White Paper

OptiRamp® Model-Based Multivariable Predictive Control

Advanced Methodology for Intelligent Control Actions

Vadim Shapiro
Dmitriy Khots, Ph.D.
Introduction

The OptiRamp Model-Based Multivariable Predictive Control (MVPC) Submodule is a software function within the OptiRamp Advanced Dispatch Control System (ADCS), Operator Training Simulator (OTS), and Dynamic Simulation (DS) suites that is designed to control processes with multiple interdependent input and output signals. This state-of-the-art tool complements the Statistics & Control, Inc., (S&C) market offering as it encompasses a 360 degree view of the process by incorporating predictive models, real-time optimization, and control to maximize plant performance while maintaining a favorable cost equation.

Existing single-dimension control methods, such as the PID controller, have shown instability in regulating highly volatile processes as well as an inability to accommodate the complex nonlinear interactions within process variables. MVPC overcomes both of these challenges by integrating mathematical process models into the plant control structure, where control variable set points are proactively maintained by issuing predictive control decisions given a fixed set of ambient conditions. Furthermore, having a process model of the entire plant allows MVPC to make global control actions that benefit the overall process and that account for each process object’s constraints, which sets MVPC apart from the PID controller environment.

The OptiRamp Model-Based MVPC Submodule directly interacts with the OptiRamp Real-Time Optimization (RTO) Submodule to ensure the most profitable and optimal operation. RTO generates transfer functions (discussed later in this white paper) for the MVPC that represent a dynamic relationship between input and output signals.

The ultimate goal of the OptiRamp Model-Based MVPC Submodule is to calculate desirable values of manipulated variables (MVs) to ensure that targets for controlled variables (CVs) are reached and account for chosen optimization criteria. MVPC achieves this goal by examining past changes in MVs and disturbance variables (DV{s}), such as ambient conditions, and by using modeled relationships between all process variables produced by the OptiRamp Modeling Submodule to ascertain the impact that past changes in MVs and DVs had on controlled variables, accounting for process delay times.

MVPC is also equipped with an operator interface that allows users to monitor processes within the submodule as well as control actions trending and forecasted controlled variable behavior based on a predefined time horizon.
MVPC vs. PID Control Comparison

The proportional–integral–derivative (PID) controller is one of the most proliferated control methods in the control system industry. The basic principle of PID controller work is to measure the difference between the controlled variable value (output) and its set point (called the error) and to adjust the manipulated variable (input) to minimize this error. Depending on the desired outcome, the error can be adjusted proportionately (for boundary results), integrally (for gradual results), or using error derivatives to damp resulting oscillations. Adjusting PID coefficients is both an art and a science for optimal PID tuning and control.

Once optimal parameters are found, the main advantages of implementing a PID controller are its speed and robustness. The main disadvantages are susceptibility to oscillation in highly volatile process conditions (for example, when set points are suddenly changed) as well as a complete lack of “knowledge” of the rest of the process. Using a PID controller can be thought of as driving a car by only using the rear-view mirror. It is a reactive control method; the consequences of control actions remain unknown until a new control action is required to adjust for previous errors.

The OptiRamp Model-Based MVPC Submodule is a proactive control method. In the driving example, using this submodule can be thought of as looking forward and being able to anticipate future turns based on current and past actions.

Figure 1 shows the advantage of using MVPC over a standard PID controller, where the controlled variable (flow) drop and eventual recovery is avoided given certain ambient disturbances by proactively changing manipulated variable values to accommodate for smooth process operation.
**MVPC Concept**

MVPC algorithms are based on model predictive control (MPC) concepts. Fundamentally, the MVPC optimization routines use steady-state and dynamic process models to predict how the process will respond to changes in each of the independent variables. It is then able to calculate future moves that will maintain the operation at specified targets. The future is typically given by a parametrically defined horizon \( N \), which is set depending on the underlying process.

