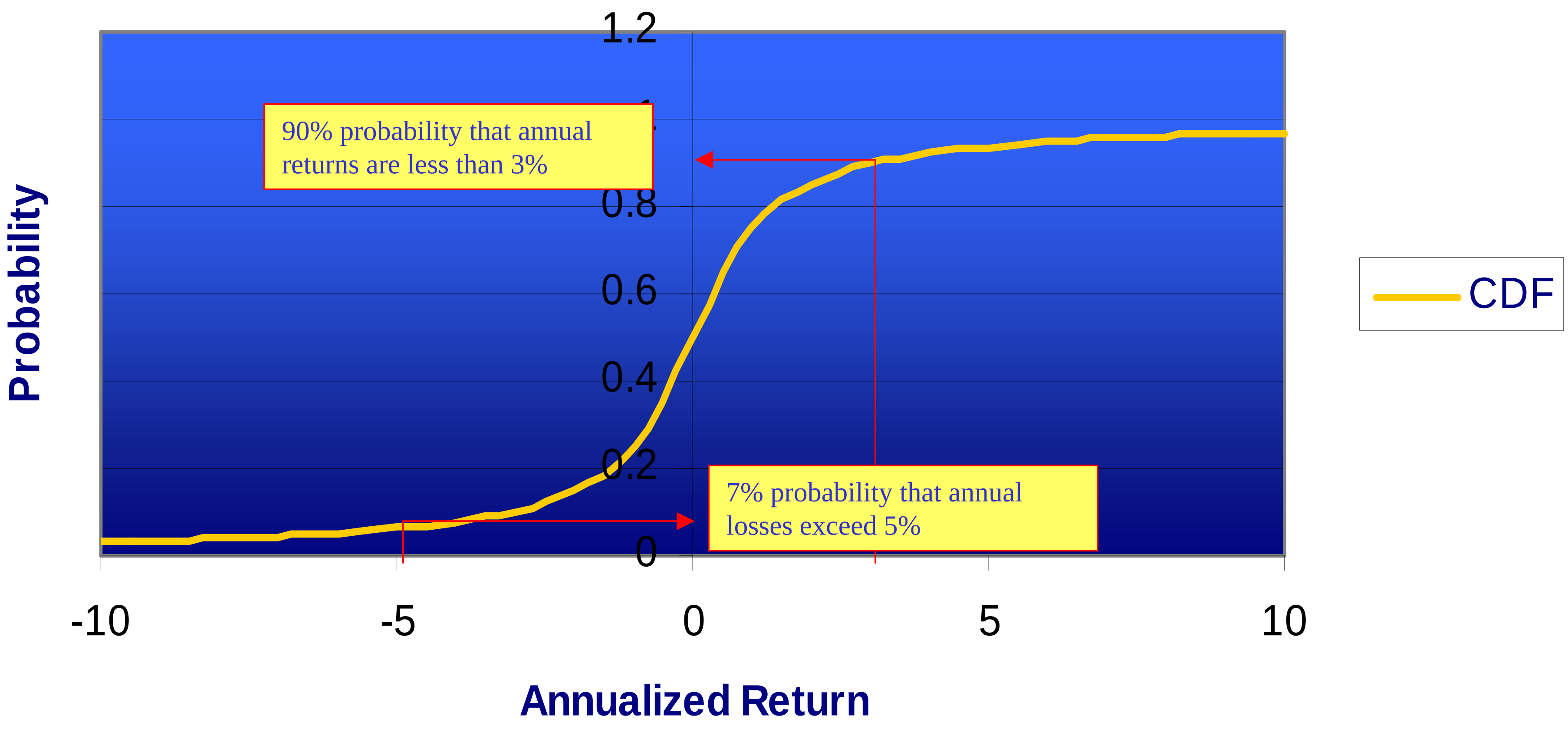
Before Omega there was...

- Markowitz's efficient frontier and Sharpe's ratio.
  - Portfolio behavior is described by its expected return and standard deviation.
- Dembo's reward/regret model.
  - Portfolio behavior is described by the reward (over-performance to a benchmark), and regret (underperformance to a benchmark).

