Optimal Income Taxation

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Public Finance RED - Lecture 6

Outline

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3. Optimal Linear Income Taxes
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5. Mirrlees Tax Problem: Full Setup
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Tax Instruments

Optimal Income Taxation

Main Goal: derive the properties of optimal taxes/subsidies in different contexts

First, we define instruments that the government can use

Define the income tax as a function $T(z)$, where z is the income reported by the agent.

Retention Function and Marginal Tax

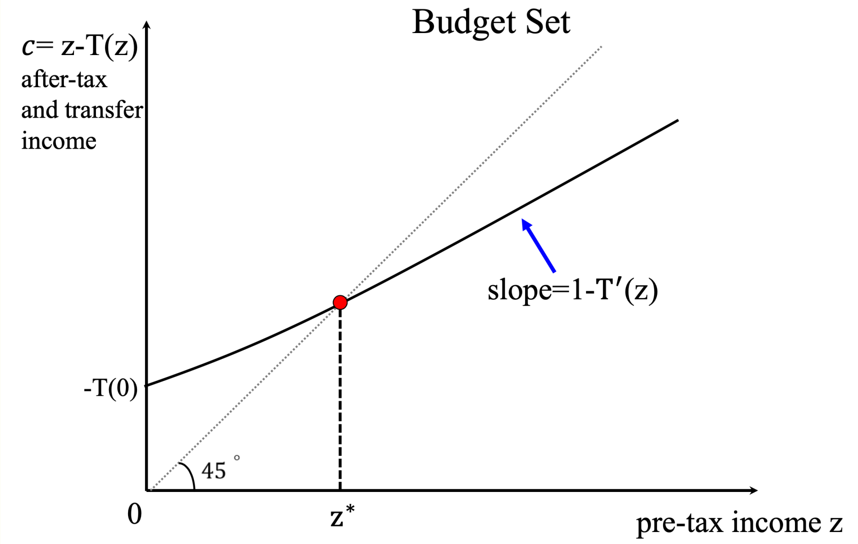
Using $T(z)$ we define:

- retention function: $R(z) = z - T(z)$, how much agent can retain out of total income z
- $-T(z)$: transfers to income z
- $-T(0)$: transfer to non-working individuals (intercept of the retention function)
- $T'(z)$: marginal tax rate. It measures how much agent gets taxed out of one additional dollar of income

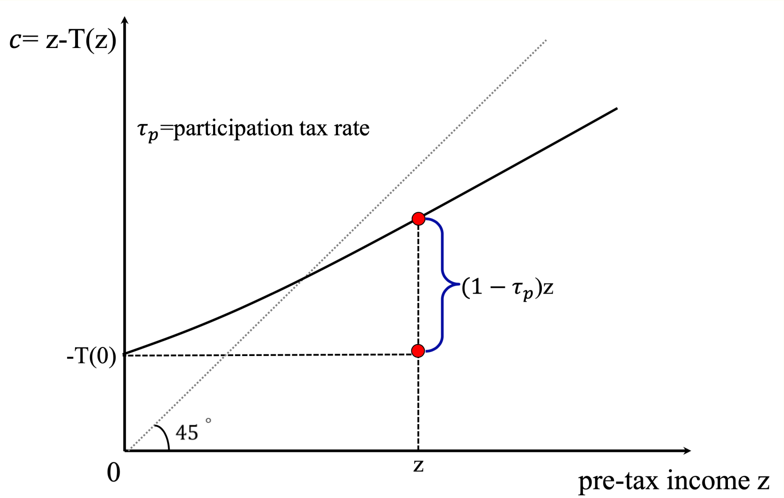
Participation Tax Rate

- $\tau_p = \frac{T(z)-T(0)}{z}$: participation tax rate
  - fraction of income that agent pays in taxes when she moves from 0 income to z.
- Useful if we study extensive margin decision between working and remaining unemployed

Retention Function and Marginal Tax



Retention Function and Participation Rate



Taxation with No Behavioral Responses

Taxation With No Behavioral Responses - Setup

Model with following assumptions:

- No labor supply response to taxation
- Agent has utility $u(c)$ such that $u'(c) > 0$ and $u''(c) \leq 0$.
- Labor does not enter the utility function and it is supplied inelastically.
- The agent consumes everything that is left after taxes: $c = z - T(z)$
- Income distribution $h(z)$, with support $[0, \infty]$.

Government Problem

Government goal: maximize the total utility of the economy.

Utilitarian SWF: every agent in the economy is equally weighted

$$\int_0^\infty u\left(z - T\left(z\right)\right) h\left(z\right) dz$$

E: revenues target. The budget constraint is:

$$\int_0^\infty T\left(z\right) h\left(z\right) dz \geq E$$

Solving the Model

- The Lagrangian for the problem reads:

$$L = [u(z - T(z)) + \lambda T(z)]\, h(z)$$

λ: value of government revenues in equilibrium

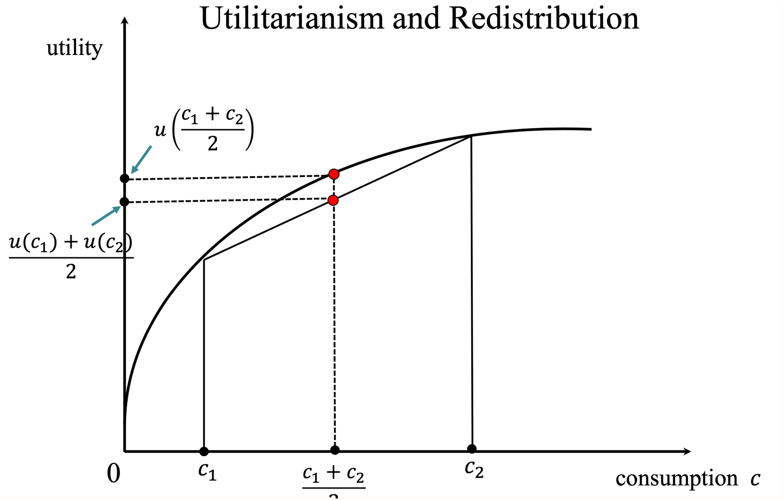
- Optimal choice of $T(z)$ delivers FOC:

$$\frac{\partial L}{\partial T(z)} = [-u'(z - T(z)) + \lambda]\, h(z) = 0$$

Rearranging:

$$u'(z - T(z)) = \lambda$$

Utilitarianism and Redistribution



The Implications of the Optimal Tax

$$u'(z - T(z)) = \lambda$$

λ is constant and all agents have the same preferences: consumption is equalized across all individuals.

