The measurement of the distances of the Universe II.

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In the first article, we introduced and described some methods used by astronomers to measure the distance of astronomical objects in our universe. By using a historical perspective in this second article, we proceed to study some of these methods in more depth. We also introduce new mensuration methods and conclude with a mixture of theoretical and practical student exercises. So let our exploration commence with some historical measurements of the closest stars.

Early estimations of the distance of Sun and Moon Our ancestors - including the ancient Greeks, were particularly fascinated by astronomy and even attempted to measure the distance of the Sun and the Moon. Indeed, two of these early Greek pioneers: Aristharcus and Hipparchus, played a major role in the development of early astronomy. In his book *On the sizes and distances*, Aristharcus gave an estimate of the relative dimension[1] of the distance of the Earth and the Moon and that of the Earth and the Sun. Although in this treatise our Solar system was considered as geocentric, the mathematical process[2] was nevertheless correct. As it was only because of the imprecise initial data, that the obtained estimates were very inaccurate.

After Aristharcus, another astronomer who made huge contributions to the study of the solar system was Hipparchus, who among other achievements compiled a star catalogue and observed that the distance between the Earth and Moon changes. He measured the distance between the Earth and the Moon, by exploiting the fact that during a solar eclipse, the solar disc is covered differently in different parts of the

[1] In terms of Earth radii.
[2] Involving properties of right triangles.

world. In his subsequent mathematical calculations, he determined a more precise but still inaccurate measure of the distance to the Moon. Also in order to determine the distance from the Earth to the Sun, he argued that, since during a Solar eclipse the apparent dimensions of Sun and Moon are similar, the distances between the Earth and them had to be in proportion to their real dimensions.

As a consequence of these measurements, it is possible to determine the dimensions of the Moon once we know its distance from the Earth using a property of isosceles triangles:

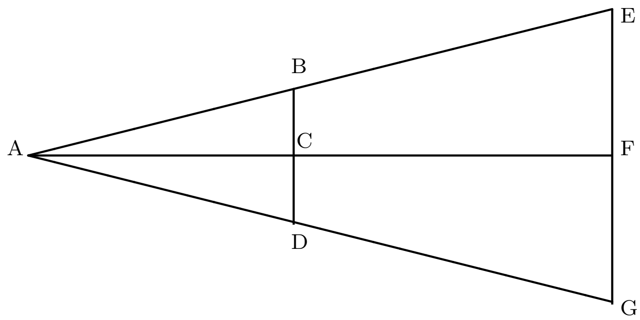


Image 1: Moon diameter measurement scheme.

This property tell us that, in an isosceles triangle like the one above:

$$\frac{\overline{AC}}{\overline{BD}} = \frac{\overline{AF}}{\overline{EG}} \tag{1}$$

So if in equation 1, $\overline{BD}$ is the diameter measurement of the Moon on a ruler, held with fully extended arms as it appears from our eye[3], $\overline{AC}$ the distance between our eye and the ruler and $\overline{AF}$ the distance from our eye to the Moon, we can then compute the diameter of the Moon $\overline{EG}$ by using similar triangles.

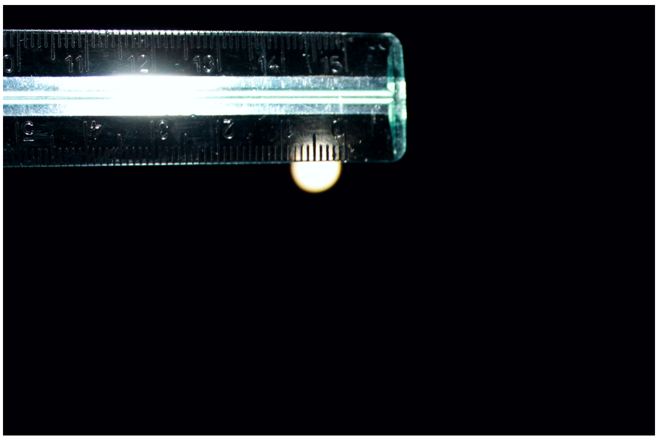


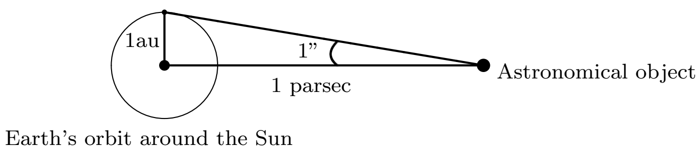
Image 2: The measurement of $\overline{BD}$.

These first evaluations of distance covered only some of the closest objects of our solar system. Yet it is still fascinating to reflect that even in antiquity, these Greek astronomers were making serious attempts

[3]It will appear in the order of magnitude of centimeters.

- despite their (by our modern standards) somewhat crude methods - to reach out beyond Earth to discover the unknown. But ultimately the methods used by the Greeks were not suitable to obtain accurate measurements on a scale as big as the one we will define in the next paragraph.

Parallax II In the first article, we described the *parallax* method. We now introduce a length size which is closely linked to it, the *parsec*.



Picture 3: The parsec.