MVPC is an iterative process, where control decisions are adjusted based on current information at every time instance and where the horizon is shifted according to a moving window methodology. For successful operation, MVPC requires three fundamental elements:

- Process Model—produced by the *OptiRamp* Modeling Submodule
- Transfer Function—produced by the *OptiRamp* RTO Submodule
- Cost Function—derived within the MVPC itself

Both dynamic and steady state process model construction are described in detail in the *OptiRamp* Modeling Submodule white paper. The transfer function describes the relationship between input and output system signals. The derivation and the types of transfer functions are discussed in detail in the MVPC Transfer Functions section.

Once the input/output transfer function is known, it is possible to predict the system’s reaction after any disturbance and at any given time. Also, it is possible to compute the manipulated variable value so that the integrated (over time) deviation of CVs from the set point would be minimal (Cost Function). The MVPC determines the optimal value for each manipulated variable with the purpose of upholding the optimization criteria while simultaneously maintaining the main CVs at a given level.

Equation (1) provides a typical cost function.

\[
C = \sum_{i=1}^{M} \alpha_{y_i} (SP_i - y_i)^2 + \sum_{j=1}^{N} \beta_{x_j} \Delta x_j^2 ,
\]

where \( y_i \) represents CVs, \( SP_i \) is the corresponding controlled variable set points, \( \Delta x_j \) is the change in MVs, and \( \alpha_{y_i} \) and \( \beta_{x_j} \) are relative weights for \( y_i \) and \( \Delta x_j \), respectively. MVPC uses RTO to estimate \( y_i \) as a transfer function of \( x_j \)s and to minimize the cost function \( C \). The MVPC Algorithm section details the cost function setup and optimization.

MVPC output is a set of manipulated variable values necessary to minimize the variance of CVs from their set points while preserving required constraints. The output is iteratively recalibrated when new information becomes available; thus, MVPC outputs the best possible scenario for a specified horizon and current operating conditions.
MVPC Transfer Functions

One of crucial components of MVPC functionality is a set of robust and reliable transfer functions, which describe the transitional process from system input to system output. Given a time-dependent input variable $MV(t)$ and output variable $CV(t)$, the transfer function is shown in equation (2).

$$H[L\{MV(t)\}] = L\{CV(t)\},$$  \hspace{1cm} (2)

where $H$ is a linear operator and $L\{MV(t)\}$ and $L\{CV(t)\}$ are the Laplace transforms of $MV(t)$ and $CV(t)$, respectively, as shown in equations (3) and (4).

$$F(s) = L\{CV(t)\} = \int_0^\infty e^{-st}MV(t)\,dt$$  \hspace{1cm} (3)

$$G(s) = L\{CV(t)\} = \int_0^\infty e^{-st}CV(t)\,dt$$  \hspace{1cm} (4)

$H$ can be thought of as a ratio of $L\{CV(t)\}$ to $L\{MV(t)\}$.

OptiRamp RTO empirically generates input/output transfer functions using data obtained from the simulated open-loop step performed on the current model structure. In case the transitional process can be characterized by ideal delay (given by $\delta(t-\nu)$, where $\delta$ is the Dirac delta function and $\nu$ is process time delay), the Laplace transform is given in equation (5).

$$w(s) = L\{\delta(t-\nu)\} = e^{-\nu s}$$  \hspace{1cm} (5)

Then, considering a first-order system, the transform function is given in equation (6).

$$W(s) = \frac{k_p e^{-\nu s}}{\tau_p s + 1},$$  \hspace{1cm} (6)

where $k_p$ is the process gain and $\tau_p$ is the process time constant. This transfer function is illustrated in Figure 2a. All coefficients are optimally derived using genetic algorithms. Overall, the input/output transfer functions may assume a number of structural forms. Figure 2b shows a parallel structure given by $CV = W1[MV1] + W2[MV2]$. Figure 2c shows a serial structure given by $CV2 = W1[W2[MV1]]$. For systems with more than one output, the input/output transfer function has the form shown in Figure 2d. The outputs are related to the inputs via equations (7) and (8).
\[ CV_1 = W_{11}[MV_1] + W_{22}[MV_2] \]  

(7) \[ CV_2 = W_{12}[MV_1] + W_{21}[MV_2], \]  

(8)

where \( W[\bullet] \) is the transfer function corresponding to input \( \bullet \).

