Network Science
PHYS 5116, Fall 2019

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Assignment 2, due by 6PM Friday, November 1st

Write your name at the top of your assignment before handing it in. Staple all pages together. If you hand in a digital/scanned copy, please be nice to your instructors and hand in a ***single****, combined file. This is to include your code for computational questions. If you hand in a digital copy, please name your file as LASTNAME.PDF, (e.g.* Barabasi.pdf*).*

1. **Structural cutoffs**
   Calculate the structural cutoff k_s for the undirected networks listed in Table 4.1 in the textbook. Based on the plots in Figure 7.10, predict for each network whether k_s is larger or smaller than the maximum expected degree k_{max}. Confirm your prediction by calculating k_{max}.

2. **Network visualization**
   Download the dataset *eu_airports.zip* from the website. This is a network of airports in Europe, derived from the operations of 37 airlines, compiled in 2013. Two airports are connected if an airline flies between them, and the weight is the number of airlines which do so. You are also supplied with airport codes, and geographical coordinates. Visualize the network (Gephi may be the easiest choice of software, but you are free to use your favorite), considering the following:

   - Degree
   - Other measures of centrality
   - Community structure

   Make appropriate use of color, size, and layout to create a clear and informative visualization. Describe your approach and comment on your observations (about a paragraph for each).

3. **Accelerated growth**
   Calculate the degree exponent of the directed Barabási-Albert model with accelerated growth, assuming that the degree of the newly arriving nodes increases in time as $m(t) = t^{\Theta}$.

4. **Avalanches in networks**
   Generate a random network with the Erdős-Réyni $G(N, p)$ model and a scale-free network with a configuration model, with $N = 10^3$ nodes and average degree $\langle k \rangle = 2$. Assume that on each node there is a bucket which can hold as many sand grains as the node degree. Simulate then the following process:

   (a) At each time step add a grain to a randomly chosen node i.

   (b) If the number of grains at node i reaches or exceeds its bucket size, then it becomes unstable and all of the grains at the node topple to the buckets of its adjacent nodes.

   (c) If this toppling causes any of the adjacent nodes' buckets to be unstable, subsequent topplings follow on those nodes, until there is no unstable bucket left. We call this sequence of topplings an avalanche, its size s being equal to the number of nodes that turned unstable following an initial perturbation (adding one grain).

   Repeat (a)-(c) 10^4 times. Assume that at each time step a fraction 10^{-4} of sand grains is lost in the transfer, so that the network buckets do not become saturated with sand. Study the avalanche distribution $p(s)$.