The harmful effects of correlation breakdown

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Basic example: CFO

A CDO with a fund-of-hedge fund collateral.

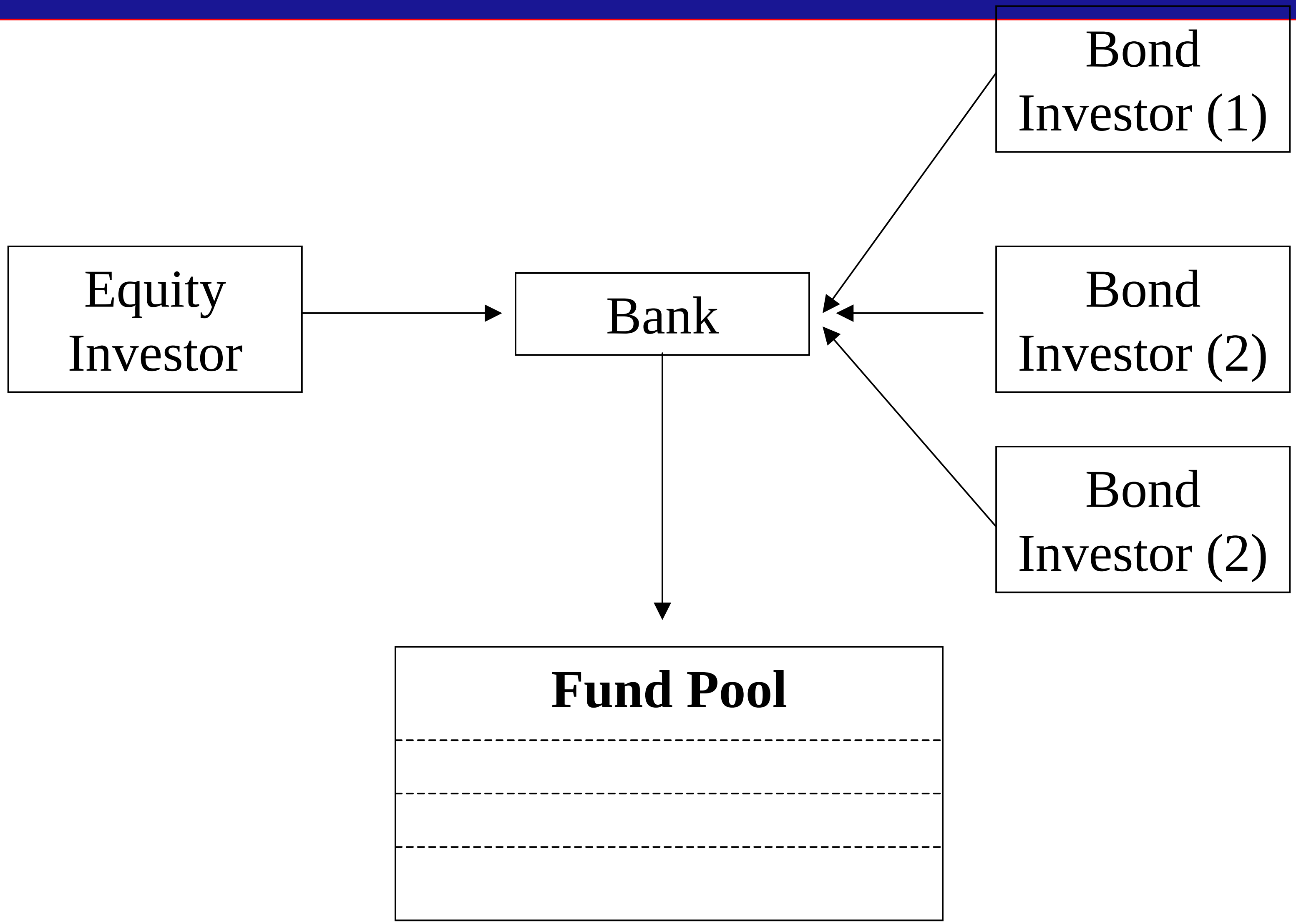
A financial structure with equity investors, and lenders; all the assets (equity and bonds) are invested in a portfolio of hedge funds

The lenders earn a spread over interest rates

The equity holder earn the total return of the fund, minus the financing fees.