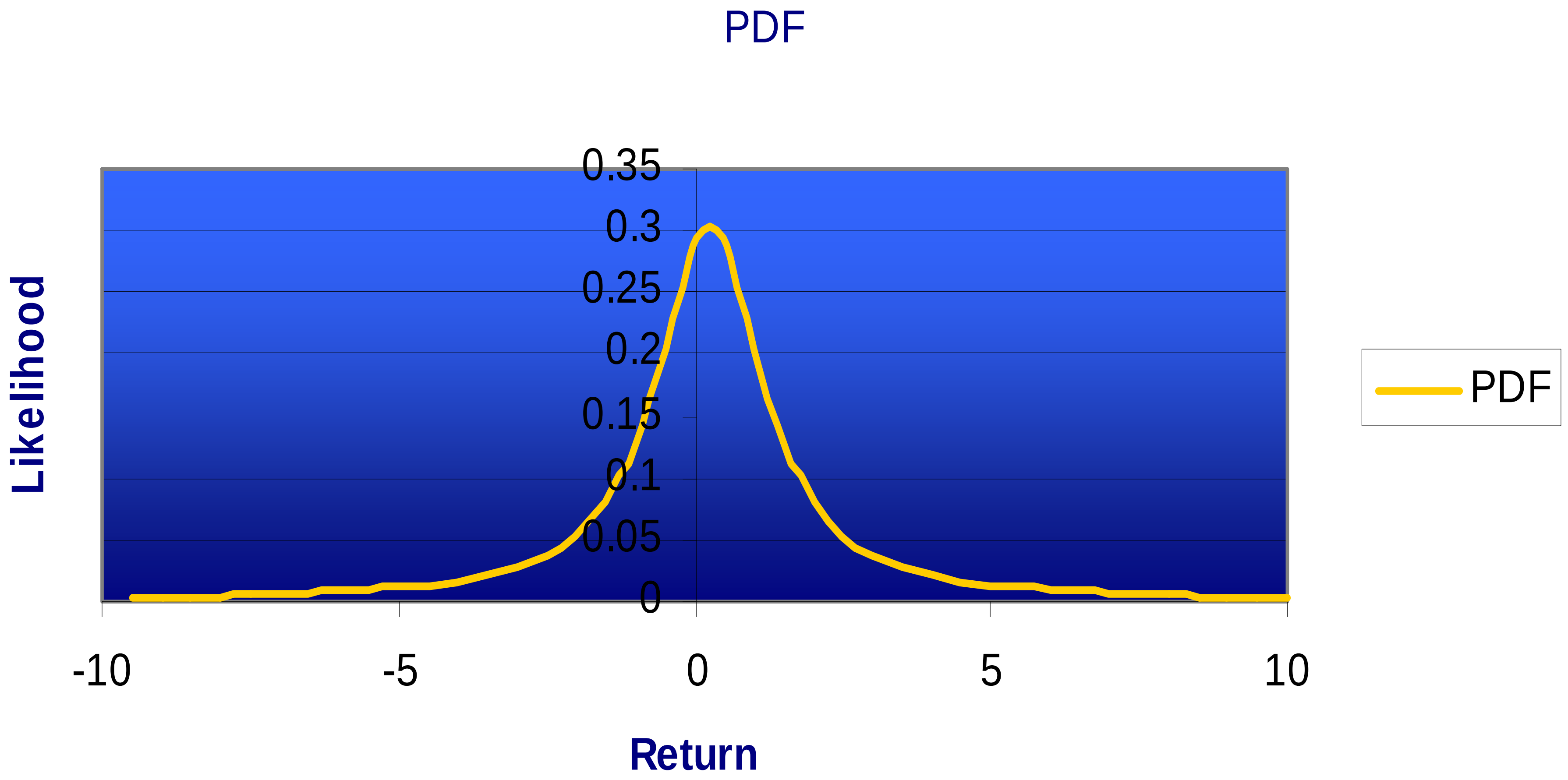
Omega

- Shadwick introduced the concept of "Omega" a few years ago, as the replacement of the Sharpe ratio when returns are not normally distributed.
- His aim was to capture the "fat tail" behavior of fund returns.
- Once the "fat tail" behavior has been captured, one then needs to optimize investment portfolios to *maximize the upside, while controlling the downside*.