As we can see from the previous picture, a parsec (often shortened to pc), is the distance of an astronomical object that has a *parallax angle* of one *arcsecond*[4], measured from two points which are distant one *astronomical unit*[5].

It should be noted that despite one of the constructions shown in the previous article is slightly different from this one, it is nevertheless equivalent to it. In fact, we can deduce the following identity from either of these approaches equivalently:

$$1pc = \frac{1au}{\tan 1''} \qquad (2)$$

From identity (2) we can evaluate $1pc \approx 3.08568 \cdot 10^{13} km = 30.856.800.000.000 km$. Even though this parsec unit is surprisingly big, it is tiny while compared to the immense vastness of the universe, so astronomers have to use large multiples of it, such as the *megaparsec (Mpc)* or the *gigaparsec*[6] *(Gpc)* to measure properly the distances.

Redshift and Hubble’s law Now that we have given the parsec definition, we can learn more about Hubble’s law and the related concept of redshift. As we mentioned in the previous article, in Hubble’s

[4] $\frac{1}{3600}$ degree.

[5] One astronomical unit (au or ua) is the average distance between the Earth and the Sun, often considered as $149.597.870.700m \approx 150.000.000km$.

[6] Resp. 30.856.800.000.000.000.000 and $30.856.800.000.000.000.000.000km$.

law the speed v of a deep space object and its distance D, are bound by the law:

$$v = H_0 D \qquad (3)$$

In this relation, H_0 plays a significant role and it is considered to be a constant[7]. Despite the SI unit for H_0 being $\frac{1}{s}$, the unit $\frac{km/s}{Mpc}$ is used instead. In this way, H_0 represents the average speed (in km/s) between two points that are $1Mpc$ apart. Today we consider $H_0 \approx 70\frac{km/s}{Mpc}$ [8].

In the application of the law in (3), another phenomenon comes into play, the Doppler Effect. We experience this effect in everyday life, when for example a car drives toward and then away from us at a high speed while emitting sound, such as a horn. We can clearly hear the pitch of the horn changes dramatically as the car approaches and then moves beyond us[9]. In this galactic context, the Doppler effect acts on the colours of the observed objects, changing them with their wavelenghts[10] according to the formula:

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{observed}} \qquad (4)$$

where z is the redshift, $\lambda_{observed}$ and $\lambda_{emitted}$ are respectively the observed and the emitted wavelength[11]. We may now ask ourselves what kind of information does z give us ? The answer is that z gives us a measure of the receding speed v of an object. If we denote the speed of light[12] by c, then the related formula is amazingly simple:

$$v = zc. \qquad (5)$$

Formula (5) enables us to determinate the speed of an object from its redshift, and subsequently its distance with the Hubble's law.

[7] Even if it can vary with the time, but only in the ultra-long term.

[8] In the last 90 years, scientists have tried with great efforts to determinate the current value of H_0, starting from Edwin Hubble who gave a measurement of $500\frac{km/s}{Mpc}$. This quantity is very important because it tells us many things about the Universe, in particular its size.

[9] In this context, this phenomenon is quantified by the formula $f' = \frac{v_0}{v_0 \pm v} f$, where f' is the perceived frequency, f is the emitted frequency, v_0 is the speed of sound and v is the speed of the source. Note that the sign considered in the denominator of the previous formula is $-$ve if the source approaches and $+$ve otherwise.

[10] See the first article for a visual example of this phenomenon.

[11] Sometimes the formula $z = \frac{f_{emitted} - f_{observed}}{f_{observed}}$ is used instead, which uses the observed and emitted light frequencies.

[12] $c \approx 299.792 km/s$.

Conclusions In the last two thousand and more years, astronomy has made huge steps. We went from thinking that we were at the centre of universe, to thinking that we might colonize it, and this progress is still ongoing. In fact many astronomers believe that in the next years we will witness new astonishing discoveries that will both challenge our knowledge about the Universe, as well as stimulate our curiosity to scratch the surface of knowledge a bit more.

Exercises We will now propose some student exercises based on the previous topics.

- Using the method described in the first paragraph, determine the distance between the Earth and the Moon with a ruler. Perform three measurements of the initial data and evaluate the consistency or the inconsistency of the final data obtained. The average distance between Earth and Moon is $384.000Km$. How does your answer compare?

- Give a more precise value of the parsec unit using (2). Then evaluate the values of the distances of some hypothetical objects having the following parallax angles: $0.2'', 0.5'', 2'', 3''$ and compare these values to the parsec.

- Evaluate the parallax angle of an hypothetical object distant 3Mpc from the Sun.

- The Galaxy IC 1101[13] has $z = 0.077947$, determine its distance using the Hubble law.

- The Galaxy NGC 262[14] has a distance of $62Mpc$, evaluate its speed and its redshift.

- A star emits light with wavelength $\lambda = 700nm$. If its wavelength perceived from Earth is $\lambda = 703nm$, determinate its redshift, speed and distance.

- Give the expression of a formula which relates perceived and emitted wavelengths of stellar objects with their distance.

[13] See IC 1101 (Wikipedia).
[14] See NGC 262 (Wikipedia).