If the fund drops in value, shareholders lose first, followed by the different bond holders according to the seniority of the bond issues.

The CFO


Bond
Investor (1)
Equity
Investor
Bank
Bond
Investor (2)
Bond
Investor (2)
Fund Pool

2004: $1 trillion in hedge fund investments

Investors are developing interests in “alternative Investments”, which offer returns non-correlated to traditional investments. Those investments consist in the purchase of fund shares offered by hedge fund companies.

A fund share purchase is used by the hedge fund to increase its asset base which is invested in a variety of management strategies, passing the gains (or losses) to the investors. Strategies include:

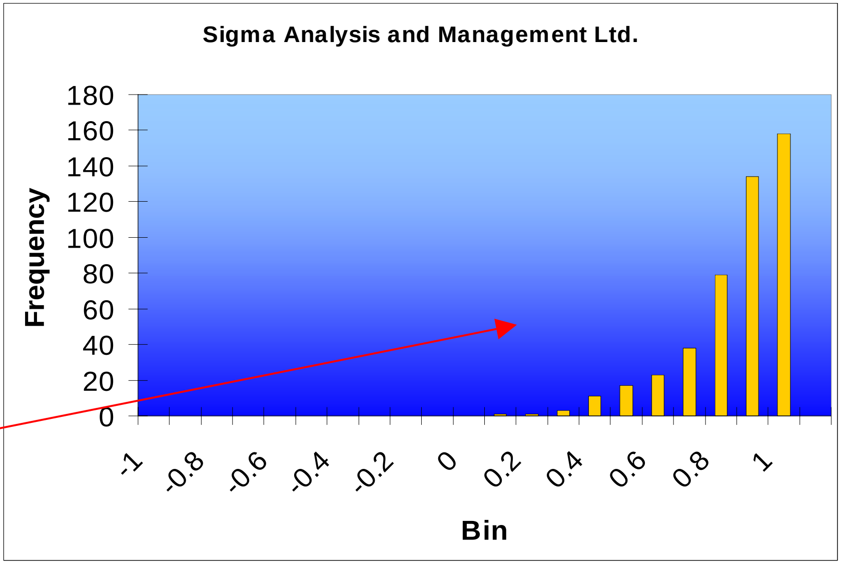
- Equity Long/short portfolios
- Relative value trades
- Merger, covertible or statistical arbtrage, etc.

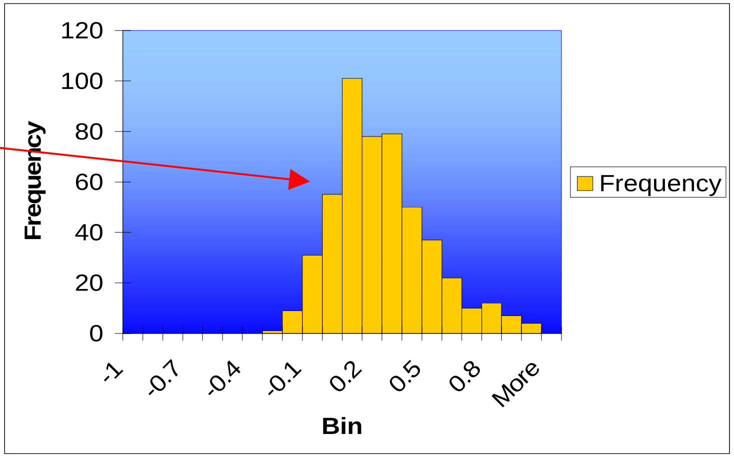
A new investment universe

Hedge funds are uncorrelated to traditional markets, so they constitute excellent diversification strategies….