The portfolio distribution function （CDF）



Probability density: histogram



Do moments capture tail behavior?

- Common wisdom: "Moments capture tail behavior"

$$m_n = \mathrm{E}\left[X^n\right]$$

  - The more moments, the more tail information.
  - Extreme events, due to the larger power, affect the higher moments more than the lower moments.
  - Using the information coming from higher moments we can reconstruct the return probability distribution better than using means and standard deviations.

Do moments capture tail behavior?

- Moments don't capture tail behavior.
  - Large events introduce distortion in the higher moments when calculated from limited datasets; the distortion is greater the higher the moment.
  - Hosking (1990) introduced the concept of L-moment which is much less sensitive to extreme events. L-moments can be accurately estimated using the order statistics.

Wins vs. losses: the Omega

$$\Omega(r) = \frac{\int_r^{\infty} (1 - F(x))\, dx}{\int_{-\infty}^{r} F(x)\, dx}$$

$$= \frac{\int_r^{\infty} x\, \rho(x)\, dx}{\int_{-\infty}^{r} x\, \rho(x)\, dx}$$

$$= \frac{\mathrm{E}\left[R \mid R > r \right] \quad \Pr(R > r)}{\mathrm{E}\left[R \mid R < r \right] \quad \Pr(R < r)}$$

$$= \frac{\text{Dembo's reward}}{\text{Dembo's regret}}$$

Omega tries to capture tail behavior avoiding moments, using the relative proportion of wins over losses:

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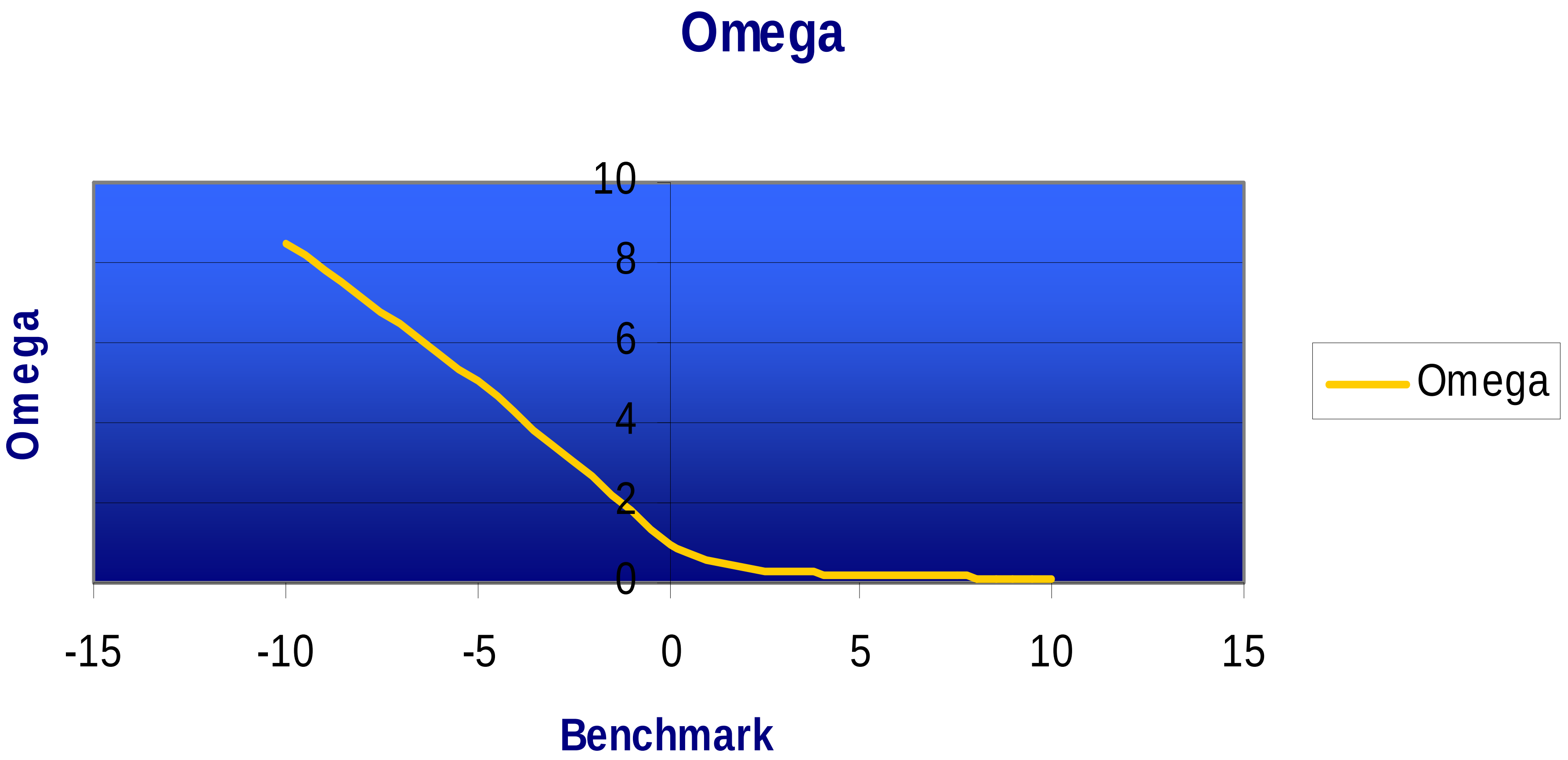
Truncated First Moments