This is a direct consequence of:

- utilitarian social welfare function: every agent has the same weight in SWF, treat all individuals equally.
- concavity of utility: until all consumption levels are equalized government can increase social welfare through "redistribution" from rich to poor

The Implications of the Optimal Tax - Continued

- Government collect revenues needed to meet requirement E
- Each individual consumes $c = \bar{z} - E$, where $\bar{z} = \int_0^\infty zh(z)\,dz$ is avg income
- Implies 100% marginal tax rate above $\tilde{z} = \bar{z} - E$.

Issues with this simple model

1. Obvious missing piece: 100% redistribution destroys incentives to work
   - Optimal income tax theory incorporates behavioral responses (Mirrlees REStud '71)
   - capture equity-efficiency trade-off
2. Issue with Utilitarianism: Even absent behavioral responses, many people would object to 100% redistribution (perceived as confiscatory)
   - Citizens' views on fairness impose bounds on redistribution
   - The issue is restricted nature of social preferences that can be captured by most SWF

Optimal Linear Income Tax

Introducing Labor Supply

We introduce labor supply:

- Preferences: $u(c,l)$, $u_c(c,l) > 0$ and $u_l(c,l) < 0$
- Each agent earns income wl when supplying l hours of labor
- Consumption: $c = wl - T(wl)$ after taxes.
- Individuals are heterogeneous in the salary w (can be interpreted as ability)
- Salaries distribution: $f(w)$

Social Marginal Welfare Weights

- Individual welfare aggregated through a SWF $G(\cdot)$
- If $G(\cdot)$ is concave the government wants to redistribute.
- Social marginal welfare weight:

$$g_i = \frac{G'(u^i)\, u_c^i}{\lambda}$$

- Government marginal utility from giving a dollar to individual i.
- Scaled by marginal value of revenues to the government (λ), that converts the marginal utility in money metric.
- Concave utility implies that g_i is decreasing in z_i.

Optimal Linear Income Tax - Setup

- Restrict instrument government can use
- Focus on linear tax τ
- Assumptions:
  - Revenues rebated through lump-sum transfers.
  - The individual consumes: $c_i = (1 - \tau) w_i l_i + \tau Z$
  - Z: total income level in equilibrium
  - τZ: total tax revenue from the tax

The Government Problem

- Government maximizes the following:

$$\int_i G\left(u_i\left((1-\tau)\, w_i l_i + \tau Z, l_i\right)\right)$$

- No government budget constraint, revenue is rebated
- Applying Envelope theorem we get:

$$\begin{aligned} \int_i G'(u_i)\, u_i' \left[-w_i l_i + Z - \tau \frac{dZ}{d(1-\tau)}\right] &= 0 \\ \int_i G'(u_i)\, u_i' \left[-z_i + Z - \frac{\tau}{(1-\tau)} Z \varepsilon_{z,1-\tau}\right] &= 0 \end{aligned}$$

Envelope Theorem: Interpretation

$$\int_i G'(u_i)\, u_i' \left[-z_i + Z - \frac{\tau}{(1-\tau)} Z \varepsilon_{z,1-\tau} \right] = 0$$

- Differentiate Z since individual does not maximize over Z
  - they take transfer as given
  - do not internalize the effect of labor supply choice on revenues and transfers
- Hence, Envelope theorem does not apply to Z, but only to z_i.

Optimality Condition: Interpretation

$$\int_i G'(u_i)\, u_i' \left[\overbrace{Z - z_i}^{\text{Mechanical Effect}} - \overbrace{\frac{\tau}{(1-\tau)} Z \varepsilon_{z,1-\tau}}^{\text{Behavioral Effect}} \right] = 0$$

Two terms above are central in the optimal taxation literature:

- $Z - z_i$: mechanical effect of the tax
  - If labor supply unchanged, increase in τ generates:
    - drop in income of z_i, and
    - mechanical increase in transfers of Z due to higher revenues
- $\frac{\tau}{(1-\tau)} Z \varepsilon_{z,1-\tau}$: behavioral effect of the tax
  - If individuals adjust labor supply, fiscal externality on revenues:
    - when work less, government collects lower revenues

Optimality Condition: Envelope Theorem (Again!)

- Why no utility consequence of change in labor supply?
  - Labor changes and no marginal disutility of labor.

- Because if tax change is small, can neglect the utility effect of a change in labor supply invoking the envelope theorem

- Envelope theorem: when we shift a parameter (the tax in this case) the agent is moving to a new bundle on the same indifference curve

The Optimal Linear Tax

$$\tau^* = \frac{1 - \bar{g}}{1 - \bar{g} + \varepsilon_{z,1-\tau}}$$

- $\bar{g} = \frac{\int_i g_i z_i}{Z \int_i g_i}$: measure of inequality in the economy.
  - low when income is extremely polarized
- Efficiency: τ^* decreases in $\varepsilon_{z,1-\tau}$
  - when income very elastic, avoid negative effects on revenues from distortions to the labor supply
- Equity: τ^* decreases in $\bar{g}$
  - the government increases taxes when inequality is high

