

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read and write numbers up to 1000 in numerals and in words
- count from 0 in multiples of 1, 2, 3, 4, 5, 8, 10, 50 and 100, forwards and backwards
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- identify, represent and estimate numbers using different representations
- find 10 or 100 more or less than a given number
- compare and order numbers up to 1000

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- continue to recall and use addition facts to 20 fluently, and derive and use related facts up to 100, e.g. $130 + 50 = 180$
- continue to add numbers mentally, including:
 - two two-digit numbers
 - three or more one-digit numbers
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- add numbers with up to three digits, using the formal written method of columnar addition
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve problems, including missing number problems, using number facts, place value, and more complex addition

Mental strategies

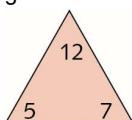
- Use of models and images:

- Trios $7 + 5 = 12$

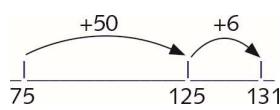
$$5 + 7 = 12$$

$$12 - 5 = 7$$

$$12 - 7 = 5$$



- the empty number line



ADDITION Continued

Mental strategies continued

- To multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- Recognise and use the inverse relationship between addition and subtraction
- Use knowledge that addition can be done in any order (commutative), e.g.
 - put the larger number first and count on in steps of 1, 10 or 100
 - partition additions into hundreds, tens and ones, then recombine, e.g. $75 + 56 = 75 + 50 + 6 = 125 + 6 = 131$
- Identify near doubles, using doubles already known, e.g. $70 + 71$
- Add the nearest multiple of 10 or 100, and adjust
- Use patterns of similar calculations, e.g. $13 + 5 = 18$ and $130 + 50 = 180$
- Use knowledge of the associative law when adding more than two numbers, e.g. $4 + 7 + 6 = (4 + 6) + 7 = 10 + 7 = 17$

ADDITION Continued

Written methods

- Add numbers with up to three digits (HTO + HTO)
- Estimate and check the answer to a calculation

Expanded written method

548+387

$$\begin{array}{r} 548 \\ + 387 \\ \hline 15 \\ 120 \\ 800 \\ \hline 935 \end{array}$$

The first stage in the written method shows separately the addition of the ones to the ones, the tens to the tens and the hundreds to the hundreds. To find the partial sums either the ones or the hundreds can be added first, and the total of the partial sums can be found by adding them in any order. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method of columnar addition.

The addition of the tens in the calculation 548 + 387 is described in the words 'forty add eighty equals one hundred and

'twenty', stressing the link to the related fact 'four add eight equals twelve'. The addition of the hundreds is described as

'five hundred add three hundred equals eight hundred', stressing the link to the related fact 'five add three equals eight'.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of columnar addition

548+387

$$\begin{array}{r} 548 \\ + 387 \\ \hline 935 \\ 11 \end{array}$$

The expanded written method leads to the formal written method of columnar addition so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- continue to recall and use subtraction facts to 20 fluently, and derive and use related facts, e.g. $120 - 70 = 50$
- continue to subtract numbers mentally, including:
 - two two-digit numbers
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- subtract numbers with up to three digits, using the formal written method of columnar subtraction
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

- solve problems, including missing number problems, using number facts, place value, and more complex subtraction

Mental strategies

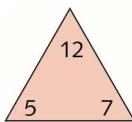
- Use of models and images:

- trios $7 + 5 = 12$

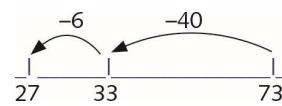
$$5 + 7 = 12$$

$$12 - 5 = 7$$

$$12 - 7 = 5$$



- the empty number line



- multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- Recognise and use the inverse relationship between addition and subtraction

- Find a difference by counting up from the smaller to the larger number

- Subtract the nearest multiple of 10 or 100, and adjust

- Use patterns of similar calculations, e.g. $18 - 5 = 13$ and $180 - 50 = 130$

- Use partitioning, e.g. $73 - 46 = 73 - 40 - 6$

$$= 33 - 6$$

$$= 27$$

SUBTRACTION Continued

Written methods

- Subtract numbers with up to three digits (HTO – HTO)
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition)

$$582 - 237$$

$$\begin{array}{r} \cancel{5} \cancel{8} 2 \\ - 2 3 7 \\ \hline 3 4 5 \end{array}$$

Start by subtracting the least significant digits first, i.e. the ones, then the tens, and finally the hundreds. Refer to subtracting the tens, for example, by saying 'seventy subtract thirty', not 'seven subtract three'.

In this example the ones to be subtracted are larger than the ones you are subtracting from.