The Omega of a heavy tailed distribution



A number, or a curve?

- The largest single misconception about the Omega, is that it is a “curve”, not a “number”;
- In other words, the Omega carries all the information about the distribution of returns, only when we have the information for all values of the benchmark return.
- Furthermore, if one is interested in tail behavior, then the benchmark returns that are relevant are in the tail of the distribution.

Omega: a "first moment" estimate

- We saw that the higher the moment, the more tail information it yields
- How can Omega give accurate tail information when it is nothing but a truncated "first moment"?
- Won't the standard deviation give more information on tail behavior than Omega?
- It all depends on the benchmark.

The role of the benchmark

- If one is interested in risk management, then the benchmark should be chosen to be large negative; the larger the Omega in that case, the safer the investment.
- However, a large Omega when r is negative does not provide any insight as to the probability of obtaining target returns in line with moderate portfolio objectives
- One again, we have to take into account the values of Omega for various values of the benchmark, and somehow balance one against the others.
- The problem is that we lack an "Omega utility theory".

Can you optimize?

- The Sharpe ratio is appealing because it can easily be optimized within the quadratic utility framework of Markowitz.
- The Omega lacks such simple framework.
- However, the Dembo optimization framework for reward and regret can be used to obtain efficient optimization algorithms for Omega.

What do you want to optimize?

- However, even if we could optimize the Omega statistic easily, it is still not clear which *value* of the benchmark to use in the optimization process; again, we would need an "Omega Utility Theory".
- The use of small benchmarks would be attractive for risk management, but would give up portfolio returns.
- The use of medium-sized benchmarks will ignore tail effects.
- An alternative would be to employ the Dembo optimization framework, in reward/regret space, as the non-gaussian analog of the Markowitz efficient frontier framework.

Conclusions

- The Omega statistic is an attempt to extend the Sharpe ratio to non-normal markets
- The Omega, as a curve, captures all the information about portfolio returns, and expresses it in a manner which is intuitive from an investment performance viewpoint.
- The Omega, as a single number, will fail to account for either extreme events, or reasonable portfolio performance benchmarks
- If used in a portfolio optimization context, one may need to make a sacrifice and choose for performance or risk management.