Correlation histogram for Dow stocks

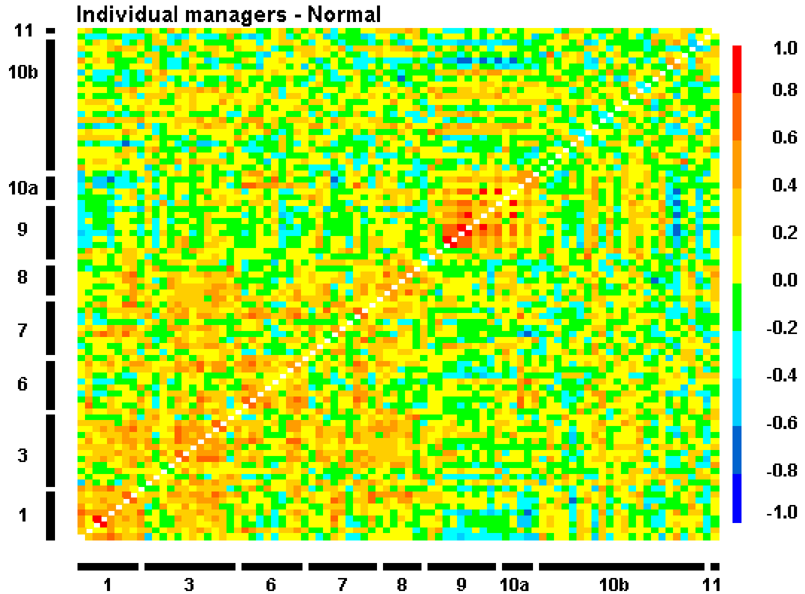
Correlation histogram for hedge funds

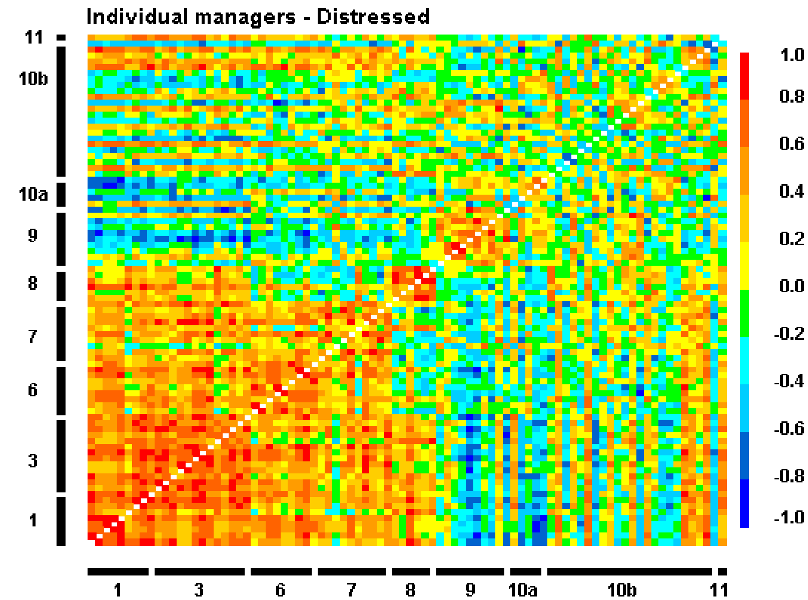




…in principle

Correlation switching





This is an example of how individual hedge funds change correlations during distress

The CFO challenge

The internal leverage within a CFO makes it more vulnerable to deviations from historical scenarios

Sensitivity analysis is crucial to get a complete risk picture.

A key sensitivity that must be used is the increase in distress situations.

Moreover, the underlying fund must be designed with objectives in mind to make the CFO work well as a structure, not just the fund portfolio.