Social Welfare Functions

- **Welfarism:** social welfare based solely on individual utilities
- Any other social objective will lead to Pareto dominated outcomes in some circumstances (Kaplow and Shavell JPE'01)
- Most widely used welfarist SWF:
  1. **Utilitarian:** $SWF = \int_i u^i$
  2. **Rawlsian** (also called Maxi-Min): $SWF = \min_i u^i$
  3. $SWF = \int_i G(u^i)$ with $G(.) \uparrow$ and concave,
     - e.g., $G(u) = u^{1-\gamma}/(1-\gamma)$ (Utilitarian: $\gamma = 0$, Rawlsian: $\gamma = \infty$)
  4. **General Pareto weights:** $SWF = \int_i \mu_i \cdot u^i$
     - with $\mu_i \geq 0$ exogenously given

Social Marginal Welfare Weight

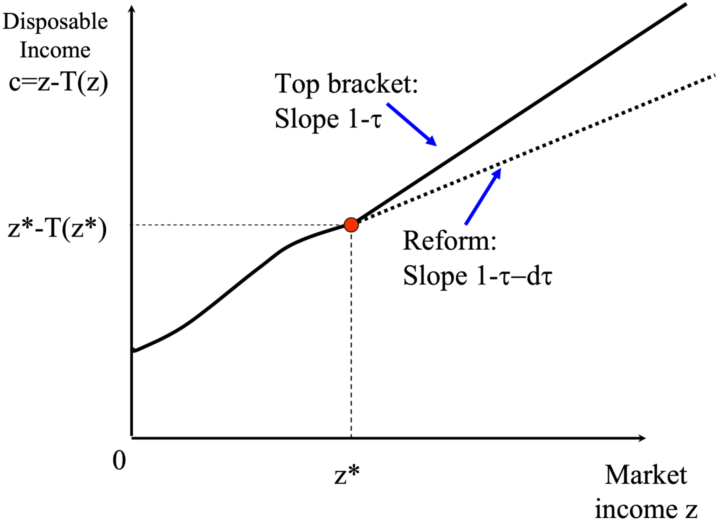
- Social Marginal Welfare Weights: key statistics in optimal tax formulas
- $g_i = G'(u^i)u^i_c/\lambda$: \$ value for govt of giving \$1 extra to i
  - λ multiplier of govt budget constraint
- No income effects: $\int_i g_i = 1$
  - giving \$1 to all costs \$1 (population has measure 1)
  - and increase SWF (in \$ terms) by $\int_i g_i$
- g_i typically depend on tax system (endogenous variable)
- Utilitarian case: g_i decreases with z_i
  - decreasing marginal utility of consumption
- Rawlsian case: g_i concentrated on most disadvantaged
  - typically those with $z_i = 0$

Variational Approach: Top and General Tax Rates

Optimal Top Income Tax: Saez (2001) Experiment

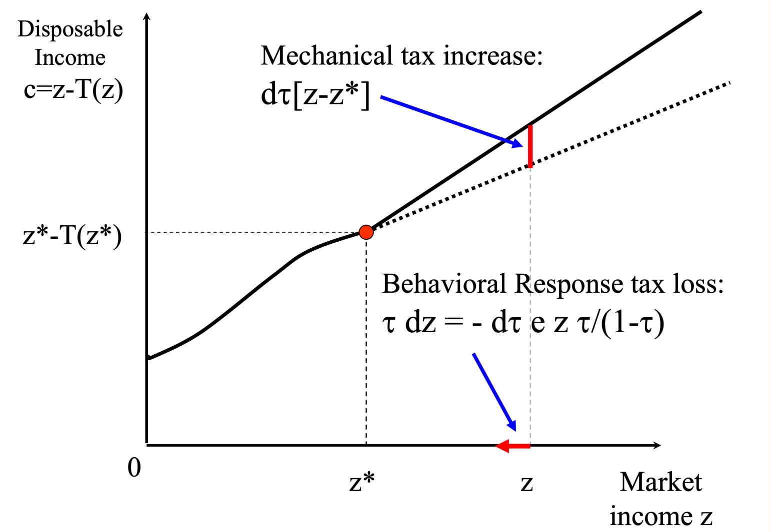
- We derive top income tax rates as in Saez (2001)
- Experiment:
  - government raises constant MTR τ above income threshold z^*
- Assumptions and notation:
  - $\bar{z}(1-\tau)$: avg income above z^* (depends on $1-\tau$)
  - $\varepsilon_{z,1-\tau}$: constant uncompensated elasticity of z for top earners

Optimal Top Income Tax: Saez (2001) Experiment



Source: Diamond and Saez JEP'11

Optimal Top Income Tax: Saez (2001) Experiment



Source: Diamond and Saez JEP'11

The Three Effects of a Tax Change

- When tax τ is raised:
  - no effects on individuals with income below z^*
  - all income above z^* are affected
- The tax has three effects:
  - Mechanical
  - Behavioral
  - Welfare

Mechanical Effect

- Suppose labor supply is inelastic
- Fixed tax base
- Mechanical increase in revenues would be:

$$dM = d\tau\,(\bar{z} - z^{*})$$

Behavioral Effect

- Suppose top earners adjust the labor supply
- We have a fiscal externality reducing revenues
- The behavioral effect is:

$$
\begin{aligned}
dB &= \tau d\bar{z} = -\tau \frac{d\bar{z}}{d(1-\tau)} d\tau \\
&= -\frac{\tau}{1-\tau} \bar{z} \frac{1-\tau}{\bar{z}} \frac{d\bar{z}}{d(1-\tau)} d\tau \\
&= -\frac{\tau}{1-\tau} \varepsilon_{\bar{z},1-\tau} \bar{z} d\tau
\end{aligned}
$$

- Proportional to the elasticity of labor supply:
  - more elastic labor, higher revenue loss (efficiency)

Welfare Effect

- Tax mechanically raises revenues on top income individuals:
$$dW = d\tau\bar{g}\left(\bar{z} - z^*\right)$$
- $\bar{g}$: constant social marginal welfare weight for those above z^*
- No behavioral response in welfare effect:
  - after tax change people reoptimize at the margin, utility is unaffected (Envelope theorem)

Optimal Tax

In equilibrium, the three effects must sum to zero:

$$dM + dB + dW = d\tau \left[(1 - \bar{g}) [\bar{z} - z^*] - \varepsilon_{\bar{z},1-\tau} \frac{\tau}{1-\tau} \bar{z} \right] = 0$$

Rearranging:

$$\tau^* = \frac{1 - \bar{g}}{1 - \bar{g} + a\varepsilon_{\bar{z},1-\tau}}$$

where $a = \frac{\bar{z}}{\bar{z} - z^*}$ measures the thinness of the tail of income distribution.