**Figure 2. MVPC transfer function structures**

Processes are influenced by external disturbances, such as changes in ambient conditions and changes in the fuel quality. To accommodate these effects, the \textit{OptiRamp} RTO Submodule incorporates process disturbances into the model with the disturbance transfer functions shown in Figure 2e.
MVPC Algorithm

The MVPC algorithm requires the following inputs: cost function and input/output transfer functions. MVPC uses an enhanced version of the cost function, which incorporates weights for the set point and manipulated variable change components and uses the least squares method for standard error modification.

The MVPC cost function is given in equation (9).

\[
C = \phi_1 \sqrt{\sum_{i=1}^{M} \alpha_{CV_i} \left( SP_i - \sum_{q=1}^{L} W_{q}^{i} [MV_{q}^{i}, DV_{q}^{i}] \right)^2} + \phi_2 \sqrt{\sum_{j=1}^{R} \beta_{x_j} \Delta MV_{j}^2},
\]

(9)

where \( MV_{q}^{i} \) is the \( q \)th manipulated variable and \( DV_{q}^{i} \) is the \( q \)th disturbance variable present in the transfer function \( W_{q}^{i} \) for every controlled variable \( CV_i \), with \( q = 1,...,L \), \( i = 1,...,M \), \( p = 1,...,P \) and \( j = 1,...,R \); \( \phi_1 + \phi_2 = 1 \) are weights regulating the importance of each component in the cost function; and \( \alpha_{CV_i} \) represents the relative weights of each controlled variable, such that \( \sum_{i=1}^{M} \alpha_{CV_i} = 1 \) and \( \beta_{x_j} \) are the relative weights penalizing large changes in the MVs so that \( \sum_{j=1}^{R} \beta_{x_j} = 1 \). Also, \( \Delta MV = MV_{j} (t+1) - MV_{j} (t) \) is the change in the manipulated variable \( MV_{j} \) from time instance \( t \) to time instance \( t+1 \), where \( t \in \{0,...,N\} \).

The constraints in cost function optimization are the boundary conditions for every \( MV_{q}^{i} \) given in equation (10).

\[
LL_{q}^{i} \leq MV_{q}^{i} \leq UL_{q}^{i},
\]

(10)

where \( LL_{q}^{i} \) is the parametrically defined lower limit and \( UL_{q}^{i} \) is the parametrically defined upper limit of \( MV_{q}^{i} \) values. The process model adds additional constraints, relating \( MV_{q}^{i} \) to other process variables.

To solve this optimization problem, MVPC uses techniques described in the OptiRamp RTO Submodule white paper. For a given time horizon \( N \), the solution is a list of values given by set (11).

\[
\{ MV_{q}^{i} \}_{i=1,...,M; q=1,...,L}.
\]

(11)
Once the control action is taken according to the calculated manipulated variables, the horizon is shifted by one time increment and the algorithm is repeated, ensuring that an optimal solution is acquired at any given time.
About Statistics & Control, Inc.

S&C—an engineering consulting and technology company headquartered in West Des Moines, IA—solves complex challenges for customers through its unique technology and its highly seasoned team of professionals. The company has a global portfolio spanning the energy, oil and gas, utility, and digital oil field industry sectors. S&C provides clients with turbomachinery control solutions that easily integrate with the existing system as well as OptiRamp® solutions, which focus on process and power analytics to optimize processes and, in turn, reduce costs and increase reliability. S&C also provides consulting, dynamic system studies, modeling, automation, training and OTS, and support services.