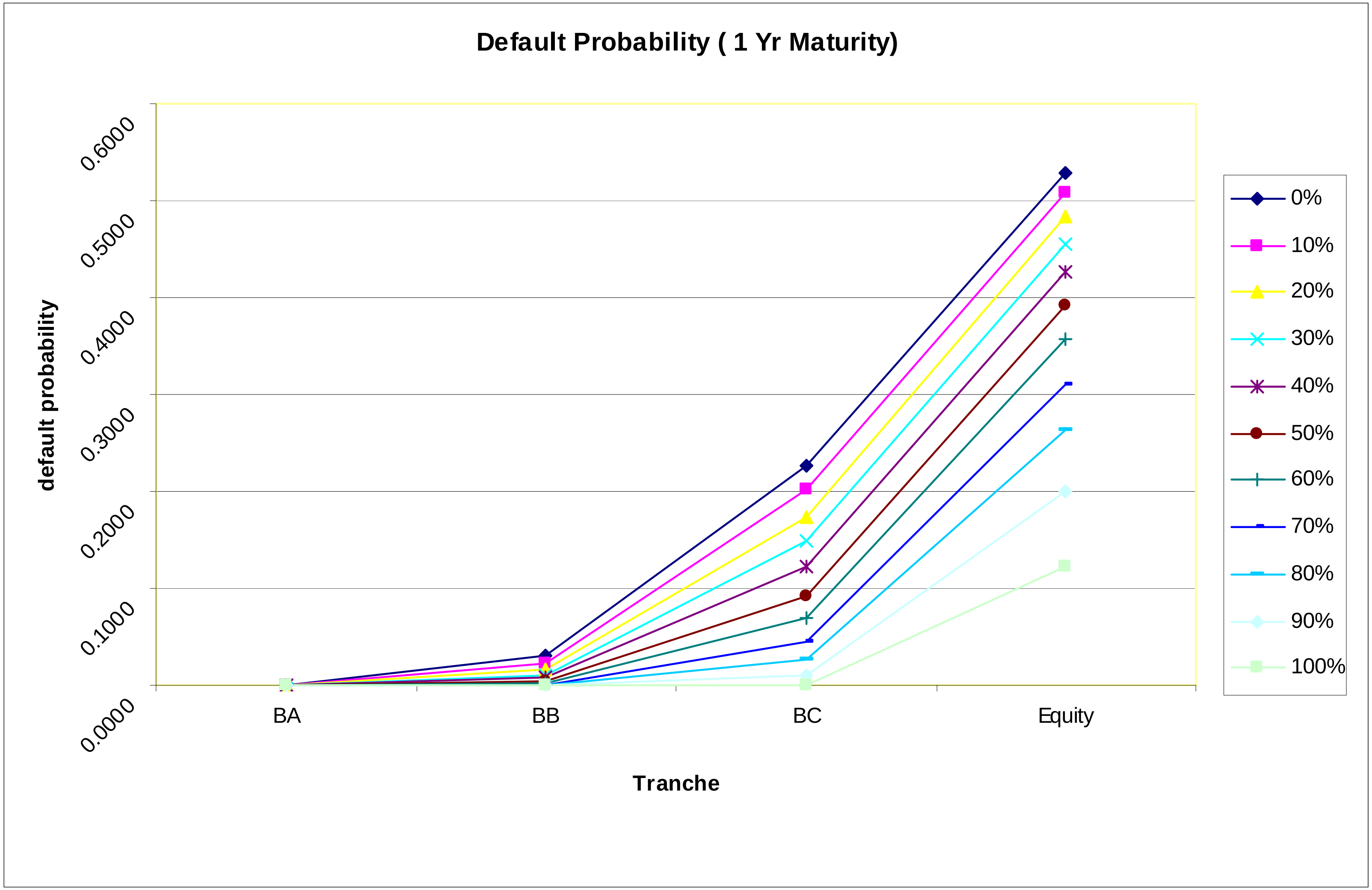
The rating of the underlying bond issues must also take into account potential deviations from historical means of distress components of future markets.