Optimal Tax - Interpretation

$$\tau^* = \frac{1 - \bar{g}}{1 - \bar{g} + a\varepsilon_{\bar{z},1-\tau}}$$

- τ^* decreases in $\bar{g}$:
  - more government cares about top income individuals, the less they will be taxed (equity)
- τ^* decreases in $\varepsilon_{\bar{z},1-\tau}$:
  - higher elasticity implies larger efficiency costs (efficiency)
- τ^* decreases in a:
  - shape of income distribution matters
  - Higher top income taxes if thicker tail

Zero Top Earner Tax

- Suppose top earner earns z^T
- When $z^* \to z^T \Rightarrow \bar{z} \to z^T$

$$dM = d\tau[\bar{z} - z^*] << dB = d\tau \cdot e \cdot \frac{\tau}{1-\tau}\bar{z} \quad \text{when} \quad z^* \to z^T$$

- Intuition:
  - extra tax applies only to earnings above z^*,
  - behavioral response applies to full $\bar{z}$
- Optimal τ should be zero when z^* close to z^T
  - (Sadka-Seade zero top rate result)
  - but result applies only to top earner

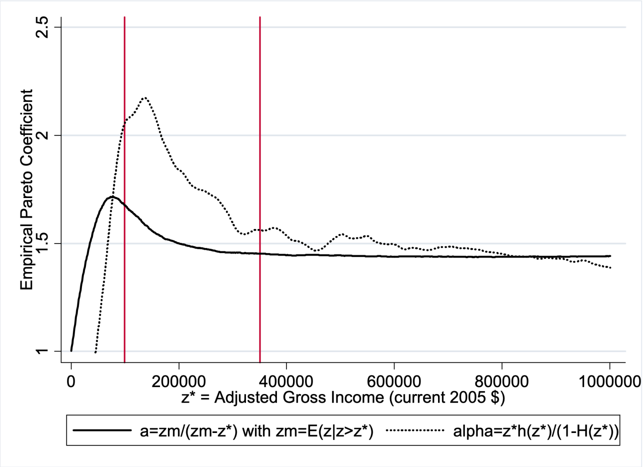
Calibrating optimal linear top tax rate

- Need estimates of $\bar{z}$ and α
- Assume Pareto distribution
  - CDF: $1 - F(z) = (k/z)^{\alpha}$,
  - PDF: $f(z) = \alpha \cdot k^{\alpha}/z^{1+\alpha}$
  - α Pareto parameter
- Implies

$$\bar{z}(z^*) = \frac{\int_{z^*}^{\infty} sf(s)ds}{\int_{z^*}^{\infty} f(s)ds} = \frac{\int_{z^*}^{\infty} s^{-\alpha}ds}{\int_{z^*}^{\infty} s^{-\alpha-1}ds} = \frac{\alpha}{\alpha - 1} \cdot z^*$$

$\alpha = \bar{z}/(\bar{z} - z^*) =$ a measures thinness of top tail of distribution

Thinnes of tail (a) in the data



Source: Diamond and Saez JEP'11

Calibrating optimal linear top tax rate

- Empirically: $a = \bar{z}/(\bar{z} - z^*)$ very stable above $z^* = \$400K$
- $a \in (1.5, 3)$, US has $a = 1.5$, Denmark has $a = 3$
- Difficult parameter to estimate: e. Try different
  - e.g. $e = 0.25$

Calibrating optimal linear top tax rate

- Implement the formula

$$\tau = \frac{1-\bar{g}}{1-\bar{g}+a \cdot e}$$

- Which g do we use?
  - Utilitarian criterion: $u_c \to 0$ as $c \to \infty$, so $\bar{g} \to 0$ as $z^* \to \infty$
  - Rawlsian criterion: only care about $\min(z)$, $\bar{g} = 0$ for $z^* > \min(z)$

- $\bar{g} = 0$ is tax revenue maximizing top tax rate

$$\tau = \frac{1}{1+a \cdot e}$$

Example: $a = 2$ and $e = 0.25$ then $\tau = 2/3 = 66.7\%$

- Laffer linear rate (flat tax maximizing revenues) is given by
  - $z^* = 0$, $a = 1$, $\tau = 1/(1+e)$

Extensions and Limitations

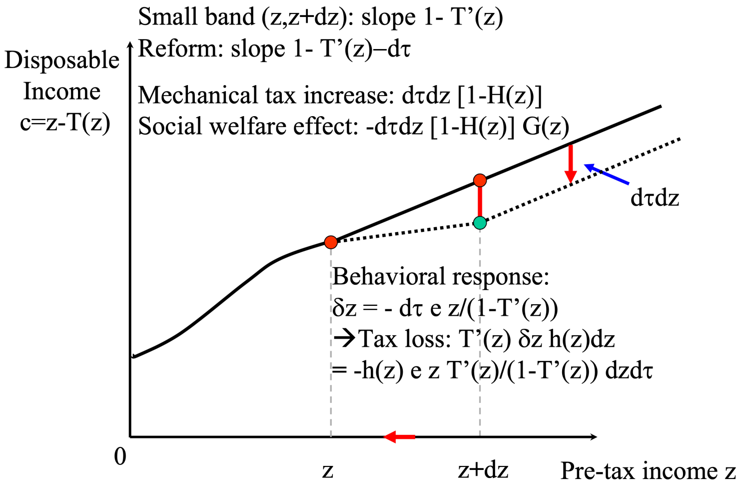
- Only includes intensive margin responses
  - extensive earnings responses: participation, enterpreneurship, migration
  - formulas can be extended
- No fiscal externality from other taxes
  - there might be income shifting that affects revenues from other taxes
  - can modify formulas
- Exclude classic externalities
  - positive spillovers (trickle-down, top earners underpaid) (Stiglitz 1982)
  - negative spillovers (top earners overpaid)
- Classical general equilibrium effects on prices are not externalities, do not affect formulas
  - Diamond Mirrlees (1971), Saez (2004)

