Network Science
PHYS 5116, Fall 2021

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Assignment 1, due by Oct. 8th, 6pm.

You are only allowed to hand in a single file containing all your work, either a PDF or a stapled paper copy. If you attempt to hand in more than one file, we will grade only one of the files, selected at random. Please name your single file as LASTNAME.pdf, *e.g.* Shekhtman.pdf

1. *Matrix Formalism.*

Let $\mathbf{A}$ be the $N \times N$ adjacency matrix of an undirected unweighted network, without self-loops. Let $\mathbf{1}$ be a column vector of N elements, all equal to 1. In other words, $\mathbf{1} = (1, 1, \ldots, 1)^T$, where the superscript T indicates the transpose operation. Use the matrix formalism (multiplicative constants, multiplication row by column, matrix operations like transpose and trace, etc, but avoid the sum symbol $\sum$) to write expressions for:

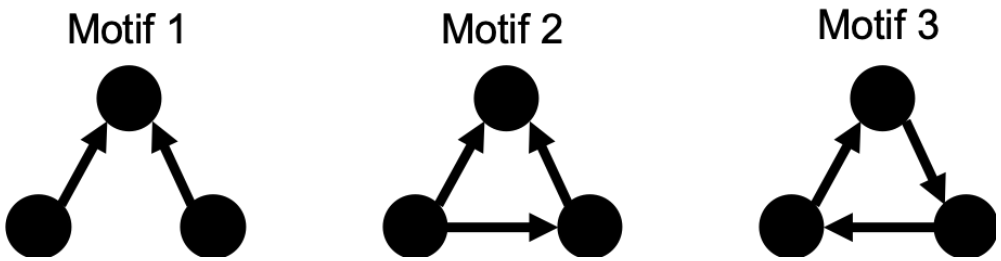
(a) The vector $\mathbf{k}$ whose elements are the degrees k_i of all nodes $i = 1, 2, \ldots, N$.

(b) The total number of links, L, in the network.

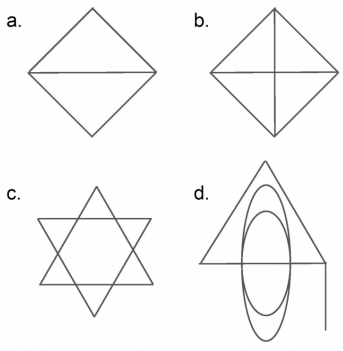
(c) The number of triangles T present in the network, where a triangle means three nodes, each connected by links to the other two. (Hint: you can use the trace of a matrix)

(d) The vector k_{nn} whose element i is the sum of the degrees of node i's neighbors.

2. *Motifs in networks.* Using the network data provided in the form of an adjacency list at `https://www.dropbox.com/s/c4tadqiwxz6trth/Question4.adjlist?dl=0`, read the network into networkx and count the number of appearances of Motifs 1, 2, and 3 (shown below). Note that you should be sure to read in the network as a directed graph as the motifs are directed. The motifs are all on groups of 3 nodes, so you will have to find a way to select groups of 3 nodes (perhaps look at the itertools package in python) to check if they have the given motif. Also, nx.is_isomorphic will check if two subgraphs are isomorphic (meaning there is a direct correspondence between the nodes).



3. *Eulerian Paths.* Which of the icons in Image below can be drawn without raising your pencil from the paper, and without drawing any line more than once? Why? Draw any networks that you use to justify your logic.



4. *Centrality and Random Networks.* Load in networkx the two networks "network1.gml" and "network2.gml" found in `https://www.dropbox.com/sh/immtaes7lmetmj0/AABc6sao-gO-tvJAeWA2hNMFa?dl=0` using the function `nx.read_gml`. Compute betweenness and closeness centrality for all nodes and plot the distribution.

(a) Based on your results, what network seems to be random? Why?

(b) The other network is a collaboration network between scientists in the field on network science, what can you say about the nodes having high values of betweenness centrality?