# Network Science
# PHYS 5116, Fall 2020

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## Assignment 3, due by Nov. 24th, 6pm.

**1**. *Random Failure: Beyond Scale-Free Networks. (25 points)*

Calculate the critical threshold $f_c$ for networks with

- Power law with exponential cutoff.
- Lognormal distribution.
- Delta distribution (all nodes have the same degree).

Assume that the networks are uncorrelated and infinite. Refer to Table 4.2 for the functional form of the distribution and the corresponding first and second moments. Discuss the consequences of the obtained results for network robustness.

**2**. *Conspiracy in Social Networks. (25 points)*

In a Big Brother society, the thought police wants to follow a "divide and conquer" strategy by fragmenting the social network into isolated components. You belong to the resistance and want to foil their plans. There are rumours that the police wants to detain individuals that have many friends and individuals whose friends tend to know each other. The resistance puts you in charge to decide which individuals to protect: those whose friendship circle is highly interconnected or those with many friends. To decide you simulate two different attacks on your network, by removing (i) the nodes that have the highest clustering coefficient and (ii) the nodes that have the largest degree. Study the size of the giant component in function of the fraction of removed nodes for the two attacks on the following networks:

(a) A network with $N = 15,625$ nodes generated with the configuration model and power-law degree distribution with $\gamma = 2.5$.

(b) A network with $N = 15,625$ nodes generated with the hierarchical model described in Figure 9.13 and Advanced Topic 9.B (clique of size 5, six iterations).

Which is the most sensitive topological information—clustering coefficient or degree—which, if protected, limits the damage best? Would it be better if all individuals' information (clustering coefficient, degree, etc.) could be kept secret? Why?

**3**. *Modularity (40 points)* Calculate the degree exponent of the hierarchical network shown in Image 9.33 (replicated below).

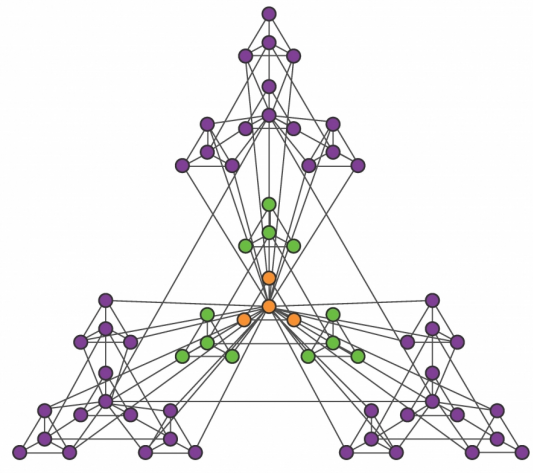

FIG. 1. Image 9.33

**4**. *Epidemics (20 points)*

Choose four networks from Table 4.1 (assume that directed networks behave like undirected and uncorrelated networks with $p_k = p_{kin}$ ) and consider an epidemic process spreading on them. Remember: not only pathogens, but also ideas or opinions can spread on a network! Determine for each network the critical fraction $g_c$ necessary to stop the epidemic if we randomly immunise a $g$-fraction of the nodes. How would the epidemic threshold $\lambda_c$ change if all nodes with degree higher than 1,000 are immunized?