Generalizing variational approach: non-linear tax

- Lumpsum grant given to everybody equal to $-T(0)$
- Marginal tax rate schedule $T'(z)$ describing how
  - lump-sum grant is taxed away,
  - how tax liability increases with income
- $H(z)$ income CDF [population normalized to 1]
- $h(z)$ income density (endogenous to $T(.)$)
- $g(z)$: social marginal value of consumption for income z
  - in terms of public funds $g(z) = G'(u) \cdot u_c / \lambda$
  - if no income effects $\Rightarrow \int g(z)h(z)dz = 1$
- Redistribution valued: $g(z)$ decreases with z
- $G(z)$: average social marginal value of c for those above z

$$G(z) = \frac{\int_z^\infty g(s)h(s)ds}{(1 - H(z))}$$

Tax Change Experiment



Source: Diamond and Saez JEP'11

General Non-Linear Tax Rate

- Assume away income effects $\varepsilon^c = \varepsilon^u = e$
  - Diamond AER'98: key theoretical simplification
  - Saez (2001) derives formulas with income effects as well
- Small reform: increase T' by $d\tau$ in small band $[z, z + dz]$
- Mechanical effect: $dM = dzd\tau[1 - H(z)]$
- Welfare effect: $dW = -dzd\tau[1 - H(z)]G(z)$
- Behavioral effect: substitution eff δz inside small band $[z, z + dz]$

$$dB = h(z)dz \cdot T' \cdot \delta z = -h(z)dz \cdot T' \cdot d\tau \cdot z \cdot e_{(z)} / (1 - T')$$

- Optimum: $dM + dW + dB = 0$

General Non-Linear Tax Rate

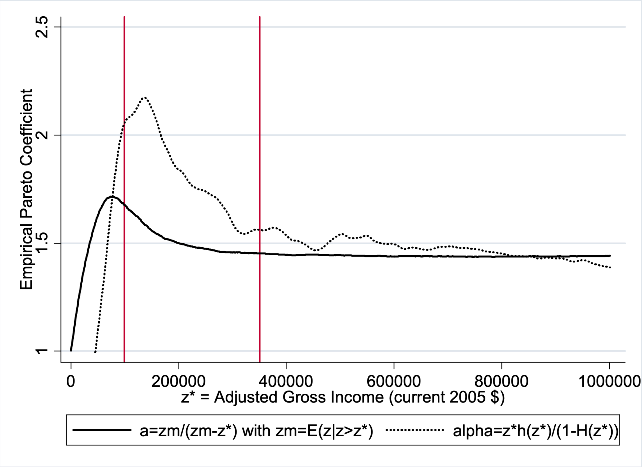
$$T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e_{(z)}}$$

- $T'(z)$ decreases with $e_{(z)}$ (elasticity efficiency effects)
- $T'(z)$ decreases with $\alpha(z) = (zh(z))/(1 - H(z))$ (local Pareto parameter)
- $T'(z)$ decreases with $G(z)$ (redistributive tastes)
- Asymptotics:
  - $G(z) \to \bar{g}$, $\alpha(z) \to a$, $e_{(z)} \to e$
  - Recover top rate formula $\tau = (1 - \bar{g})/(1 - \bar{g} + a \cdot e)$

Negative Marginal Tax Rates Are Never Optimal

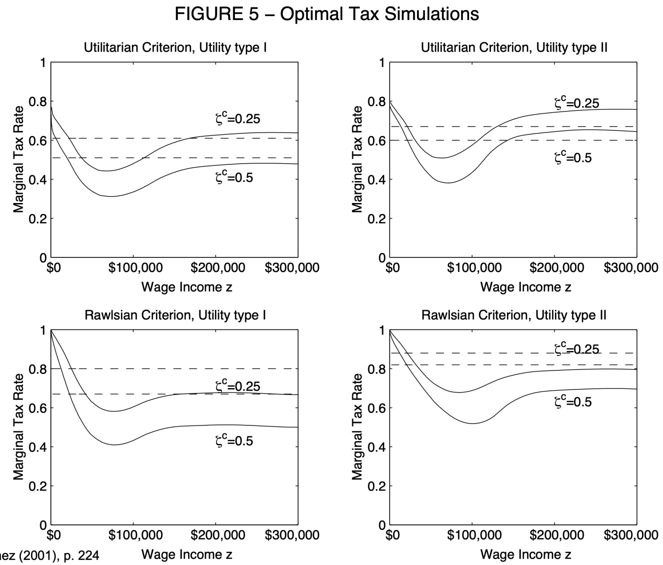
- Suppose $T' < 0$ in band $[z, z + dz]$
- Increase T' by $d\tau > 0$ in band $[z, z + dz]$:
  - $dM + dW > 0$ and $dB > 0$ because $T'(z) < 0$
- This is a desirable reform!
- Hence, $T'(z) < 0$ cannot be optimal
- EITC schemes are not desirable in Mirrlees '71 model
- Can justify $T'(z) < 0$ with participation responses (Saez 2002)

Saez (2001) - Implementation



Source: Diamond and Saez JEP'11

Saez (2001) - Implementation



Source: Saez (2001), p. 224

Mirrlees Tax Problem: Full Setup

Model Assumptions

Assumptions:

- individuals are heterogeneous
- government tax individuals, but does not observe their types
- individuals behaviorally respond to taxation

Model Setup

- Preferences: $u(c, l)$
- Separable and quasi-linear: $u(c, l) = c - v(l)$, $v'(l) > 0$ and $v''(l) \geq 0$
- Agent earn income $z = nl$
- Consumption: $c = nl - T(nl)$
- Individuals are heterogeneous in the salary n (ability)
- $n \sim f(n)$, with $n \in [\underline{n}, \bar{n}]$
- Welfare is aggregated through a social welfare function $G(\cdot)$, that we assume differentiable and concave.