Probabilities of bond defaults

In the chart to follow, we present the probability of default of the different component of the CFO;

- The probability of default of the different bond issues is the annualized probability that the structure will default on its respective bond obligations.
- The probability of default of the investor's part means the probability that the investor will lose on its investment.

S&P CTA CFO. A case study.



The need for a generalized portfolio theory

Many investments do not satisfy the gaussian hypothesis of Markowitz model

There are many ways of doing this:

- Scenario based optimization
- Moment-based parametric models
- Etc.

We would like a model that, in addition to relax the gaussian assumption, keeps the nice, simple, intuitive concepts which arise from the gaussian case.

Correlation sensitivity

We will deal with correlation sensitivity from a mixtures of multivariate gaussian approach

Its density is given by:

$$\frac{pe^{-\frac{1}{2}(X-M_1)^t A(X-M_1)}}{\sqrt{\det(2\pi A)}}+\frac{(1-p)e^{-\frac{1}{2}(X-M_2)^t B(X-M_2)}}{\sqrt{\det(2\pi B)}}$$

Non-gaussian portfolio theory

Each portfolio is described by four performance numbers: mean and standard deviation, each under normal and distressed market assumptions. They are given by

$$\mu_{\mathrm{N}} = \vec{\mu}_{\mathrm{N}} \cdot \vec{\theta}, \quad \mu_{\mathrm{D}} = \vec{\mu}_{\mathrm{D}} \cdot \vec{\theta}$$

and

$$\sigma_{\mathrm{N}} = \sqrt{\vec{\theta} \cdot \mathrm{V}_{\mathrm{N}} \cdot \vec{\theta}^{t}}, \quad \sigma_{\mathrm{D}} = \sqrt{\vec{\theta} \cdot \mathrm{V}_{\mathrm{D}} \cdot \vec{\theta}^{t}}$$