You exchange one of the 8 tens for 10 ones, crossing out 8 and writing a superscript 7, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 subtract 7, 70 subtract 30 and finally 500 subtract 200.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of multiplication facts for the 2, 5 and 10 multiplication tables
- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- use known multiplication facts to derive related facts involving multiples of 10, e.g. $2 \times 30 = 60$
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods and progressing to formal written methods
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

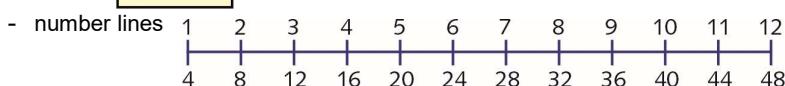
Mental strategies

- Use of models and images:



$$3 \times 4 = 12$$

$$4 \times 3 = 12$$



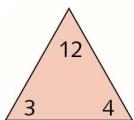
- trios

$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

$$12 \div 4 = 3$$

$$12 \div 3 = 4$$



- multiplication square

\times	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division
- Use doubling, e.g. connect the 2, 4 and 8 multiplication tables

MULTIPLICATION Continued

Mental strategies continued

- Use the ‘key multiplication facts’ of $\times 1$, $\times 2$, $\times 5$, and $\times 10$ to work out the answers to unknown multiplication facts, e.g. $7 \times 4 = (5 \times 4) + (2 \times 4)$
 $= 20 + 8$
 $= 28$
- Use patterns of similar calculations, e.g. $8 \times 6 = 48$ and $8 \times 60 = 480$
- Show that multiplication of two numbers can be done in any order (commutative), e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12$
 $= 20 \times 12$
 $= 240$
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit number by a one-digit number,
 $63 \times 8 = (60 \times 8) + (3 \times 8)$
 $= 480 + 24$
 $= 504$

Written methods

- Short multiplication:
 - Multiply a two-digit number by a one-digit number (TO x O)
- Estimate and check the answer to a calculation

Grid method

$$\begin{array}{r} 63 \times 8 \\ \times 60 \quad 3 \\ \hline 8 \quad \boxed{480 \quad 24} = 504 \end{array}$$

Expanded written method

$$\begin{array}{r} 63 \times 8 \\ \times \quad 8 \\ \hline 24 \quad (3 \times 8) \\ 480 \quad (60 \times 8) \\ \hline 504 \end{array}$$

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 63×8 it is ‘sixty multiplied by eight’, not ‘six multiplied by eight’, although the relationship 6×8 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

$$\begin{array}{r} 63 \\ \times \quad 28 \\ \hline 504 \end{array}$$

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of division facts for the 2, 5 and 10 multiplication tables
- recall and use division facts for the 3, 4 and 8 multiplication tables
- use known division facts to derive related facts involving multiples of 10, e.g. $60 \div 3 = 20$
- write and calculate mathematical statements for division using the multiplication tables that they know
- develop reliable written methods for division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written method of short division (without a remainder)
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems, including missing number problems, involving division

Mental strategies

- Use of models and images:

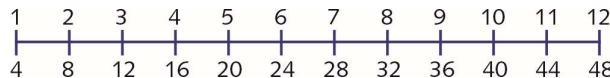
- arrays



$$12 \div 4 = 3$$

$$12 \div 3 = 4$$

- number lines

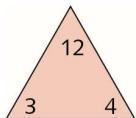


- trios $3 \times 4 = 12$

$$4 \times 3 = 12$$

$$12 \div 4 = 3$$

$$12 \div 3 = 4$$



- multiplication square

\times	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division
- Use halving, e.g. find quarters by halving halves

DIVISION Continued

Mental strategies continued

- Use patterns of similar calculations, e.g. $48 \div 8 = 6$ and $480 \div 80 = 6$
- Understand and use the distributive law, e.g. partitioning when dividing a two-digit number by a one-digit number,

$$\begin{aligned} 92 \div 4 &= (80 \div 4) + (12 \div 4) \\ &= 20 + 3 \\ &= 23 \end{aligned}$$

Written methods

- Short division (without a remainder):
 - Divide a two-digit number by a one-digit number (T0 ÷ O)
- Estimate and check the answer to a calculation

Expanded written method

$$92 \div 4$$

$$\begin{array}{r} 2\ 3 \\ 4 \overline{)9\ 2} \\ 8\ 0 \quad (20 \times 4) \\ \hline 1\ 2 \quad (3 \times 4) \\ \hline 0 \end{array}$$

The first step is to show all of the calculations involved.

Children should describe what they are doing using phrases similar to the following: 'How many fours divide into 90 so that the answer is a multiple of 10? (20) There are 20 fours or 80, with 12 remaining. How many fours in 12? (3) So 92 divided by four is 23.'

Formal written method of short division

$$\begin{array}{r} 2\ 3 \\ 4 \overline{)9\ 12} \end{array}$$

The expanded written method leads children to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The superscript 1 represents the 1 ten that is remaining after 4 has been divided into 90. It is written in front of the 2 to show that 12 now has to be divided by 4.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.