Revelation Principle

Goal: define optimal tax schedule that delivers allocation $z(n)$, $c(n)$ for each n.

Revelation Principle:

- if allocation can be implemented through some mechanism,
- THEN can also be implemented through a direct truthful mechanism where the agent reveals her information about n.

Agents report their type n', allocations are a function of n'.

By revelation principle, the government cannot do better than $c(n)$, $z(n)$ such that:

$$c(n) - v\left(\frac{z(n)}{n}\right) \geq c(n') - v\left(\frac{z(n')}{n}\right) \; \forall n, n'$$

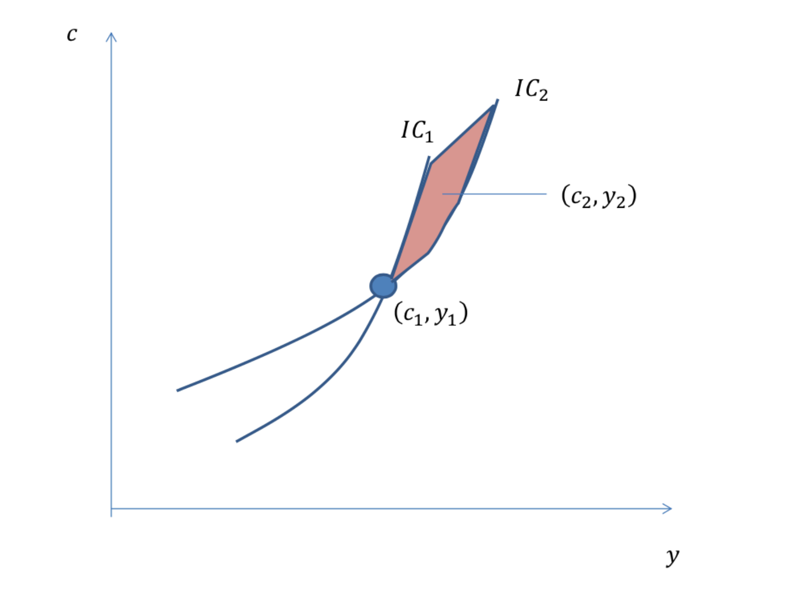
Single-Crossing Condition and Monotonicity

We assume single-crossing condition (or Spence-Mirrlees condition):

$$-MRS_{cz} = \frac{v'\left(z\left(n\right)/n\right)}{nu'\left(c\left(n\right)\right)} \text{ decreases in } n$$

- Incentive compatibility + single crossing $\Longrightarrow$ monotonicity of allocations: $c'\left(n\right), z'\left(n\right) > 0$.
- Single-Crossing Condition + monotonicity $\Longrightarrow$ local incentives constraints are sufficient conditions for the problem.
- We can replace the incentive constraint with the first-order necessary conditions providing local incentive conditions. Ignore monotonicity and verify it ex-post.

Monotonicity



Local Incentive Constraints

Reduce the dimensionality of the problem: first order approach.

When reporting, individual of type n solves:

$$\max_{n'} c(n') - v\left(\frac{z(n')}{n}\right)$$

- FOC is:

$$c'(n') - \frac{z'(n')}{n} v'\left(\frac{z(n')}{n}\right) = 0$$

- Under truth-telling:

$$c'(n) = \frac{z'(n)}{n} v'\left(\frac{z(n)}{n}\right)$$

Local Incentive Constraints - Continued

Differentiating the utility wrt n at some n, we get:

$$\frac{du(n)}{dn} = \underbrace{\left(c'(n) - \frac{z'(n)}{n} v'\left(\frac{z(n)}{n}\right)\right)}_{\text{Agent's Truthtelling FOC}} + \frac{z(n)}{n^2} v'\left(\frac{z(n)}{n}\right)$$

Therefore, at the optimum:

$$\frac{du(n)}{dn} = \frac{z(n)}{n^2} v'\left(\frac{z(n)}{n}\right)$$

- $\frac{du(n)}{dn}$: slope of utility assigned to the agent at the optimum.
- By convexity of $v(\cdot)$, always positive.
- Higher utility to high types at optimum: informational rents
  - Why? Higher types have a lower marginal disutility of labor for a given level of hours worked

Labor Supply and Labor Wedge

Individual solves the following optimization problem:

$$\max_{z} z - T(z) - v\left(\frac{z}{n}\right)$$

FOC is:

$$T'(z) = 1 - \frac{v'(l)}{n}$$

- $\frac{v'(l)}{n}$: MRS between consumption and income.
- No distortions: $\frac{v'(l)}{n} = 1$ and $T'(z) = 0$
- $T'(z)$: wedge on the optimal labor supply: if different from zero, labor supply distorted.

Elasticity of Labor Supply

Totally differentiating wrt $(1 - T'(z))\,n$, we have:

$$\frac{dl}{d\left(1 - T'(z)\right) n} = \frac{1}{v''(l)}$$

Which implies the following elasticity to the net-of-tax wage:

$$\varepsilon = \frac{dl}{d\left(1 - T'(z)\right) n} \frac{\left(1 - T'(z)\right) n}{l} = \frac{v'(z)}{l v''(z)}$$

Resource Constraint

- Exogenous revenue requirement E.
- Write the tax levied on a single agent as

$$T(z(n)) = z(n) - c(n)$$

- Sum over all the individuals:

$$\int_{\underline{n}}^{\bar{n}} c(n) f(n)\, dn \geq \int_{\underline{n}}^{\bar{n}} z(n) f(n)\, dn - E$$

Unlike incentive constraint, this constraint is unique.

The Government Problem

Government chooses allocations $(c(n), y(n))$, equivalent to choosing tax.