Benchmark satisfaction

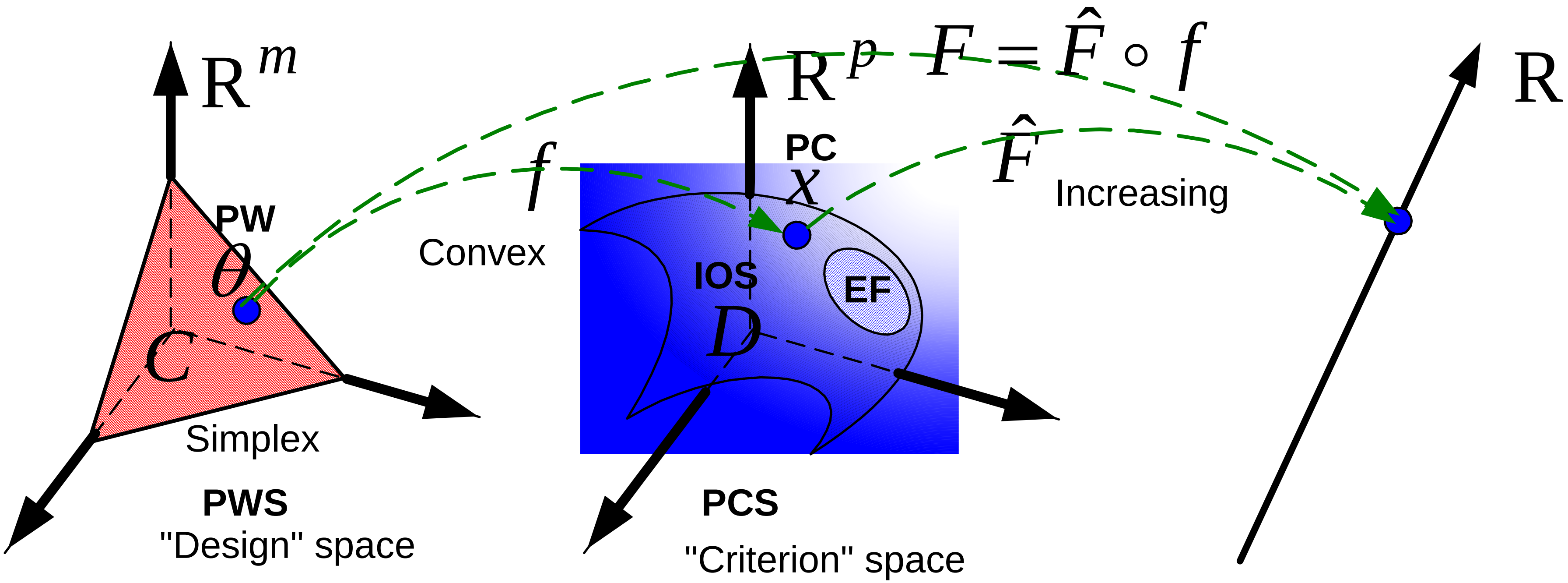
The objective function to maximize was

$$\Pr\{\Pi > c\} = p\left\{1-\phi\left(\frac{c-\mu_{\mathrm{N}}}{\sigma_{\mathrm{N}}}\right)\right\} + (1-p)\left\{1-\phi\left(\frac{c-\mu_{\mathrm{D}}}{\sigma_{\mathrm{D}}}\right)\right\}$$
$$= F_N\left(\frac{\mu_{\mathrm{N}}-c}{\sigma_{\mathrm{N}}}\right) + F_D\left(\frac{\mu_{\mathrm{D}}-c}{\sigma_{\mathrm{D}}}\right)$$

Increasing functions

It is possible to have portfolios which are *efficient* from this point of view, which however are not efficient under either normal or distressed conditions.