Solve:

$$\max_{c(n),u(n),z(n)} \int_{\underline{n}}^{\bar{n}} G(u(n)) f(n)$$

s.t.

$$\frac{du(n)}{dn} = \frac{z(n)}{n^2} v'\left(\frac{z(n)}{n}\right)$$

$$\int_{\underline{n}}^{\bar{n}} c(n) f(n)\, dn \geq \int_{\underline{n}}^{\bar{n}} z(n) f(n)\, dn - E$$

Optimal Control and Hamiltonian

- n: continuous variable
- $u(n)$ state variable, $z(n)$ control variable
- Local incentive constraint becomes the law of motion of the state variable
- Replace $c(n) = u(n) + v(z(n)/n)$ into the resource constraint

The Hamiltonian is:

$$H = \left[G(u(n)) + \lambda\left(z(n) - u(n) - v\left(\frac{z(n)}{n}\right)\right)\right] f(n) + \mu(n)\frac{z(n)}{n^2}v'\left(\frac{z(n)}{n}\right)$$

- $\mu(n)$: multiplier on the incentive constraint of type n
- λ : multiplier on the resource constraint.

Optimality Conditions

The first order conditions are:

$$\frac{\partial H}{\partial z(n)} = \lambda \left[1 - \frac{v'(l(n))}{n}\right] f(n) + \frac{\mu(n)}{n^2}\left[v'\left(\frac{z(n)}{n}\right) + \frac{z(n)}{n} v''\left(\frac{z(n)}{n}\right)\right] = 0$$

$$\frac{\partial H}{\partial u(n)} = \left[G'(u(n)) - \lambda\right] f(n) = -\mu'(n)$$

Boundary (transversality) conditions are:

$$\mu(\underline{n}) = \mu(\bar{n}) = 0$$

Boundary Conditions

Boundary (transversality) conditions are:

$$\mu(\underline{n}) = \mu(\bar{n}) = 0$$

- upper bound: it should be $\mu(\bar{n})\, u_{\bar{n}} = 0$. However, we give positive utility in equilibrium to the $\bar{n}$ individual (i.e. $u_{\bar{n}} > 0$), it must be $\mu(\bar{n}) = 0$
- lower bound: ICs are binding downwards and the $\underline{n}$ individual has no one she wants to “imitate” in equilibrium, while everyone else indifferent between her allocation and the allocation of the immediately lower type. Constraint for $\underline{n}$ is slack implies $\mu(\underline{n}) = 0$.

The Value of Public Funds

Integrate the second optimality condition equation we find:

$$\lambda = \int_{\underline{n}}^{\bar{n}} G'(u(n)) f(n) \, dn$$

Interpretation:

- value of public funds = welfare effect of transferring $1 to every individual in the economy
  - public funds more valuable the higher are the social welfare gains achievable in the economy

The Multiplier on Incentive Constraint

We can integrate the second equation to find the value of $\mu(n)$:

$$-\mu(n) = \int_{n}^{\bar{n}} \left[\lambda - G'(u(m))\right] f(m)\, dm$$

Optimal Tax

Using definition of labor elasticity:

$$\left[v'\left(\frac{z(n)}{n}\right)+\frac{z(n)}{n}v''\left(\frac{z(n)}{n}\right)\right]=v'\left(\frac{z(n)}{n}\right)\left[1+\frac{1}{\epsilon}\right]$$

Exploiting the definition of the tax wedge, we simplify the first optimality condition:

$$\lambda T'(z(n))=\frac{\mu(n)}{f(n)}\left(1-T'(z(n))\right)\left(1+\frac{1}{\epsilon}\right)$$

Using the expression for μ :

$$\frac{T'(z(n))}{1-T'(z(n))}=\left(\frac{1+\epsilon}{\epsilon}\right)\frac{\int_n^{\bar{n}}[1-g(m)]\,f(m)\,dm}{nf(n)}$$

Assumption on Welfare Weights

- Assume linear welfare weights
- distributed according to a function $\psi(\mathsf{w})$ with cdf $\Psi(\mathsf{w})$.
- The government objective function becomes:

$$\int_{\underline{n}}^{\bar{n}} u(n)\,\psi(n)\,dn$$

By assumption $\int_{\underline{n}}^{\bar{n}} \psi(n)\,dn = 1$ implies $\lambda = 1$.

Derive the Multiplier

Same first order conditions, we have:

$$-\mu'(\mathsf{n}) = \psi(\mathsf{n}) - \lambda \mathsf{f}(\mathsf{n})$$

and after integration it becomes:

$$\begin{aligned} -\mu(\mathsf{n}) &= \int_{\mathsf{n}}^{\bar{\mathsf{n}}} (\mathsf{f}(\mathsf{n}) - \psi(\mathsf{n}))\, \mathsf{dn} \\ &= \Psi(\mathsf{n}) - \mathsf{F}(\mathsf{n}) \end{aligned}$$

Optimal Tax - ABC Formula

The tax formula reads:

$$\frac{T'(z(n))}{1 - T'(z(n))} = \left(\frac{1+\epsilon}{\epsilon}\right) \frac{\Psi(n) - F(n)}{nf(n)}$$

Divide and multiply by $1 - F(n)$ to get:

$$\frac{T'(z(n))}{1 - T'(z(n))} = \underbrace{\left(\frac{1+\epsilon}{\epsilon}\right)}_{A(n)} \underbrace{\frac{\Psi(n) - F(n)}{1 - F(n)}}_{B(n)} \underbrace{\frac{1 - F(n)}{nf(n)}}_{C(n)}$$

Optimal Tax: Interpretation

$$\frac{T'(z(n))}{1-T'(z(n))} = \underbrace{\left(\frac{1+\epsilon}{\epsilon}\right)}_{A(n)} \underbrace{\frac{\Psi(n)-F(n)}{1-F(n)}}_{B(n)} \underbrace{\frac{1-F(n)}{nf(n)}}_{C(n)}$$

- $A(n)$: standard elasticity and efficiency argument
- $B(n)$: desire for redistribution. If the sum of weights below n is high relative to the mass above n, the government will tax more
- $C(n)$: thickness of the right tail of the distribution. A thicker tail implies higher tax rates.

Commodity Taxation with Non-Linear Taxes

Commodity and Income Taxation

- Mirrlees model assumes only income tax
- What about commodity taxes? Or other taxes?
- Diamond-Mirrlees (1971, AER) optimal commodity taxes in world with no lump-sum taxation
  - Leads to inverse elasticity rule

Demand Functions and Indirect Utility

- Does commodity taxation have a role if we have a nonlinear income tax (with lump-sum)?
  - Need to put commodity taxes into Mirrlees (1971) framework
  - Atkinson and Stiglitz (1976) JPubEc
  - Follow Kaplow (2006, JPubEc) for a simple proof

Kaplow (2006) - Setup

- Individuals choose commodities $\{c_1, c_2, \ldots, c_N\}$ and labor l
- Maximize utility function

$$\tilde{u}_h\left(c_1, c_2, \ldots, c_N, l\right) = u_h\left(g\left(c_1, c_2, \ldots, c_N\right), l\right)$$

  Key assumption: g same across people

- Subject to budget constraint

$$\sum \left(p_i + \tau_i\right) c_i \leq wl - T\left(wl\right)$$

  where w is an individual's wage (heterogeneous in population)

- wl is earnings and $T(wl)$ is the (nonlinear) tax on earnings

Atkinson-Stiglitz Result

- Suppose there is a commodity tax τ_i on each good
- Can welfare be improved by re-setting $\tau_i = \tau_j = \ldots = 0$ and suitably augmenting the tax schedule T?
  - Atkinson-Stiglitz/Kaplow: YES.
- Define $V(\tau, T, wl)$

$$V(\tau, T, wl) = \max g(c_1, c_2, \ldots, c_N)$$

s.t.

$$\sum (p_i + \tau_i) c_i \leq wl - T(wl)$$

- V is the value of the consumption argument of the utility function
  - holds independent of labor effort l!
- Consumption allocations don't reveal any information about labor supply type w conditional on wl.

Kaplow (2006) - Proof

Define intermediate environment:

- Start with commodity taxes τ
- Define new taxes at zero $\tau_i^* = 0$
- Augment the tax schedule on income
  - Define T^* to offset the impact on utility so utility held constant in this intermediate world
- Specifically, T^* satisfies

$$V(\tau, T, wl) = V(\tau^*, T^*, wl)$$

for all wl

Kaplow (2006) - Proof

- **Lemma 1**: Every type w chooses same level of labor effort under τ^*, T^* as under τ, T
- Proof:
  - Note that

$$\begin{aligned} U(\tau, T, w, l) &= u(V(\tau, T, wl), l) \\ &= u(V(\tau^*, T^*, wl), l) = U(\tau^*, T^*, w, l) \end{aligned}$$

    so utility same in both environments for a given h, for any choice of l

  - Hence, l that maximizes utility in original world maximizes utility in intermediate world

Kaplow (2006) - Proof

- Lemma 2: The augmented world raises more revenue than the original world
- Proof:
  - No individual in intermediate regime can afford the original consumption vector
    - Show that implies they pay more taxes in intermediate regime
  - Suppose type w can afford original vector
    - Then she strictly prefers a different vector because of change in relative price
    - Implies intermediate environment is strictly better off $\rightarrow$ contradicting definition of intermediate environment holding utilities constant

Kaplow (2006) - Proof

- Why does this imply aggregate tax revenue is higher in the intermediate environment?
- Since cannot afford old bundle, we have:

$$\sum p_i c_i > wl - T^* (wl)$$

for all wl (note $\tau^* = 0$)

- Budget constraint in initial regime implies

$$\sum (p_i + \tau_i) c_i = wl - T (wl)$$

so that

$$\sum p_i c_i = -\sum \tau_i c_i + wl - T (wl)$$

- then, using inequality above

$$-\sum \tau_i c_i + wl - T (wl) > wl - T^* (wl)$$

$$T^* (wl) > \sum \tau_i c_i + T (wl)$$

Kaplow (2006) - Proof

- Intermediate world generates more tax revenue and holds utility constant
- Rebate some revenue, make everyone better off relative to initial world
- This proves the result!

Application of Atkinson Stiglitz - Production Efficiency

- Diamond and Mirrlees (1971)
- Suppose C produced with intermediate goods x_i

$$C = f(x_1, x_2, \ldots, x_n)$$

- Do you want to tax inputs?
- Agent's utility

$$u(x, l) = U(C(x), l)$$

- Production function C is same across agents
  - weak separability
  - no taxes on intermediate inputs!

When does Atkinson Stiglitz Fail?

- Mirrlees information logic:
  - When commodity choices have desirable information about type conditional on earnings!
- What constitutes "desirable information"? (Saez 2002 JPubEc)
  - Information about social welfare weights: Society likes people that consume x_1 more than x_2 conditional on earnings
    - Implement subsidy on good x_1 financed by tax on x_2
    - First order welfare gain (b/c of difference in social welfare weights)
    - Second order distortionary cost starting at $\tau = 0$
- Information about latent productivity: More productive types like x_1 more than x_2 conditional on earnings
  - e.g. x_1 is books; x_2 is surf boards
  - Then, tax the goods rich people like but reduce the marginal tax rate
  - Leads to increase in earnings!
  - Depends on covariance

Remarks on Atkinson Stiglitz

- Diamond Mirrlees (1971): optimal commodity taxation
- Consider model without lump-sum transfers
- Result: tax more inelastic goods

Why?

- Because no lump-sum $\rightarrow$ desire to tax inelastic goods as they replicate the lump-sum
- With lump-sum this desire goes away
- Optimal commodity taxes depend on whether
  - commodity choice provides systematic information about latent productivity
  - allows for a relaxation of the income distribution