Spaces, functions, IOSs & EFs



MultiV. InvOppSets w' SharpeRatio contours

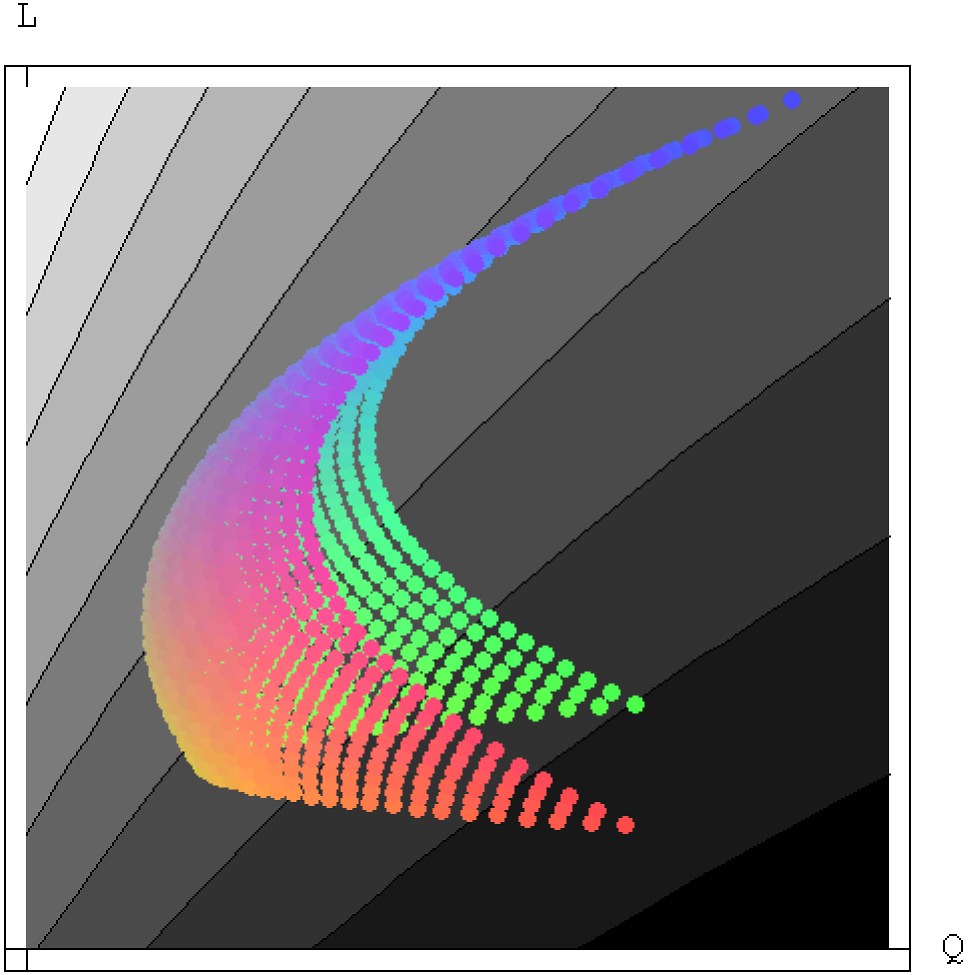
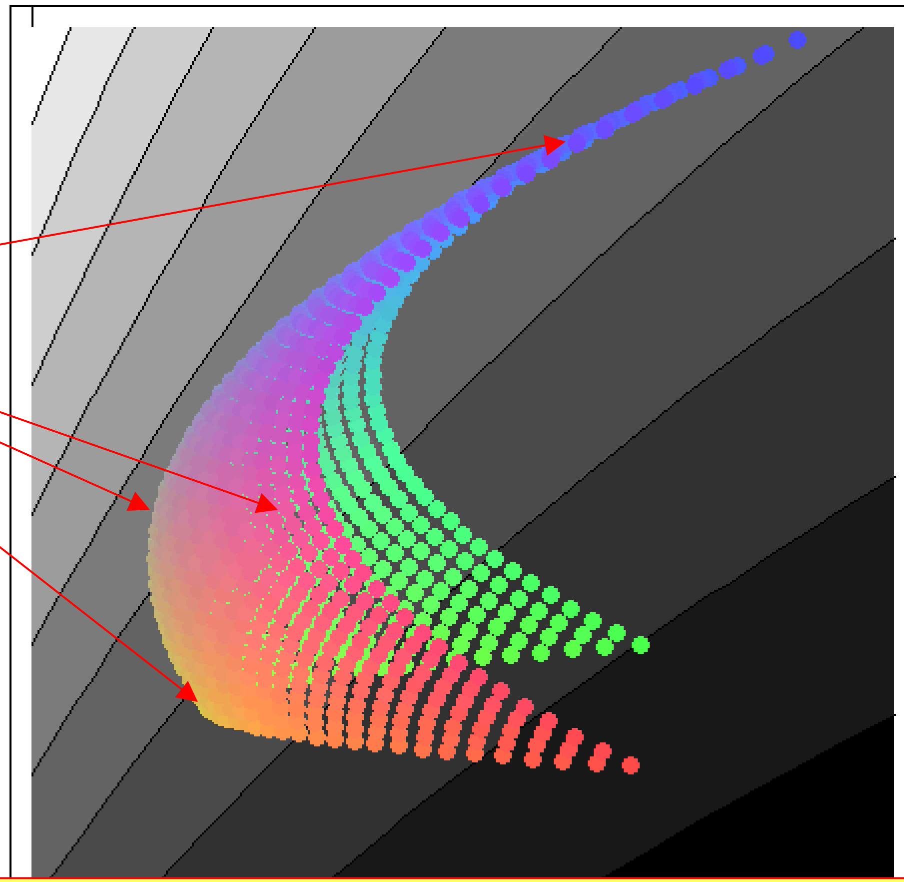


Figure 1

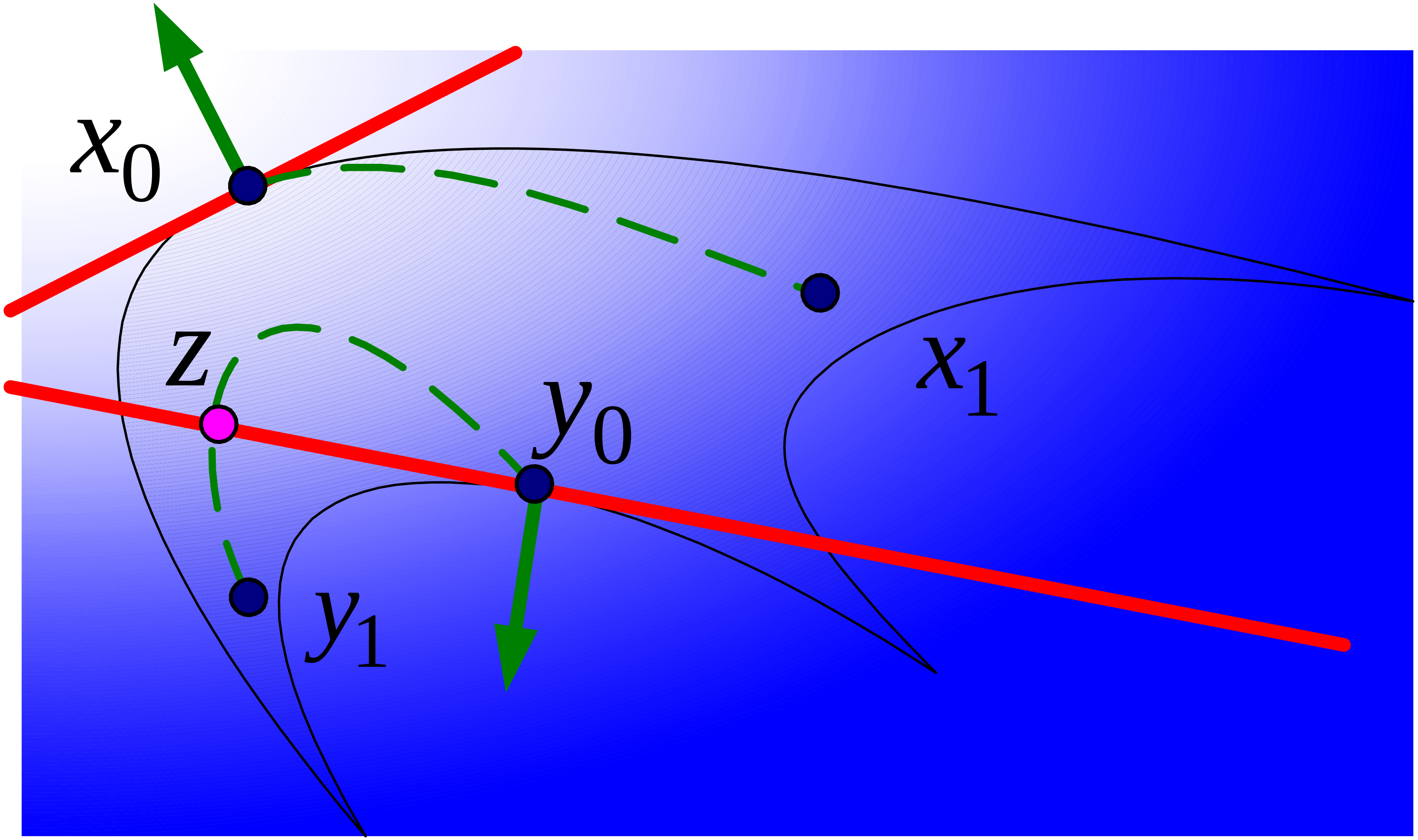
Theorem:

Under suitable conditions, the optimal portfolio is here:



These are the points we can easily calculate using Quadratic Programming; if the optimal portfolios could be somewhere else, we would have a very hard time finding them

What could be the problem?



Application: allocation sensitivities